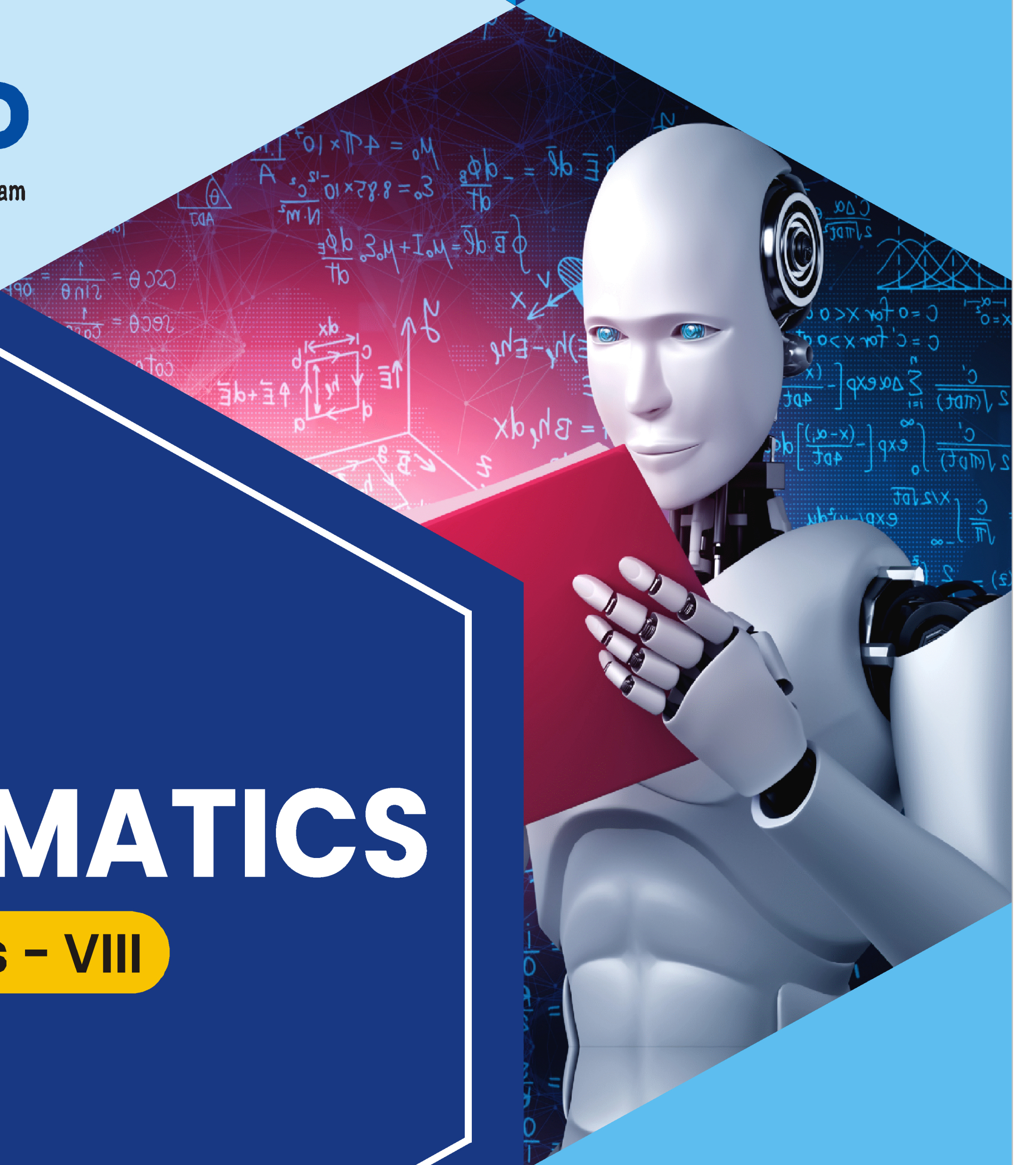
MATRIX OLYMPIAD

The Most Innovative Talent Recognition Exam



MATHEMATICS

Class - VIII

MATRIX

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Few words for the Readers

Dear Reader,

"*Matrix Olympiad is established to encourage school students to go a step further than their regular studies, and get a chance and exposure to competition on a wide scale. It also helps students enhance their learning of basic cognitive skills and deeper knowledge of subjects like Science, Mathematics, English, Mental Ability, Social Studies.*"*Matrix Olympiad helps students nurture their minds for higher targets of tomorrow and enables them to study School for JEE, NEET, CLAT, NDA, Olympiads , NSEJS, NTSE , STSE etc.*"

The above thought has been our guiding principle while designing and collating the study material for **Matrix Olympiad** . And hence, we hope that this particular material will be helpful towards your preparation for **Matrix Olympiad**.

Our team at **MATRIX** has put in their best efforts for making this particular module interesting and relevant for you. Additional efforts have been made to ensure that the content is easy to understand and error free to the extent possible. However, there might remain some inadvertent errors in answer keys and theoretical portion and we would welcome your valuable feedback regarding the same.
If there are any suggestions for corrections, please write to us at **smd@matrixacademy.co.in** and we would be highly grateful.

Finally, we would like to end this message by a famous quote by Ernest Hemingway - "*There is no friend as loyal as a book*." So, please give your study material the time and attention it deserves, and it will surely help you reach newer heights in your fight with competition examinations.

With love and best wishes !
Team MATRIX

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INTRODUCTION

In Mathematics, we frequently come across simple equations to be solved. For example,

the equation $x + 2 = 13$ (1)

is solved when x = 11, because this value of x satisfies the given equation. The solution 11 is a natural number. On the other hand, for the equation

$x + 5 = 5$ (2)

the solution gives the whole number 0 (zero). If we consider only natural numbers, equation (2) cannot be solved. To solve equations like (2), we added the number zero to the collection of natural numbers and obtained the whole numbers. Even whole numbers will not be sufficient to solve equations of type

$x + 18 = 5$ (3)

Do you see 'why'? We require the number –13 which is not a whole number. This led us to think of integers, (positive and negative). Note that the positive integers correspond to natural numbers. One may think that we have enough numbers to solve all simple equations with the available list of integers. Now consider the equations

$2x = 3$ (4)

$5x + 7 = 0$ (5)

for which we cannot find a solution from the integers.(Check this)

We need the numbers $\frac{3}{2}$ to solve equation (4) and $\frac{-7}{5}$ to solve equation (5).

This leads us to the collection of **rational numbers.**

We have already seen basic operations on rational numbers. We now try to explore some properties of operations on the different types of numbers seen so far.

1. NATURAL NUMBERS

The numbers from 1 onwards i.e., 1, 2, 3 and so on are called natural numbers. The set of natural number is denoted by N.

1.1 PROPERTIES OF NATURAL NUMBERS

(i) For every natural number, there is a successor.

(ii) 1 is the smallest natural number.

(iii) There are finite number of natural numbers between two natural numbers.

(iv) Every natural number (except 1) can be obtained by adding 1 to the previous natural number.

(v) For the natural number 1, there is no 'Previous' natural number.

(vi) There are infinite number of natural numbers.

1.2 LAWS OF NATURAL NUMBERS

If a, b and c are three natural numbers :

1. Closure Law :

Addition	a + b is a natural number.
Subtraction	If a > b, then a – b is a natural number.
Multiplication	a × b is a natural number.
Division	a ÷ b need not be a natural number.

Example 1

4, 3 are natural numbers belongs to N, 4 + 3 = 7 ∈ N, 4 × 3 = 12 ∈ N, 3 – 4 ∉ N, 3/4 ∉ N.

2. Commutative Law :

Addition	a + b = b + a	Commutative
Subtraction	a – b ≠ b – a	Not commutative
Multiplication	ab = ba	Commutative
Division	a ÷ b ≠ b ÷ a	Not commutative

Example 2

5 and 6 are natural numbers, 5 + 6 = 6 + 5 = 11, 5 – 6 ≠ 6 – 5, 5 × 6 = 6 × 5 = 30, $\frac{5}{6} \neq \frac{6}{5}$.

3. Associative Law :

Addition	a + (b + c) = (a + b) + c	Associative
Subtraction	a – (b – c) ≠ (a – b) – c	Not associative
Multiplication	a × (b × c) = (a × b) × c	Associative
Division	a ÷ (b ÷ c) ≠ (a ÷ b) ÷ c	Not associative

Example 3

7, 8 and 9 are natural numbers, (7 + 8) + 9 = 7 + (8 + 9) = 24, (7 – 8) – 9 ≠ 7 – (8 – 9), (7 × 8) × 9 = 7 × (8 × 9) = 504, (7 ÷ 8) ÷ 9 ≠ 7 ÷ (8 ÷ 9)

4. Distributive Law :

a × (b + c) = ab + ac (over addition)

a × (b – c) = ab – ac (over subtraction)

5. Multiplicative Identity :

a × 1 = 1 × a = a; '1' is called the multiplicative identity of 'a'.

2. WHOLE NUMBERS

The natural numbers along with zero forms the collection of whole numbers.

2.1 PROPERTIES OF WHOLE NUMBERS

(i) The number 0 is the first and the smallest whole number.

(ii) There is no last or greatest whole number.

(iii) There are infinitely many or uncountable number of whole numbers.

(iv) All natural numbers are whole numbers.

(v) All whole numbers are not natural numbers.

2.2 LAWS OF WHOLE NUMBERS

If a, b and c are three whole numbers :

1. Closure Law :

Addition	a + b is a whole number.
Subtraction	If a > b, then a – b is a whole number.
Multiplication	a × b is a whole number.
Division	a ÷ b need not be a whole number.

Example 1

(i) 0 + 5 = 5, 5 is a whole number.

(ii) 0 – 8 = – 8, – 8 is not a whole number but 8 – 0 = 8, 8 is whole number.

(iii) 0 × 9 = 0, 0 is a whole number.

(iv) $8 \div 11 = \frac{8}{11}, \frac{8}{11}$ is not a whole number.

2. Commutative Law :

Addition	$a + b = b + a$	Commutative
Subtraction	$a - b \neq b - a$	Not commutative
Multiplication	$a \times b = b \times a$	Commutative
Division	$a \div b \neq b \div a$	Not commutative

Example 2

(i) 5 + 3 = 3 + 5 = 8

(ii) 3 – 5 ≠ 5 – 3

(iii) 7 × 3 = 3 × 7 = 21

(iv) $\frac{3}{7} \neq \frac{7}{3}$

3. Associative Law :

Addition	$a + (b + c) = (a + b) + c$	Associative
Subtraction	$a - (b - c) \neq (a - b) - c$	Not associative
Multiplication	$a \times (b \times c) = (a \times b) \times c$	Associative
Division	$a \div (b \div c) \neq (a \div b) \div c$	Not associative

Example 3

(i) 5 + (3 + 8) = (5 + 3) + 8 = 16.

(ii) 5 – (3 – 8) ≠ (5 – 3) – 8 ⇒ 5 – (– 5) ≠ (2 – 8) ⇒ 10 ≠ – 6.

(iii) $5 \times (7 \times 3) = (5 \times 7) \times 3 \Rightarrow 5 \times 21 = 35 \times 3 \Rightarrow 105 = 105$

(iv) $5 \div (7 \div 3) \neq (5 \div 7) \div 3$

4. Distributive Law :

$a \times (b + c) = ab + ac$ (over addition)

$a \times (b - c) = ab - ac$ (over subtraction)

5. Existence of Additive Identity :

$a + 0 = 0 + a = a$; '0' is called additive identity of 'a'.

6. Multiplicative Identity :

$a \times 1 = 1 \times a = a$; '1' is called the multiplicative identity of 'a'.

3. INTEGERS

The collection of whole numbers and negative of natural numbers together are called integers.

The collection 1, 2, 3, ... are said to be positive integers and – 1, – 2, – 3, ... are said to be negative integers.

3.1 PROPERTIES OF INTEGERS

(i) '0' is neither negative integer nor positive integer.

(ii) There is no smallest and greatest integer.

(iii) Negative integers are smaller than positive integers.

(iv) There are infinitely many or uncountable integers.

(v) All natural and whole numbers are integers.

(vi) All integers are neither natural nor whole numbers.

(vii) Multiplication of a positive and a negative integer is negative.

(viii) Multiplication of two positive integers is positive.

(ix) Multiplication of two negative integers is positive.

(x) Multiplication of odd number of negative integers is negative.

(xi) Multiplication of even number of negative integers is positive.

3.2 LAWS OF INTEGERS

If a, b and c are three integers :

1. Closure Law :

Addition	$a + b$ is an integer.
Subtraction	$a - b$ is an integer.
Multiplication	$a \times b$ is an integer.
Division	$a \div b$ need not be an integer.

Example 1

(i) – 7 + (– 3) = – 10, an integer.

(ii) 4 + 3 = 7, an integer.

(iii) 7 – 3 = 4, an integer.

(iv) 5 – 10 = – 5, an integer.

(v) – 8 × 5 = – 40, an integer.

(vi) $5 \div 8 = \frac{5}{8}$, which is not an integer.

2. Commutative Law :

Addition	a + b = b + a	Commutative
Subtraction	a – b ≠ b – a	Not commutative
Multiplication	a × b = b × a	Commutative
Division	a ÷ b ≠ b ÷ a	Not commutative

Example 2

(i) 5 + (– 3) = (– 3) + 5 = 2

(ii) 5 – (– 2) ≠ (– 2) – 5 ⇒ 7 ≠ – 7.

(iii) 5 × (– 3) = (– 3) × 5 = – 15

(iv) 5 ÷ (– 3) ≠ (– 3) ÷ 5

3. Associative Law :

Addition	a + (b + c) = (a + b) + c	Associative
Subtraction	a – (b – c) ≠ (a – b) – c	Not associative
Multiplication	a × (b × c) = (a × b) × c	Associative
Division	a ÷ (b ÷ c) ≠ (a ÷ b) ÷ c	Not associative

Example 3

(i) (– 6) + [(– 4) + (– 3)] = [(– 6) + (– 4)] + (– 3) ⇒ (– 6) + (– 7) = [(– 10)] + (– 3) ⇒ – 13 = – 13.

(ii) 6 – (8 – 4) ≠ (6 – 8) – 4 ⇒ 6 – 4 ≠ (– 2) – 4 ⇒ 2 ≠ – 6.

(iii) 5 × [(– 2) × (– 3)] = [5 × (– 2)] × (– 3) ⇒ 5 × 6 = (– 10) × (– 3) ⇒ 30 = 30.

(iv) [(– 10) ÷ 2] ÷ (– 5) ≠ (– 10) ÷ [2 ÷ (– 5)] ⇒ [(– 5) ÷ (– 5)] ≠ (–10) ÷ (– 0.4) ⇒ 1 ≠ 25.

4. Distributive Law :

a × (b + c) = ab + ac (over addition)

a × (b – c) = ab – ac (over subtraction)

5. Additive Identity :

a + 0 = 0 + a = a; '0' is called the additive identity of 'a'.

6. Additive Inverse :

a + (– a) = (– a) + a = 0; '– a' is called additive inverse of 'a'.

7. Multiplicative Identity :

a × 1 = 1 × a = a; '1' is called the multiplicative identity of 'a'.

4. RATIONAL NUMBERS

A number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number.

Example 1

$\frac{-2}{3}, \frac{6}{7}$.

4.1 PROPERTIES OF RATIONAL NUMBERS

(i) All whole number, integers and natural numbers are rational numbers.

(ii) All rational numbers are not whole numbers, integers and natural numbers.

(iii) There are infinitely many rational numbers between two rational numbers.

Focus Point

Absolute Value of a Rational Number :

- Absolute value is denoted by the symbol '| |'.
- Absolute value of a positive number remains the same.
- Absolute value of a negative number is always positive.

Thus, for any rational number x, we write $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$.

4.2 LAWS OF RATIONAL NUMBERS

If a, b and c are three rational numbers :

1. Closure Law :

Addition	a + b is a rational number.
Subtraction	a – b is a rational number.
Multiplication	a × b is a rational number.
Division	a ÷ b need not be a rational number.

Example 1

(i) $\left(\frac{-5}{12} + \frac{-1}{4}\right) = \frac{\{-5 + (-3)\}}{12} = \frac{-8}{12} = \frac{-2}{3}$, a rational number.

(ii) $\frac{-3}{4} + \frac{5}{6} = \frac{-9}{12} + \frac{10}{12} = \frac{-9 + 10}{12} = \frac{1}{12}$, a rational number.

(iii) $\frac{-27}{36} - \left(\frac{14}{36}\right) = \frac{-27 - 14}{36} = \frac{-41}{36}$, a rational number.

(iv) $\frac{-4}{5}\times\frac{-6}{11}=\frac{24}{55}$, a rational number.

(v) $\frac{2}{5}\div\frac{5}{3}=\frac{2}{5}\times\frac{3}{5}=\frac{6}{25}$, a rational number.

(vi) $\frac{2}{0}$ = not defined and hence, not a rational number.

2. Commutative Law :

Addition	$a + b = b + a$	Commutative
Subtraction	$a - b \neq b - a$	Not commutative
Multiplication	$a \times b = b \times a$	Commutative
Division	$a \div b \neq b \div a$	Not commutative

Example 2

(i) $\frac{5}{6}+\frac{-4}{9}=\frac{15}{18}+\frac{-8}{18}=\frac{15+(-8)}{18}=\frac{7}{18}$ and $\frac{-4}{9}+\frac{5}{6}=\frac{-8}{18}+\frac{15}{18}=\frac{-8+15}{18}=\frac{7}{18}$ $\therefore \frac{5}{6}+\frac{-4}{9}=\frac{-4}{9}+\frac{5}{6}$

(ii) $\frac{2}{3}-\frac{1}{6}=\frac{2}{3}+\frac{-1}{6}=\frac{2\times2+(-1)\times1}{6}=\frac{4-1}{6}=\frac{3}{6}=\frac{1}{2}$ and $\frac{1}{6}-\frac{2}{3}=\frac{1}{6}+\frac{-2}{3}=\frac{1+(-2)\times2}{6}=\frac{1-4}{6}=\frac{-3}{6}=\frac{-1}{2}$

$\therefore \frac{2}{3}-\frac{1}{6}\neq\frac{1}{6}-\frac{2}{3}$

(iii) $\frac{3}{4}\times\frac{5}{7}=\frac{3\times5}{4\times7}=\frac{15}{28},\frac{5}{7}\times\frac{3}{4}=\frac{5\times3}{7\times4}=\frac{15}{28}$ $\therefore \frac{3}{4}\times\frac{5}{7}=\frac{5}{7}\times\frac{3}{4}$

(iv) $\frac{-5}{4}\div\frac{3}{7}=\frac{-5}{4}\times\frac{7}{3}=\frac{-35}{12}$ and $\frac{3}{7}\div\left(\frac{-5}{4}\right)=\frac{3}{7}\times\left(\frac{-4}{5}\right)=\frac{-12}{35}$ $\therefore \frac{-5}{4}\div\frac{3}{7}\neq\frac{3}{7}\div\left(\frac{-5}{4}\right)$

3. Associative Law :

Addition	$a + (b + c) = (a + b) + c$	Associative
Subtraction	$a - (b - c) \neq (a - b) - c$	Not associative
Multiplication	$a \times (b \times c) = (a \times b) \times c$	Associative
Division	$a \div (b \div c) \neq (a \div b) \div c$	Not associative

Example 3

(i) $\frac{-3}{4}+\left(\frac{5}{6}+\frac{-4}{9}\right)=\frac{-3}{4}+\left(\frac{15}{18}+\frac{-8}{18}\right)=\frac{-3}{4}+\left(\frac{15-8}{18}\right)=\frac{-3}{4}+\frac{7}{18}=\frac{-27}{36}+\frac{14}{36}=\frac{-27+14}{36}=\frac{-13}{36}$ and

$\left(\frac{-3}{4}+\frac{5}{6}\right)+\left(\frac{-4}{9}\right)=\left(\frac{-9}{12}+\frac{10}{12}\right)+\frac{-4}{9}=\frac{-9+10}{12}+\frac{-4}{9}=\frac{1}{12}+\frac{-4}{9}=\frac{3}{36}+\frac{-16}{36}=\frac{3+(-16)}{36}=\frac{-13}{36}$

$\therefore \frac{-3}{4}+\left(\frac{5}{6}+\frac{-4}{9}\right)=\left(\frac{-3}{4}+\frac{5}{6}\right)+\frac{-4}{9}$

(ii) $\frac{-2}{3}-\left(\frac{-4}{5}-\frac{1}{2}\right)=\frac{-2}{3}-\left(\frac{-8-5}{10}\right)\Rightarrow\frac{-2}{3}-\left(\frac{-13}{10}\right)=\frac{-2}{3}+\frac{13}{10}=\frac{-20+39}{30}=\frac{19}{30}$ and $\left[-\frac{2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$

$=\left[\frac{-2}{3}+\frac{4}{5}\right]-\frac{1}{2}=\left(\frac{-10+12}{15}\right)-\frac{1}{2}=\left(\frac{2}{15}\right)-\frac{1}{2}=\frac{4-15}{30}=\frac{-11}{30}$ so, $\frac{-2}{3}-\left[\frac{-4}{5}-\frac{1}{2}\right]\neq\left[\frac{-2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$

(iii) $\left(\frac{-5}{4}\right)\times\left(\frac{-6}{11}\times\frac{2}{7}\right)=\frac{-5}{4}\times\frac{-12}{77}=\frac{(-5)\times(-12)}{4\times77}=\frac{(-5)\times(-3)}{1\times77}=\frac{15}{77}$ and $\left(\frac{-5}{4}\times\frac{-6}{11}\right)\times\frac{2}{7}=\left\{\frac{(-5)\times(-6)}{4\times11}\right\}\times\frac{2}{7}$

$=\left\{\frac{(-5)\times(-3)}{2\times11}\right\}\times\frac{2}{7}=\frac{15}{22}\times\frac{2}{7}=\frac{15\times2}{22\times7}=\frac{15}{77}\therefore\frac{-5}{4}\times\left(\frac{-6}{11}\times\frac{2}{7}\right)=\left(\frac{-5}{4}\times\frac{-6}{11}\right)\times\frac{2}{7}$

(iv) $\left[\frac{1}{2}\div\left(\frac{-1}{3}\right)\right]\div\frac{2}{5}=\left[\frac{1}{2}\times(-3)\right]\div\frac{2}{5}=\frac{-3}{2}\div\frac{2}{5}=\frac{-3}{2}\times\frac{5}{2}=\frac{-15}{4}$ and $\frac{1}{2}\div\left[\frac{-1}{3}\div\frac{2}{5}\right]=\frac{1}{2}\div\left[\frac{-1}{3}\times\frac{5}{2}\right]$

$=\frac{1}{2}\div\left[\frac{-5}{6}\right]=\frac{1}{2}\times\left(\frac{6}{-5}\right)=\frac{-3}{5}\quad\therefore\left[\frac{1}{2}\div\left(\frac{-1}{3}\right)\right]\div\frac{2}{5}\neq\frac{1}{2}\div\left[\frac{-1}{3}\div\frac{2}{5}\right]$

4. Distributive Law :

$a\times(b+c)=ab+ac$ (over addition)

$a\times(b-c)=ab-ac$ (over subtraction)

Example 4

Consider any three rational number, say, $\frac{2}{3},\frac{-3}{5}$ and $\frac{7}{10}$. We have, $\frac{2}{3}\times\left(\frac{-3}{5}+\frac{7}{10}\right)=\frac{2\times(-3)}{3\times5}+$

$\frac{2\times7}{3\times10}=\frac{2\times(-1)}{1\times5}+\frac{1\times7}{3\times5}=\frac{-2}{5}+\frac{7}{15}=\frac{-2\times3+7}{15}=\frac{-6+7}{15}=\frac{1}{15}\quad\therefore\frac{2}{3}\times\left(\frac{-3}{5}+\frac{7}{10}\right)=\frac{2}{3}\times\left(\frac{-3}{5}\right)+\frac{2}{3}\times\frac{7}{10}$

5. Existence of Additive Identity (Role of 0) :

$a+0=0+a=a$; '0' is called the additive identity of 'a'.

Example 5

$\frac{2}{3}+0=\frac{2}{3}=0+\frac{2}{3}$

6. Existence of Additive Inverse (Negative of a Number) :

$a+(-a)=(-a)+a=0$; '–a' is called additive inverse of 'a'.

Example 6

The additive inverse (negative) of $\frac{3}{5}$ is $-\frac{3}{5}$ and $-\frac{3}{5}$ is written as $\left(-\frac{3}{5}\right)$.

Therefore, $-\frac{3}{5}$ and $-\frac{3}{5}$ represent the same rational number, that is, $-\frac{3}{5}=-\frac{3}{5}$. or $-\left(\frac{-3}{5}\right)=\frac{3}{5}\Rightarrow-\left(-\frac{3}{5}\right)=\frac{3}{5}$

Similarly, we have $-\left(-\frac{5}{7}\right)=\frac{5}{7}, -\left(-\frac{15}{13}\right)=\frac{15}{13}, -\left(-\frac{7}{12}\right)=\frac{7}{12}$ and so on.

Note : The rational number which is equal to its negative is zero.

7. Multiplicative Identity (Role of 1) :

a × 1 = 1 × a = a; '1' is called multiplicative identity of 'a'.

8. Multiplicative Inverse (Reciprocal) :

$a \times \frac{1}{a} = \frac{1}{a} \times a = 1; \left(\frac{1}{a}\right)$ is called multiplicative inverse of 'a' or reciprocal of 'a'.

Example 7

$\frac{8}{21}$ should be multiplied by $\frac{21}{8}$ to get the product 1.

Similarly, $\frac{-5}{7}$ must be multiplied by $\frac{7}{-5}$ so as to get the product 1.

So, we say that $\frac{21}{8}$ is the reciprocal of $\frac{8}{21}$ and $\frac{7}{-5}$ is the reciprocal of $\frac{-5}{7}$.

A rational number $\frac{c}{d}$ is called the reciprocal or multiplicative inverse of another rational number $\frac{a}{b}$ if $\frac{a}{b} \times \frac{c}{d} = 1$.

Focus Point

- The reciprocal of 1 is 1 and the reciprocal of – 1 is – 1.

Also 1 and – 1 are the only rational numbers which are their own reciprocals.

- There is no rational number which when multiplied with 0, gives 1.

Therefore rational number 0 has no reciprocal or multiplicative inverse.

Example 1

Express each of the following as a rational number :

(i) $\frac{-2}{5}+\left(\frac{11}{5}+\frac{-3}{5}\right)$ (ii) $\left(\frac{-2}{5}+\frac{11}{5}\right)+\frac{-3}{5}$

What do you see ?

Solution :

(i) $\frac{-2}{5}+\left(\frac{11}{5}+\frac{-3}{5}\right)=\frac{-2}{5}+\left(\frac{11+(-3)}{5}\right)=\frac{-2}{5}+\frac{8}{5}=\frac{-2+8}{5}=\frac{6}{5}$

(ii) $\left(\frac{-2}{5}+\frac{11}{5}\right)+\frac{-3}{5}=\left(\frac{-2+11}{5}\right)+\frac{-3}{5}=\frac{9}{5}+\frac{-3}{5}=\frac{9+(-3)}{5}=\frac{6}{5}$

The answer obtained is same in both the cases. This verifies additive associativity of rational numbers.

Example 2

Simplify : $\frac{3}{8}+\frac{7}{2}+\frac{-3}{5}+\frac{9}{8}+\frac{-3}{2}+\frac{6}{5}$

Solution :

Re-arranging and grouping the number in pairs in such a way that each group contains a pair of rational numbers with a same denominator, we have.

$$\frac{3}{8}+\frac{7}{2}+\frac{-3}{5}+\frac{9}{8}+\frac{-3}{2}+\frac{6}{5}=\left(\frac{3}{8}+\frac{9}{8}\right)+\left(\frac{7}{2}+\frac{-3}{2}\right)+\left(\frac{-3}{5}+\frac{6}{5}\right)=\frac{3+9}{8}+\frac{7+(-3)}{2}+\frac{(-3)+6}{5}=\frac{12}{8}+\frac{4}{2}+\frac{3}{5}$$

$$=\frac{3}{2}+2+\frac{3}{5}=\frac{3\times5}{2\times5}+\frac{2\times10}{1\times10}+\frac{3\times2}{5\times2}=\frac{15}{10}+\frac{20}{10}+\frac{6}{10}=\frac{15+20+6}{10}=\frac{41}{10}$$

Example 3

Verify : $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$ for $\frac{a}{b}=\frac{-2}{3},\frac{c}{d}=\frac{5}{7}$ and $\frac{e}{f}=\frac{-1}{6}$

Solution :

We have, $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{-1}{6}=\frac{(-2)\times7+3\times5}{21}+\frac{-1}{6}=\frac{(-14)+15}{21}+\frac{-1}{6}=\frac{1}{21}+\frac{(-1)}{6}$

$$=\frac{1\times2+(-1)\times7}{42}=\frac{2+(-7)}{42}=\frac{(-5)}{42}=\frac{-5}{42} \text{ and } \frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{-2}{3}+\left(\frac{5}{7}+\frac{-1}{6}\right)=\frac{-2}{3}+\left(\frac{5\times6+7\times(-1)}{42}\right)$$

$$=\frac{-2}{3}+\frac{30+(-7)}{42}=\frac{(-2)}{3}+\frac{23}{42}=\frac{(-2)\times14+23\times1}{42}=\frac{(-28)+(23)}{42}=\frac{-5}{42}\quad\therefore\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$$

Example 4

The product of two rational numbers is $\frac{-28}{81}$. If one of the number is $\frac{14}{27}$, find the other.

Solution :

We have, product of two numbers $=\frac{-28}{81}$. One number $=\frac{14}{27}$ so, the other number is obtained by dividing the product by the given number.

$$\therefore \text{ Other number}=\frac{-28}{81}\div\frac{14}{27}=\frac{-28\times27}{81\times14}=\frac{-(28\times27)}{81\times14}=\frac{-(2\times1)}{3\times1}=\frac{-2}{3}$$

Example 5

Express each of the following expression in its lowest terms.

(i) $\left(\frac{12}{5}\times\frac{3}{4}\right)+\left(\frac{12}{5}\times\frac{7}{2}\right)$ (ii) $\left(\frac{2}{3}\times\frac{-5}{7}\right)-\left(\frac{2}{3}\times\frac{4}{5}\right)$

Solution :

(i) $\left(\frac{12}{5}\times\frac{3}{4}\right)+\left(\frac{12}{5}\times\frac{7}{2}\right)=\left(\frac{12\times3}{5\times4}\right)+\left(\frac{12\times7}{5\times2}\right)=\frac{9}{5}+\frac{42}{5}=\frac{9+42}{5}=\frac{51}{5}$

(ii) $\left(\frac{2}{3}\times\frac{-5}{7}\right)-\left(\frac{2}{3}\times\frac{4}{5}\right)=\frac{-10}{21}-\frac{8}{15}=\frac{-50-56}{105}=\frac{-106}{105}$

Example 6

Simplify: $\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1}{2}\times\frac{3}{4}\right)$

Solution :

$\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1}{2}\times\frac{3}{4}\right)=\frac{-3\times4}{2\times5}+\frac{9\times(-10)}{5\times3}-\frac{1\times3}{2\times4}=\frac{-3\times2}{1\times5}+\frac{3\times(-2)}{1\times1}-\frac{3}{8}=\frac{-6}{5}+\frac{-6}{1}-\frac{3}{8}$

$=\frac{-6}{5}+\frac{-6}{1}+\frac{-3}{8}=\frac{(-6)\times8+(-6)\times40+(-3)\times5}{40}=\frac{-48+(-240)+(-15)}{40}=\frac{-303}{40}$

5. REPRESENTATION OF RATIONAL NUMBERS ON NUMBER LINE

Step – 1 : Draw a line and mark a point 'O' on it to represent 0.

Step – 2 : Mark points at equal distance on the left and right side of 0, such that distance between two adjacent points is 1 unit.

Step – 3 : Then divide each unit into equal parts such that the number of parts is equal to the denominator of rational number.

Step – 4 : The numerator tells the number of parts to be considered.

Example 1

Represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line.

Solution :

$\frac{5}{3}$ and $\frac{-5}{3}$ can be written as $1\frac{2}{3}$ and $-1\frac{2}{3}$.

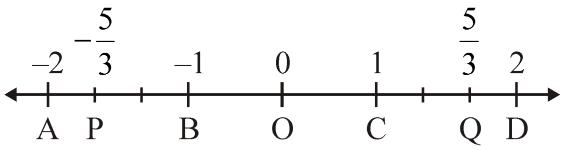
Step – 1 : In order to represent $\frac{5}{3}$ and $\frac{-5}{3}$ on the number line, we draw a number line and mark a point O on it to represent zero.

Step – 2 : Since $1\frac{2}{3}$ and $-1\frac{2}{3}$ lie between 1 and 2 and – 1 and – 2 respectively, therefore mark the points A and B on left and C and D on right side of O, such that A, B, C and D represent – 2, – 1, 1 and 2 respectively.

Step – 3 : Now, the denominator of rational number is 3.

∴ Divide the intervals in 3 equal parts.

Step – 4 : Since the numerator is 2, then mark second point on the parts. P and Q are the required points.

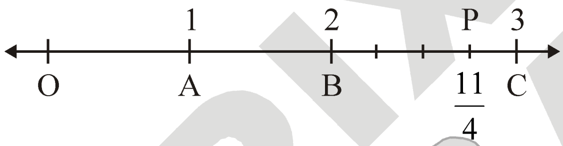


Example 2

Represent $\frac{11}{4}$ on the number line.

Solution :

$\frac{11}{4} = 2\frac{3}{4}$ lies between 2 and 3.



6. RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

Between two rational numbers there are infinitely many rational numbers.

Method – 1 : Let a and b be two rational numbers, then $q_1 = \frac{a+b}{2} \Rightarrow a < q_1 < b$

q_1 is the rational number between a and b.

$q_2 = \frac{a+q_1}{2} \Rightarrow a < q_2 < q_1 < b$

q_2 is the rational number between a and q_1.

$q_3 = \frac{q_1+b}{2} \Rightarrow a < q_2 < q_1 < q_3 < b$

q_3 is the rational number bwtween q_1 and b.

In this manner we can find infinite rational numbers between two given distinct rational numbers.

Method – 2 : $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers.

Step – 1 : Make denominators equal in both rational numbers.

Step – 2 : If we have to find n rational numbers between $\frac{ad}{bd}$ and $\frac{cb}{bd}$, then multiply numerators and denominators of both numbers by such a number so that there are n numbers between the numerators.

Example 1

Write three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Solution :

Method – 1 : The rational number between $\frac{1}{3}$ and $\frac{1}{2}$ is $\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{2}{6}+\frac{3}{6}\right)=\frac{1}{2}\times\frac{5}{6}=\frac{5}{12}\Rightarrow\frac{1}{3}<\frac{5}{12}<\frac{1}{2}$

The rational number between $\frac{1}{3}$ and $\frac{5}{12}$ is $\frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right)=\frac{1}{2}\left[\frac{4}{12}+\frac{5}{12}\right]=\frac{1}{2}\times\frac{9}{12}=\frac{9}{24}=\frac{3}{8}\Rightarrow\frac{1}{3}<\frac{3}{8}<\frac{5}{12}<\frac{1}{2}$

The rational number between $\frac{5}{12}$ and $\frac{1}{2}$ is $\frac{1}{2}\left(\frac{5}{12}+\frac{1}{2}\right)=\frac{1}{2}\left(\frac{5}{12}+\frac{6}{12}\right)=\frac{1}{2}\times\frac{11}{12}=\frac{11}{24}\Rightarrow\frac{1}{3}<\frac{3}{8}<\frac{5}{12}<\frac{11}{24}<\frac{1}{2}$

Hence, $\frac{3}{8},\frac{5}{12},\frac{11}{24}$ are the required three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

Method – 2 : Make the denominator equal by multiplying 2 and 3. $\frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$ and $\frac{1}{2}\times\frac{3}{3}=\frac{3}{6}$

To insert 3 rational numbers we multiply the numerators and denominators by such a number so that difference between the numerators is at least 3. Multiplying the numerators and denominators of both rational numbers by 4, we get $\frac{8}{24}$ and $\frac{12}{24}$. Hence, the required 3 rational numbers are $\frac{9}{24},\frac{10}{24},\frac{11}{24}$.

Example 2

Find three rational numbers between – 2 and 5.

Solution :

Three rational numbers between – 2 and 5 are – 1, 0, 1, $\Rightarrow -2<-1<0<1<5$

SOLVED EXAMPLES

SE. 1

Write the additive inverse of each of the following rational numbers :

(i) $\frac{5}{-11}$ (ii) $\frac{-11}{-14}$

Ans. (i) We have, $\frac{5}{-11}=\frac{-5}{11}$

The additive inverse of $\frac{-5}{11}$ is $-\left(\frac{-5}{11}\right)=\frac{5}{11}$

(ii) We have, $\frac{-11}{-14}=\frac{11}{14}$

The additive inverse of $\frac{11}{14}$ is $-\left(\frac{11}{14}\right)=\frac{-11}{14}$

SE. 2

Verify that – (– x) is the same as x for :

(i) $x=\frac{13}{17}$ (ii) $x=\frac{-21}{31}$

Ans. (i) $x=\frac{13}{17}$

So, $-(-x)=-\left(\frac{-13}{17}\right)=\frac{13}{17}$

(ii) $x=\frac{-21}{31}$

So, $-(-x)=-\left(-\left(\frac{-21}{31}\right)\right)=-\left(\frac{21}{31}\right)=-\frac{21}{31}$

SE. 3

What number should be subtracted from $\frac{27}{13}$ to get $\frac{-3}{7}$?

Ans. We have, difference of the given number and the required number is $\frac{-3}{7}$. Given number = $\frac{27}{13}$.

Other number = $\frac{27}{13}-\frac{-3}{7}=\frac{27}{13}+\frac{3}{7}=\frac{189+39}{13\times7}$

$=\frac{228}{13\times7}=\frac{228}{91}$. So, $\frac{228}{91}$ should be subtracted from $\frac{27}{13}$ to get $\frac{-3}{7}$.

SE. 4

The sum of two rational numbers is $\frac{-1}{2}$. If one of them is $\frac{-9}{10}$, find the other.

Ans. Given that the sum of numbers is $\frac{-1}{2}$.

One number = $\frac{-9}{10}$

∴ Other number = Sum of the numbers – One number

$=\frac{-1}{2}-\left(\frac{-9}{10}\right)=\frac{-1}{2}+\frac{9}{10}=\frac{-5+9}{10}=\frac{4}{10}=\frac{2}{5}$

SE. 5

Using commutativity of addition of rational numbers, express the following $\frac{4}{3}+\frac{-4}{5}+\frac{-2}{3}+\frac{7}{5}-2$ as a rational number.

Ans. Re-arranging and grouping the numbers in such a way that each group contains a pair of rational numbers with equal denominators, we have

$\frac{4}{3}+\frac{-4}{5}+\frac{-2}{3}+\frac{7}{5}-2=\left(\frac{4}{3}+\frac{-2}{3}\right)+\left(\frac{-4}{5}+\frac{7}{5}\right)-2$

$=\frac{4+(-2)}{3}+\frac{(-4)+7}{5}-2=\frac{2}{3}+\frac{3}{5}-2$

$=\left(\frac{2}{3}+\frac{3}{5}\right)-2=\frac{2\times5+3\times3}{15}+(-2)$

$$= \frac{10+9}{15} + (-2) = \frac{19}{15} + \frac{(-2)}{1} = \frac{19+(-2)\times 15}{15}$$

$$= \frac{19+(-30)}{15} = \frac{-11}{15}$$

SE. 6

Re-arrange suitably and find the sum of $\frac{-4}{7} + \frac{7}{6} + \frac{2}{7} + 3 + \frac{-11}{6}$.

Ans. Re-arranging and grouping the numbers in such a way that each group contains a pair of rational numbers with equal denominators, we have

$$\frac{-4}{7} + \frac{7}{6} + \frac{2}{7} + 3 + \frac{-11}{6}$$

$$= \left(\frac{-4}{7} + \frac{2}{7}\right) + \left(\frac{7}{6} + \frac{-11}{6}\right) + 3$$

$$= \frac{(-4)+2}{7} + \frac{7+(-11)}{6} + 3 = \frac{-2}{7} + \frac{(-4)}{6} + 3$$

$$= \frac{(-2)}{7} + \frac{(-2)}{3} + 3 = \left(\frac{(-2)}{7} + \frac{(-2)}{3}\right) + 3$$

$$= \frac{(-2)\times 3 + (-2)\times 7}{21} + 3 = \frac{(-6)+(-14)}{21} + 3$$

$$= \frac{(-20)}{21} + \frac{3}{1} = \frac{(-20)+3\times 21}{21} = \frac{-20+63}{21} = \frac{43}{21}$$

SE. 7

What number should be added to $\frac{-5}{8}$ to get $\frac{5}{9}$?

Ans. Let x is the rational number to be added to $\frac{-5}{8}$ to get $\frac{5}{9}$. Then, $\frac{-5}{8} + x = \frac{5}{9}$

$$\Rightarrow x = \frac{5}{9} - \left(\frac{-5}{8}\right)$$

$$\Rightarrow x = \frac{5}{9} + \frac{5}{8} \Rightarrow x = \frac{5\times 8 + 5\times 9}{72} = \frac{40+45}{72} = \frac{85}{72}$$

SE. 8

Subtract $\frac{-8}{33}$ from $\frac{-5}{11}$.

Ans. $\frac{-5}{11} - \left(\frac{-8}{33}\right) = \frac{-5}{11} + \frac{8}{33} = \frac{-5\times 3}{11\times 3} + \frac{8}{33}$

$$= \frac{-15}{33} + \frac{8}{33} = \frac{-15+8}{33} = \frac{-7}{33}$$

SE. 9

Subtract the sum of $\frac{-4}{7}$ and $\frac{5}{14}$ from the sum of $\frac{9}{14}$ and $\frac{23}{14}$.

Ans. Sum of $\frac{-4}{7}$ and $\frac{5}{14} = \frac{-4}{7} + \frac{5}{14} = \frac{-8+5}{14} = \frac{-3}{14}$

Sum of $\frac{9}{14}$ and $\frac{23}{14} = \frac{9}{14} + \frac{23}{14} = \frac{9+23}{14} = \frac{32}{14}$

$= \frac{16}{7}$. Now, subtracting $\frac{-3}{14}$ from $\frac{16}{7}$, we get

$$\frac{16}{7} - \left(\frac{-3}{14}\right) = \frac{16}{7} + \frac{3}{14} = \frac{32+3}{14} = \frac{35}{14} = \frac{5}{2}$$

SE. 10

Evaluate : $\frac{-12}{5} + \frac{-7}{20} + \frac{3}{14} + \frac{1}{7} + \frac{-1}{10}$

Ans. L.C.M. of 5, 20, 14, 7, 10 = 2 × 2 × 5 × 7 = 140

$$\therefore \frac{-12}{5} + \frac{-7}{20} + \frac{3}{14} + \frac{1}{7} + \frac{-1}{10}$$

$$= \frac{(-12)\times(28) + (-7)\times 7 + 3\times 10 + 1\times 20 + (-1)\times 14}{140}$$

$$= \frac{(-336)+(-49)+30+20+(-14)}{140}$$

$$= \frac{(-399)+50}{140} = \frac{-349}{140}$$

SE. 11

Simplify : $\left(\frac{-7}{18}\times\frac{15}{-7}\right)-\left(1\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)$

Ans. $\left(\frac{-7}{18}\times\frac{15}{-7}\right)-\left(1\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)$

$=\left(\frac{-7}{18}\times\frac{15}{-7}\right)-\left(\frac{1}{1}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)$

$=\frac{1\times5}{6\times1}-\frac{1\times1}{1\times4}+\frac{1\times1}{2\times4}=\frac{5}{6}-\frac{1}{4}+\frac{1}{8}$

$=\frac{5}{6}+\frac{-1}{4}+\frac{1}{8}=\frac{5\times4+(-1)\times6+1\times3}{24}$

$=\frac{20+(-6)+3}{24}=\frac{17}{24}$

SE. 12

Divide :

(i) $\frac{-16}{21}$ by $\frac{-4}{3}$ (ii) $\frac{-8}{13}$ by $\frac{3}{-26}$

Ans. (i) $\frac{-16}{21}\div\frac{-4}{3}=\frac{-16}{21}\times\frac{3}{-4}=\frac{-16\times3}{21\times(-4)}=\frac{4\times1}{7\times1}=\frac{4}{7}$

(ii) $\frac{-8}{13}\div\frac{3}{-26}=\frac{-8}{13}\times\frac{-26}{3}=\frac{(-8)\times(-26)}{13\times3}$

$=\frac{8\times26}{13\times3}=\frac{8\times2}{1\times3}=\frac{16}{3}$

SE. 13

The product of two numbers is $\frac{14}{15}$. If one of the numbers is $\left(\frac{-20}{17}\right)$, then find the other.

Ans. Product of two numbers = $\frac{14}{15}$.

One number = $\frac{-20}{17}$

$\therefore$ Other number $=\frac{14}{15}\div\frac{-20}{17}=\frac{14}{15}\times\frac{17}{-20}$

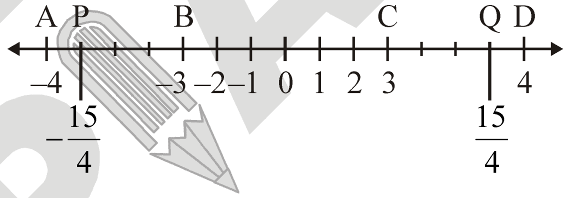
$=\frac{14\times17}{15\times(-20)}=\frac{7\times17}{15\times(-10)}=\frac{119}{-150}=\frac{-119}{150}$

SE. 14

Represent $\frac{15}{4}$ and $\frac{-15}{4}$ on the number line.

Ans. $\frac{15}{4}$ and $\frac{-15}{4}$ can be written as $3\frac{3}{4}$ and $-3\frac{3}{4}$

$\therefore 3\frac{3}{4}$ lies between 3 & 4 and $-3\frac{3}{4}$ lies between – 3 and – 4.



SE. 15

Express $\left(\frac{1}{2}+\frac{3}{4}\right)\div2$ as a rational number and show that it lies between $\frac{1}{2}$ and $\frac{3}{4}$.

Ans. $\left(\frac{1}{2}+\frac{3}{4}\right)\div2=\left(\frac{1\times2}{2\times2}+\frac{3}{4}\right)\div2=\left(\frac{2}{4}+\frac{3}{4}\right)\div2$

$=\frac{5}{4}\div2=\frac{5}{4}\times\frac{1}{2}=\frac{5}{8}$

Let us now arrange the number $\frac{1}{2},\frac{3}{4}$ and $\frac{5}{8}$ in ascending order of magnitude.

L.C.M. of denominators 2, 4 and 8 = 8

Now, $\frac{1}{2}=\frac{1\times4}{2\times4}=\frac{4}{8},\frac{3}{4}=\frac{3\times2}{4\times2}=\frac{6}{8},\frac{5}{8}=\frac{5}{8}$

Since, $4<5<6$, it follows that $\frac{4}{8}<\frac{5}{8}<\frac{6}{8}$

i.e., $\frac{1}{2}<\frac{5}{8}<\frac{3}{4}$.

We find that, $\frac{5}{8}$ lies between $\frac{1}{2}$ and $\frac{3}{4}$.

Therefore, $\left(\frac{1}{2}+\frac{3}{4}\right)\div 2$ lies between $\frac{1}{2}$ and $\frac{3}{4}$.

SE. 16

Find 10 rational numbers between $\frac{-2}{11}$ and $\frac{9}{11}$.

Ans. The given rational numbers are $\frac{-2}{11}$ and $\frac{9}{11}$.

Since the denominators of both the rational numbers are equal and positive. Finding 10 rational numbers between $\frac{-2}{11}$ and $\frac{9}{11}$ is similar to finding 10 integers between – 2 and 9. We know that

$-2 < -1 < 0 < 1 < 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9$

So, $\frac{-1}{11},\frac{0}{11},\frac{1}{11},\frac{2}{11},\frac{3}{11},\frac{4}{11},\frac{5}{11},\frac{6}{11},\frac{7}{11}$ and $\frac{8}{11}$ are the required 10 rational numbers between $\frac{-2}{11}$ and $\frac{9}{11}$.

Space for Notes :

EXERCISE – I

ONLY ONE CORRECT TYPE

1. The standard form of $\frac{192}{-168}$ is :

(A) $\frac{-2}{3}$ (B) $\frac{-8}{7}$

(C) $\frac{-1}{7}$ (D) $\frac{-6}{7}$

2. Standard form of $\frac{-24}{36}$ is a rational number with denominator :

(A) 3 (B) 4

(C) 1 (D) 2

3. 0.75 when represented as rational number, is :

(A) $\frac{75}{99}$ (B) $\frac{75}{90}$

(C) $\frac{3}{4}$ (D) $\frac{5}{4}$

4. The additive inverse of $\frac{-a}{b}$ is :

(A) $\frac{a}{b}$ (B) $\frac{b}{a}$

(C) $\frac{-b}{a}$ (D) $\frac{-a}{b}$

5. The number which is subtracted from $\frac{27}{13}$ to get $\frac{-3}{7}$, is :

(A) $\frac{228}{91}$ (B) $\frac{1}{91}$

(C) $\frac{200}{91}$ (D) $\frac{198}{91}$

6. The sum of the additive inverse and multiplicative inverse of 2 is :

(A) $\frac{3}{2}$ (B) $\frac{-3}{2}$

(C) $\frac{1}{2}$ (D) $\frac{-1}{2}$

7. Which of the following statements is false ?

(A) Every fraction is a rational number.

(B) Every rational number is a fraction.

(C) Every integer is a rational number.

(D) All of these

8. Which of the following statements is true ?

(A) $\frac{5}{7}<\frac{7}{9}<\frac{9}{11}<\frac{11}{13}$ (B) $\frac{11}{13}<\frac{9}{11}<\frac{7}{9}<\frac{5}{7}$

(C) $\frac{5}{7}<\frac{11}{13}<\frac{7}{9}<\frac{9}{11}$ (D) $\frac{5}{7}<\frac{9}{11}<\frac{11}{13}<\frac{7}{9}$

9. Multiplicative inverse of '0' is :

(A) – 1 (B) 0

(C) Does not exist (D) 1

10. The value of x for which the two rational numbers $\frac{3}{7}$ and $\frac{x}{42}$ are equivalent, is :

(A) 18 (B) 15

(C) 12 (D) 10

11. Which of the following illustrates the inverse property of addition ?

(A) 3 + (– 3) = 0 (B) 3 – (– 3) = 6

(C) 3 + 0 = 3 (D) 3 – 0 = 3

12. What number should be added to $\frac{7}{12}$ to get $\frac{4}{15}$?

(A) $-\frac{19}{60}$ (B) $-\frac{11}{30}$

(C) $\frac{51}{60}$ (D) $\frac{1}{20}$

13. The difference between the largest and the smallest of the rationals, $\frac{5}{8}, \frac{7}{12}, \frac{1}{3}, \frac{2}{5}$, is :

(A) $\frac{1}{4}$ (B) $\frac{-5}{24}$

(C) $\frac{7}{24}$ (D) $\frac{13}{21}$

14. The additive inverse of sum of the rational numbers $-\frac{5}{16}$ and $\frac{7}{12}$ is :

(A) $-\frac{7}{48}$ (B) $\frac{1}{24}$

(C) $-\frac{13}{48}$ (D) $\frac{13}{48}$

15. Which of the following rational numbers is the smallest ? $-\frac{5}{16}, \frac{-3}{4}, \frac{-13}{24}$ and $\frac{7}{-12}$

(A) $-\frac{5}{16}$ (B) $\frac{-3}{4}$

(C) $\frac{-13}{24}$ (D) $\frac{-7}{12}$

16. What number should be subtracted from $-\frac{3}{5}$ to get -2 ?

(A) $-\frac{7}{5}$ (B) $-\frac{13}{5}$

(C) $\frac{13}{5}$ (D) $\frac{7}{5}$

17. Name the law of multiplication illustrated by the statement, $\frac{-15}{8} \times \frac{-12}{7} = \frac{-12}{7} \times \frac{-15}{8}$.

(A) Associative law (B) Closure law

(C) Commutative law (D) None of these

18. Which of the following forms a pair of equivalent rational numbers ?

(A) $\frac{14}{35}$ and $\frac{21}{45}$ (B) $\frac{-12}{26}$ and $\frac{18}{39}$

(C) $\frac{-3}{7}$ and $\frac{-21}{56}$ (D) $\frac{-7}{28}$ and $\frac{-5}{20}$

19. The reciprocal of $\left(-\frac{9}{16} \times \frac{8}{15}\right)$ is

(A) $-\frac{3}{10}$ (B) $-\frac{4}{150}$

(C) $-\frac{10}{3}$ (D) $-\frac{2}{50}$

20. By what number should we multiply $\frac{3}{-14}$, so that the product is $\frac{5}{12}$?

(A) $\frac{-35}{18}$ (B) $\frac{34}{19}$

(C) $\frac{35}{18}$ (D) $\frac{-34}{19}$

21. The sum of two rational numbers is $\frac{-3}{5}$. If one of the numbers is $\frac{-9}{20}$, find the other.

(A) $\frac{7}{20}$ (B) $\frac{27}{100}$

(C) $\frac{-21}{20}$ (D) $\frac{-3}{20}$

22. The area of a rectangle is $45\frac{5}{16} cm^2$. If its length is $7\frac{1}{4}$ cm, then find its breadth.

(A) $6\frac{1}{4}$ cm (B) $4\frac{1}{6}$ cm

(C) $328\frac{33}{64}$ cm (D) $38\frac{1}{16}$ cm

23. If $\frac{3}{7}+x+\left(\frac{-8}{21}\right)+\frac{5}{22}=\frac{-125}{462}$, then x is

(A) $\frac{6}{11}$ (B) $\frac{-5}{11}$

(C) $\frac{-6}{11}$ (D) $\frac{5}{11}$

24. The product of two numbers is $-\frac{16}{35}$. If one of the numbers is $-\frac{15}{14}$, find the additive inverse of other.

(A) $-\frac{2}{5}$ (B) $\frac{-32}{75}$

(C) $\frac{32}{75}$ (D) $-\frac{8}{3}$

25. Simplify: $\left(\frac{3}{5}\times\frac{-15}{21}\right)+\left(\frac{-9}{14}\div\frac{45}{28}\right)-\left(\frac{2}{3}\times\frac{30}{12}\right)$

(A) $-\frac{17}{35}$ (B) $-2\frac{52}{105}$

(C) $-1\frac{4}{11}$ (D) $\frac{-40}{41}$

PARAGRAPH TYPE

PASSAGE # I

If two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are such that $\frac{c}{d}\neq 0$ and $\frac{a}{b}\times\frac{c}{d}=1$, then $\frac{a}{b}$ and $\frac{c}{d}$ are called multiplicative inverse of each other.

1. $\frac{73}{-29}$ is the multiplicative inverse of ________.

(A) $\frac{29}{73}$ (B) $\frac{-29}{73}$

(C) $\frac{73}{29}$ (D) 1

2. Find the multiplicative inverse of $\frac{1}{2}\left(2+\frac{3}{2}\right)$.

(A) $\frac{4}{7}$ (B) $\frac{8}{3}$

(C) $\frac{7}{2}$ (D) $\frac{2}{3}$

3. If $(x+y)z=1$, then z is a multiplicative inverse of ________.

(A) x (B) y

(C) x + y (D) $\frac{x+y}{2}$

PASSAGE # II

If $\frac{a}{b},\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then

$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}$.

4. If $\frac{2}{3}\times\frac{-7}{10}+\frac{-2}{3}\times\frac{8}{9}=p\times\left[\frac{-7}{10}+q\right]$, then p and q are

(A) $\frac{2}{3},\frac{8}{9}$ (B) $\frac{-2}{3},\frac{-8}{9}$

(C) $\frac{-2}{3},\frac{8}{9}$ (D) $\frac{2}{3},\frac{-8}{9}$

5. Name the property used above.

(A) Commutativity of multiplication over addition.

(B) Commutativity of addition over multiplication.

(C) Distributivity of multiplication over addition.

(D) Distributivity of addition over multiplication.

6. If $\frac{2}{5}\times\frac{-8}{9}+p\times\frac{5}{9}=\frac{2}{5}\times[q+r]$, then p, q and r are

(A) $\frac{2}{5},\frac{8}{9},\frac{5}{9}$

(B) $\frac{2}{5},\frac{8}{9},\frac{-5}{9}$

(C) $\frac{-2}{5},\frac{-8}{9},\frac{-5}{9}$

(D) $\frac{-2}{5},\frac{-8}{9},\frac{5}{9}$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the corect combination of elements from List – I and List – II are given as options (A), (B), (C) and (D) out of which one is correct.

1. Match the following :

List – I	List – II
(P) Additive identity of a rational number 'a' is	(i) 1/a
(Q) Multiplicative inverse of a rational number 'a' is	(ii) 0
(R) Multiplicative identity of a rational number 'a' is	(iii) – a
(S) Additive inverse of a rational number 'a' is	(iv) 1

(A) (P)→(ii), (Q)→(i), (R)→(iv), (S)→(iii)
(B) (P)→(iii), (Q)→(ii), (R)→(iv), (S)→(i)
(C) (P)→(iii), (Q)→(ii), (R)→(i), (S)→(iv)
(D) (P)→(ii), (Q)→(iv), (R)→(iii), (S)→(i)

2. Match the following :

List – I	List – II
(P) Associative law	(i) If a and b are rational numbers, then a + b is a rational number.
(Q) Commutative law	(ii) If a and b are rational numbers, then a + b = b + a
(R) Distributive law	(iii) If a, b and c are rational numbers, then a + (b + c) = (a + b) + c
(S) Closure law	(iv) If a, b and c are rational numbers, then a × (b + c) = ab + ac

(A) (P)→(ii), (Q)→(iii), (R)→(iv), (S)→(i)
(B) (P)→(iii), (Q)→(ii), (R)→(iv), (S)→(i)
(C) (P)→(iii), (Q)→(ii), (R)→(i), (S)→(iv)
(D) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Write additive inverse of the following number.

   (i) $\frac{-11}{-25}$ (ii) 0

2. Fill in blank : $\frac{-9}{14}+........=-1$.

3. Multiply $\frac{-3}{17}$ by $\frac{-5}{-4}$.

4. Express $\frac{2}{7}$ as a rational number whose numerator is – 6.

5. Express rational number $\frac{4}{-14}$ with positive denominator.

6. Find the value of x, if $\frac{-5}{9}=\frac{10}{x}$.

7. Are $\frac{15}{24}$ and $\frac{45}{48}$ equivalent rational numbers ?

8. Can you write $\frac{1}{2}$ with denominator equal to 5 ?

9. Write the absolute value of $\frac{-9}{-100}$.

10. Can – 2 be the absolute value of any rational number ?

SHORT ANSWER TYPE

1. If $\frac{3}{5}$ of a number exceeds its $\frac{2}{7}$ by 44, find the number.

2. By what number should $\frac{-33}{16}$ be divided to get $\frac{-11}{4}$?

3. Find (x + y) ÷ (x – y), if $x=\frac{5}{4}, y=\frac{-1}{3}$.

4. Find a rational number between $\frac{-2}{3}$ and $\frac{1}{4}$.

5. The product of two rational numbers is $\frac{63}{40}$. If one of the number is $\left(\frac{-7}{5}\right)$, find the other number.

LONG ANSWER TYPE

1. Divide the sum of $\frac{-13}{5}$ and $\frac{12}{7}$ by the product of $\frac{-31}{7}$ and $\frac{-1}{2}$.

2. Simplify : $\left[\frac{3}{11}\times\frac{5}{6}\right]-\left[\frac{9}{12}\times\frac{4}{3}\right]+\left[\frac{5}{13}\times\frac{6}{15}\right]$

3. Verify the property x × (y + z) = x × y + x × z when $x=\frac{-12}{5}, y=\frac{-15}{4}, z=\frac{8}{3}$.

4. Find four rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$.

5. For $x=\frac{1}{2}$ and $y=\frac{2}{3}$, verify that – (x + y) = (– x) + (– y).

TRUE / FALSE TYPE

1. Prime number can also be a negative integer.
2. 2 is the only even prime number.
3. 0 is a rational number.
4. $\frac{3}{5}>\frac{2}{3}$.
5. There exists infinite rational numbers between any 2 integers.

NUMERICAL PROBLEMS

1. Multiplicative identity for rational number is.
2. Find the sum of digits of numerator and denominator of reciprocal of $\left(\frac{2}{5}+\frac{5}{4}\right)$.
3. The product of two numbers is $\frac{45}{56}$. One of them is $\frac{9}{7}$ and the other number is $\frac{m}{n}$. Then m + n is.
4. The sum of two rational numbers is – 3. If one of the numbers is $-\frac{10}{3}$, then numerator of other number is.
5. Find x, if $4\times\frac{7}{9}=\frac{7}{9}\times x$.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Simplify :

$$\frac{\left(\frac{2}{3}\times\left(-\frac{5}{4}\right)\right)+\left(\left(-\frac{10}{3}\right)\times\frac{5}{2}\right)-\left(\left(-\frac{16}{3}\right)\times\left(-\frac{55}{32}\right)\right)}{\frac{3}{2}\times\left(\left(-\frac{9}{14}\right)\times\left(-\frac{1}{7}\right)\right)}$$

(A) $\frac{1082}{81}$ (B) $-\frac{1082}{81}$

(C) $-133\frac{7}{81}$ (D) $133\frac{7}{81}$

2. To reduce a rational number in its standard form, we divide its numerator and denominator by their ____________.

(A) L.C.M. (B) H.C.F.

(C) Product (D) Multiple

3. Which of the following shows distributive property of multiplication over addition for rational numbers ?

(A) $-\frac{3}{4}\times\left\{\frac{1}{3}+\left(-\frac{5}{7}\right)\right\}=\left[-\frac{3}{4}\times\frac{1}{3}\right]+\left[-\frac{3}{4}\times\left(-\frac{5}{7}\right)\right]$

(B) $-\frac{3}{4}\times\left\{\frac{1}{3}+\left(-\frac{5}{7}\right)\right\}=\left[-\frac{3}{4}\times\frac{1}{3}\right]-\left[-\frac{5}{7}\right]$

(C) $-\frac{3}{4}\times\left\{\frac{1}{3}+\left(-\frac{5}{7}\right)\right\}=\frac{1}{3}+\left[-\frac{3}{4}\right]\times\left(-\frac{5}{7}\right)$

(D) $-\frac{3}{4}\times\left\{\frac{1}{3}+\left(-\frac{5}{7}\right)\right\}=\left[\frac{1}{3}+\left(-\frac{5}{7}\right)\right]-\frac{3}{4}$

4. If A : Rational numbers are always closed under division and R : Division by zero is not defined, then ____________.

(A) Both A and R are true

(B) Both A and R are false

(C) A is true and R is false

(D) A is false and R is true

5. If a, b, c are rational numbers, then associativity of rational numbers under addition is given by

(A) a + b = b + a

(B) a + (b + c) = (a + b) + c

(C) a × (b × c) = (a × b) × c

(D) a + (b – c) = (a + b) – c

6. Which of the following statements is true ?

(i) $\frac{-5}{0}$ is a negative rational number.

(ii) The reciprocal of $\frac{1}{a}$, if a = 0 is $\frac{1}{0}$.

(iii) $1\div\left(-\frac{1}{4}\right)=-4$

(iv) x ÷ (y + z) = x ÷ y + x ÷ z

(A) Both (i) and (ii)

(B) Only (iii)

(C) (i), (ii) and (iv)

(D) (ii), (iii) and (iv)

7. Zero is ________.
(A) The identity for addition of rational numbers.
(B) The identity for subtraction of rational numbers.
(C) The identity for multiplication of rational numbers.
(D) The identity for division of rational numbers.

8. Which of the following statements is always true ?
(A) $\frac{x-y}{2}$ is a rational number between x and y.
(B) $\frac{x+y}{2}$ is a rational number between x and y.
(C) $\frac{x \times y}{2}$ is a rational number between x and y.
(D) $\frac{x \div y}{2}$ is a rational number between x and y.

9. The number 34 is divided into two parts such that $\frac{4}{7}$ th of the first part is equal to $\frac{2}{5}$ th of the second part. The numbers are respectively
(A) 20, 14 (B) 21, 13
(C) 13, 21 (D) 14, 20

10. Which of the following rational numbers does not lie between $\frac{1}{4}$ and $\frac{2}{3}$?
(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{14}{24}$ (D) $\frac{18}{24}$

11. The numerator and the denominator of a rational number are in the ratio 5 : 7. When 6 is added to both the numerator and denominator, the ratio becomes 4 : 5. What is the rational number ?
(A) $\frac{7}{5}$ (B) $\frac{5}{7}$
(C) $\frac{2}{5}$ (D) $\frac{13}{14}$

12. A water pump pumps out $14\frac{1}{6}\,l$ of water per minute from a reservoir. How many litres of water will be pumped out in $1\frac{1}{5}$ of an hour ?
(A) 1125 l (B) 6120 l
(C) 1020 l (D) 1560 l

13. Simplest form of $\cfrac{1}{3-\cfrac{1}{2-\cfrac{1}{7}}}$ is :
(A) $\frac{13}{32}$ (B) $\frac{32}{13}$
(C) $\frac{7}{13}$ (D) $\frac{13}{7}$

14. p : Every fraction is a rational number.
q : Every rational number is a fraction.
Which of the following is correct ?
(A) p is true and q is false.
(B) p is false and q is true.
(C) Both p and q are true.
(D) Both p and q are false.

15. Nine times the reciprocal of a rational number equals 6 times the reciprocal of 17. Find the rational number.
(A) $11\frac{1}{3}$ (B) $25\frac{1}{2}$
(C) $10\frac{1}{3}$ (D) None of these

EXERCISE – III

PREVIOUS YEAR QUESTIONS (NTSE)

1. If a number is divided by 45, then the remainder is 32. If the same number is divided by 15, then the remainder is **(Aryabhatta 2008)**

(A) 2 (B) 3

(C) 16 (D) 4

2. The product of x^2y and $\left(\frac{x}{y}\right)$ is equal to the quotient obtained when x^2 is divided by **(NSTSE 2010)**

(A) 0 (B) 1

(C) x (D) $\frac{1}{x}$

3. Identify a rational number between $\frac{1}{3}$ and $\frac{4}{5}$ **(NSTSE 2012)**

(A) $\frac{1}{4}$

(B) $\frac{9}{10}$

(C) $\frac{17}{30}$

(D) $1\frac{7}{10}$

4. Which of the statements is true about consecutive natural numbers ? **(NSTSE 2012)**

(A) There are 2n + 1 numbers between the difference of squares of consecutive numbers.

(B) There are 2n non-perfect square numbers between the squares.

(C) The sum of the squares of two consecutive numbers is not a perfect square.

(D) $n^2 - 1$ is the standard form of the difference between two consecutive numbers.

5. Identify the ones that is/are greater than 'm' if $m = \frac{9}{11}$. **(NSTSE 2014)**

(i) $\frac{1}{m}$ (ii) $\frac{m+1}{m}$

(iii) $\frac{m+1}{m-1}$

(A) (i) only (B) (ii) and (iii) only

(C) (i) and (iii) only (D) (i) and (ii) only

6. Which number is in the middle if $\frac{-1}{6}, \frac{4}{9}, \frac{6}{-7}, \frac{2}{5}$ and $\frac{-3}{4}$ are arranged in descending order **(NSTSE 2014)**

(A) $\frac{2}{5}$ (B) $\frac{4}{9}$

(C) $\frac{-1}{6}$ (D) $\frac{-6}{7}$

7. If the division N ÷ 5 leaves a remainder of 3, what might be the ones digit of N ? **(NSTSE 2014)**

(A) 2 (B) 3

(C) 4 (D) 6

8. Which of the following numbers does not have a multiplicative inverse ? **(NSTSE 2014)**

(A) $-\frac{1}{3}$ (B) 0

(C) 1 (D) 3

9. The difference between the place value and the face value of 6 in the numeral 856973 is ________. **(NSTSE 2014)**

(A) 973 (B) 6973

(C) 5994 (D) None of these

10. Which of the following expressions is true ? **(NSTSE 2014)**

(A) $0.09 > \frac{7}{8}$ (B) 6 % < 0.09

(C) $\frac{7}{8} < 8.0 \times 10^{-3}$ (D) $8.0 \times 10^{-3} > 6$ %

11. If x : y = 5 : 2, then (8x + 9y) : (8x + 2y) is : **(NSTSE 2014)**

(A) 22 : 29 (B) 26 : 61

(C) 29 : 22 (D) 61 : 26

12. Which of the following statements is incorrect for rational numbers ? **(NSTSE 2014)**

(A) The rational number 0 is the additive identity for rational numbers.

(B) The rational number 1 is the multiplicative identity for rational numbers.

(C) Subtraction is associative for rational numbers.

(D) There are infinite rational numbers between any two given rational numbers.

Space for Notes :

Answer Key

EXERCISE I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	A	C	A	A	B	B	A	C	A	A	A	C	C	B
16	17	18	19	20	21	22	23	24	25					
D	C	D	C	A	D	A	C	B	B					

PARAGRAPH

1. B **2.** A **3.** C **4.** D **5.** C **6.** A

MATCH THE COLUMN

1. A **2.** B

EXERCISE II

VERY SHORT ANSWER TYPE

1. $-\frac{11}{25}$ **2.** $-\frac{5}{14}$ **3.** $-\frac{15}{68}$ **4.** $\frac{-6}{-21}$ **5.** $-\frac{4}{14}$ **6.** – 18 **7.** No

8. No **9.** $\frac{9}{100}$ **10.** No

SHORT ANSWER TYPE

3. 140 **6.** $\frac{3}{4}$ **7.** $\frac{11}{19}$ **8.** $-\frac{5}{24}$ **10.** $-\frac{9}{8}$

LONG ANSWER TYPE

1. $-\frac{2}{5}$ **2.** $-\frac{177}{286}$ **4.** $\frac{41}{60},\frac{42}{60},---,\frac{47}{60}$

TRUE / FALSE

2. F **3.** T **4.** T **6.** F **10.** T

NUMERICAL PROBLEMS

3. 1 **4.** 8 **5.** 13 **8.** 1 **10.** 4

ANALYTICAL PROBLEMS & BRAIN TEASER

1. C **2.** B **3.** A **4.** D **5.** B **6.** B **7.** A
8. B **9.** A **10.** D **11.** D **12.** C **13.** A **14.** B
15. B

EXERCISE III

1	2	3	4	5	6	7	8	9	10	11	12
A	D	C	B	D	C	B	B	C	B	C	C

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : INTEGERS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put “completed” only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

LINEAR EQUATIONS IN ONE VARIABLE

2

Concepts

1. EQUATION

A statement of equality which contains one or more unknown quantity or variable is called an equation.

Example 1

(i) $3x + 7 = 12,\ \frac{5}{2}x - 9 = 1,\ x^2 + 1 = 5$ and $\frac{x}{3} + 5 = \frac{x}{2} - 3$ are equations in one variable x.

(ii) $4x + 3y = 21,\ 3x - \frac{y}{5} = 9$ are equations in two variables x and y.

2. LINEAR EQUATION IN ONE VARIABLE

An equation involving only one variable and highest degree of variable is one is called linear equation in one variable.

Example 1

$3x - 2 = 7,\ \frac{3}{2}x + 9 = \frac{1}{2},\ \frac{y}{3} + \frac{y-2}{4} = 5$ are linear equations in one variable, because the highest power of the variable in each equation is one whereas the equations $3x^2 - 2x + 1 = 0,\ y^2 - 1 = 8$ are not linear equations, because the highest power of the variable in each equation is not one.

3. SOLUTION OF A LINEAR EQUATION

A value of the variable which when substituted for the variable in an equation, makes L.H.S. = R.H.S. is said to satisfy the equation and is called a solution of the equation. In other words, a value of the variable which makes the equation a true statement, is called a solution of the equation.

Example 1

Verify that $x = 4$ is a solution of the equation $2x - 3 = 5$.

Solution :

Substituting $x = 4$ in the given equation, we get

L.H.S. = $2x - 3 = 2 \times 4 - 3 = 8 - 3 = 5$ = R.H.S.

Hence, $x = 4$ is the solution of the equation $2x - 3 = 5$.

Example 2

Verify that $x = 8$ is a solution of the equation $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$.

Solution :

Substituting $x = 8$ in the given equation, we get

L.H.S. = $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{5\times8-4}{8} - \frac{8-3}{5} = \frac{36}{8} - \frac{5}{5} = \frac{9}{2} - 1 = \frac{9-2}{2} = \frac{7}{2}$ and R.H.S. = $\frac{x+6}{4} = \frac{8+6}{4} = \frac{14}{4} = \frac{7}{2}$

Thus, for x = 8, we have L.H.S. = R.H.S.

Hence, x = 8 is a solution of the given equation.

4. METHODS FOR SOLVING LINEAR EQUATIONS IN ONE VARIABLE

Solving an equation means determining the value of x i.e., determining the value of the variable which satisfies the equation :

(i) Balancing Method (ii) Transposition Method

(iii) Cross Multiplication Method

4.1 BALANCING METHOD

Following rules can be used for balancing an equation :

Rule – I : Same quantity (number) can be added to both sides of an equation without changing the equality.

Rule – II : Same quantity can be subtracted from both sides of an equation without changing the equality.

Rule – III : Both sides of an equation may be multiplied by the same non-zero number without changing the equality.

Rule – IV : Both sides of an equation may be divided by the same non-zero number without changing the equality.

Example 1

Solve the equation $\frac{x}{5}+11=\frac{1}{15}$ and check the result.

Solution :

We have, $\frac{x}{5}+11=\frac{1}{15} \Rightarrow \frac{x}{5}+11-11=\frac{1}{15}-11$ [Subtracting 11 from both sides]

$\Rightarrow \frac{x}{5}=\frac{1}{15}-11 \Rightarrow \frac{x}{5}=\frac{1-165}{15} \Rightarrow \frac{x}{5}=-\frac{164}{15} \Rightarrow 5\times\frac{x}{5}=5\times\left(-\frac{164}{15}\right)$ [Multiplying both sides by 5]

$\Rightarrow x=-\frac{164}{3}$. Thus, $x=-\frac{164}{3}$ is the solution of the given equation.

Check : Substituting $x=\frac{-164}{3}$ in the given equation, we get

L.H.S. $=\frac{x}{5}+11=\frac{-164}{3}\times\frac{1}{5}+11=\frac{-164}{15}+11=\frac{-164+165}{15}=\frac{1}{15}$ and R.H.S. $=\frac{1}{15}$

$\therefore$ L.H.S. = R.H.S. for $x=\frac{-164}{3}$

Hence, $x=\frac{-164}{3}$ is the solution of the given equation.

Example 2

Solve $\frac{1}{3}x - \frac{5}{2} = 6$ and check the result.

Solution :

We have, $\frac{1}{3}x - \frac{5}{2} = 6$

$\Rightarrow \frac{1}{3}x - \frac{5}{2} + \frac{5}{2} = 6 + \frac{5}{2}$ [Adding $\frac{5}{2}$ on both sides]

$\Rightarrow \frac{1}{3}x = 6 + \frac{5}{2} \Rightarrow \frac{1}{3}x = \frac{12+5}{2} \Rightarrow \frac{1}{3}x = \frac{17}{2} \Rightarrow 3 \times \frac{1}{3}x = 3 \times \frac{17}{2}$ [Multiplying both sides by 3]

$\Rightarrow x = \frac{51}{2}$

Thus, $x = \frac{51}{2}$ is the solution of the given equation.

Check : Substituting $x = \frac{51}{2}$ in the given equation, we get

L.H.S. $= \frac{1}{3} \times \frac{51}{2} - \frac{5}{2} = \frac{17}{2} - \frac{5}{2} = \frac{12}{2} = 6$ and R.H.S. = 6

$\therefore$ L.H.S. = R.H.S. for $x = \frac{51}{2}$

Hence, $x = \frac{51}{2}$ is the solution of the given equation.

4.2 TRANSPOSITION METHOD

Any term of an equation may be taken to the other side with a change in its sign. This process is called transposition.

The transposition method involves the following steps :

Step – I : Obtain the linear equation.

Step – II : Identify the variable (unknown quantity) and constants (numbers).

Step – III : Simplify the L.H.S. and R.H.S. to their simplest forms by removing brackets.

Step – IV : Transpose all terms containing variable on L.H.S. and constant terms on R.H.S. The sign of the terms will change in shifting them from L.H.S. to R.H.S. and vice-versa.

Step – V : Simplify L.H.S. and R.H.S. in the simplest form so that each side contains just one term.

Step – VI : Solve the equation obtained in Step V dividing both sides by the coefficient of the variable on L.H.S.

Example 1

Solve for x : $5x - 2 = 3x - 4$

Solution :

$5x - 2 = 3x - 4 \Rightarrow 5x - 3x = -4 + 2$ (Collecting numbers and variables)

$\Rightarrow 2x = -2 \Rightarrow x = \frac{-2}{2} = -1 \therefore x = -1$

Example 2

Solve for y : $2(y + 3) + 3(y + 1) = 4(2y - 3) + 3$

Solution :

$2(y + 3) + 3(y + 1) = 4(2y - 3) + 3$

$\Rightarrow 2y + 6 + 3y + 3 = 8y - 12 + 3$

$\Rightarrow 2y + 3y - 8y = -12 + 3 - 6 - 3$ (Collecting the variables and numbers)

$\Rightarrow -3y = -18$

$\Rightarrow \frac{-3y}{-3} = \frac{-18}{-3} = 6$ [Dividing both sides by – 3]

$\therefore y = 6$

Example 3

Solve for y : $\frac{y+6}{4} + \frac{y-3}{5} = \frac{5y-4}{8}$

Solution :

$\frac{y+6}{4} + \frac{y-3}{5} = \frac{5y-4}{8}$

$\Rightarrow 10(y + 6) + 8(y - 3) = 5(5y - 4)$ [Multiplying throughout by 40, the LCM of 4, 5 and 8]

$\Rightarrow 10y + 60 + 8y - 24 = 25y - 20$

$\Rightarrow 25y - 10y - 8y = 60 - 24 + 20$ [Collecting variables and numbers]

$\Rightarrow 7y = 56 \Rightarrow \frac{7y}{7} = \frac{56}{7} = 8$ [Dividing both sides by 7]

$\therefore y = 8$

Example 4

Solve for y : $0.18(5y - 4) = 0.5y + 0.8$

Solution :

$0.18 \times 5y - 0.18 \times 4 = 0.5y + 0.8$

$\Rightarrow 0.90y - 0.72 = 0.5y + 0.8$

$\Rightarrow 0.90y - 0.5y = 0.8 + 0.72$ [Collecting variables and numbers]

$\Rightarrow 0.4y = 1.52 \Rightarrow \frac{0.4}{0.4}y = \frac{1.52}{0.4} = \frac{15.2}{4} = 3.8$ [Dividing both sides by 0.4]

$\therefore\ y = 3.8$

4.3 CROSS-MULTIPLICATION METHOD

Step – I : Let the equation be of the form : $\frac{ax+b}{cx+d} = \frac{m}{n}$

Consider the equation $\frac{3x+2}{9x+4} = \frac{3}{5}$, where a = 3, b = 2, c = 9, d = 4, m = 3 and n = 5.

Step – II : Equation is obtained directly by equating the product of numerator of L.H.S. and denominator of R.H.S. to the product of denominator of L.H.S. and numerator of R.H.S.

This can be exhibited as follows : $\frac{3x+2}{9x+4} = \frac{3}{5}$

This process of multiplying the numerator of L.H.S. with the denominator of R.H.S. and equating it to the product of the denominator of L.H.S. with the numerator of R.H.S. is called cross-multiplication.

So, we can convert an equation of the form $\frac{ax+b}{cx+d} = \frac{m}{n}$ to a linear equation n(ax + b) = m(cx + d).

Example 1

Solve $\frac{5x-7}{3x} = 2$ and check your solution.

Solution :

$\frac{5x-7}{3x} = 2 \Rightarrow \frac{5x-7}{3x} = \frac{2}{1}$

$\Rightarrow (5x - 7) = 2 \times 3x \Rightarrow 5x - 7 = 6x$

$\Rightarrow 5x - 6x = 7 \Rightarrow -x = 7$ or $x = -7$

Check : When x = – 7

L.H.S. = $\frac{5(-7)-7}{3(-7)} = \frac{-35-7}{-21} = \frac{-42}{-21} = 2$ = R.H.S.

Example 2

Solve : $\frac{3x+5}{2x+1} = \frac{1}{3}$.

Solution :

$\frac{3x+5}{2x+1} = \frac{1}{3} \Rightarrow 3(3x + 5) = (2x + 1)$

$\Rightarrow 9x + 15 = 2x + 1 \Rightarrow 9x - 2x = 1 - 15 \Rightarrow 7x = -14 \Rightarrow x = \frac{-14}{7} = -2.$

Hence, x = – 2.

Example 3

Solve : $\frac{2x-1}{x+3} + \frac{1-2x}{x-3} = \frac{4-3x}{x^2-9}$

Solution :

$\frac{(2x-1)(x-3)+(1-2x)(x+3)}{x^2-9} = \frac{4-3x}{x^2-9}$ [Using the identity (a + b) (a – b) = $a^2 - b^2$]

$\Rightarrow \frac{[2x^2-6x-x+3]+[x+3-2x^2-6x]}{x^2-9} = \frac{4-3x}{x^2-9}$

$\Rightarrow -12x + 3x = 4 - 6 \Rightarrow -9x = -2 \Rightarrow x = \frac{2}{9}$. Hence, $x = \frac{2}{9}$.

$\Rightarrow 2x^2 - 7x + 3 + 3 - 2x^2 - 5x = 4 - 3x \Rightarrow -12x + 6 = 4 - 3x$

5. APPLICATIONS OF LINEAR EQUATIONS TO PRACTICAL PROBLEMS

A word problem is first translated in the form of an equation containing unknown quantities (variables) and known quantities (numbers or constants) and then we solve it by using any one of the methods. The procedure to translate a word problem in the form of an equation is known as the formulation of the problem. Thus, the process of solving a word problem consists of two parts, namely, formulation and solution.

The following steps should be followed to solve a word problem :

Step – I : Read the problem thoroughly.

Step – II : Note what is given and what needs to be find out.

Step – III : Denote the unknown quantity (i.e., the value to be found) with any (variable), say x, y, z etc.

Step – IV : Translate the statements of the given problem into algebraic equation.

Step – V : Solve the equation for the unknown.

Step – VI : The solution of the equation becomes the value of the unknown.

Example 1

The perimeter of a rectangle is 13 cm and its width is $2\frac{3}{4}$ cm. Find its length.

Solution :

Assume the length of the rectangle to be x cm. The perimeter of the rectangle = 2 × (length + width)

$= 2 \times \left(x + 2\frac{3}{4}\right) = 2\left(x + \frac{11}{4}\right)$

The perimeter is given to be 13 cm. Therefore, $2\left(x+\frac{11}{4}\right)=13 \Rightarrow x+\frac{11}{4}=\frac{13}{2}$

$\Rightarrow x=\frac{13}{2}-\frac{11}{4} \Rightarrow x=\frac{26}{4}-\frac{11}{4}=\frac{15}{4}=3\frac{3}{4}$

$\therefore$ The length of the rectangle is $3\frac{3}{4}$ cm.

Example 2

Divide 34 into two parts in such a way that $\left(\frac{4}{7}\right)^{th}$ of one part is equal to $\left(\frac{2}{5}\right)^{th}$ of the other.

Solution :

Let one part be x. Then, other part is (34 – x). It is given that, $\left(\frac{4}{7}\right)^{th}$ of one part = $\left(\frac{2}{5}\right)^{th}$ of the other part

$\Rightarrow \frac{4}{7}x=\frac{2}{5}(34-x) \Rightarrow 20x=14(34-x) \Rightarrow 20x=14\times 34-14x \Rightarrow 20x+14x=14\times 34$

$\Rightarrow 34x=14\times 34 \Rightarrow x=\frac{14\times 34}{34} \Rightarrow x=14$

Hence, the two parts are x = 14 and (34 – x) = 34 – 14 = 20.

SOLVED EXAMPLES

SE. 1

Solve for x : $5x+\frac{7}{2}=\frac{3}{2}x-14$.

Ans. Taking L.C.M. both sides, we get

$$\frac{10x+7}{2}=\frac{3x-28}{2} \Rightarrow 10x + 7 = 3x - 28$$

$$\Rightarrow 10x - 3x = -28 - 7 \Rightarrow 7x = -35$$

$$\Rightarrow x=\frac{-35}{7} \Rightarrow x = -5$$

SE. 2

Solve for x : $5x - 2(2x - 7) = 2(3x - 1) + \frac{7}{2}$

Ans. $5x - 4x + 14 = 6x - 2 + \frac{7}{2}$

$$\Rightarrow x + 14 = 6x - 2 + \frac{7}{2}$$

$$\Rightarrow x - 6x = -2 + \frac{7}{2} - 14$$

$$\Rightarrow -\frac{5x}{1}=\frac{-16}{1}+\frac{7}{2}\Rightarrow\frac{-5x}{1}=\frac{-32+7}{2}$$

$$\Rightarrow\frac{-5x}{1}=\frac{-25}{2}\Rightarrow x=\frac{-25}{2\times(-5)}$$

$$\Rightarrow x=\frac{-25}{-10}\Rightarrow x=\frac{5}{2}$$

SE. 3

Solve for x : $6(3x + 2) - 5(6x - 1) = 6(x - 3) - 5(7x - 6) + 12x$.

Ans. $6(3x + 2) - 5(6x - 1)$

$= 6(x - 3) - 5(7x - 6) + 12x$.

$\Rightarrow 18x + 12 - 30x + 5$

$= 6x - 18 - 35x + 30 + 12x$

$\Rightarrow 18x - 30x - 6x + 35x - 12x$

$= -18 + 30 - 12 - 5$

$\Rightarrow 18x + 35x - 30x - 6x - 12x = 30 - 35$

$\Rightarrow 53x - 48x = -5 \Rightarrow 5x = -5 \Rightarrow x = -1.$

SE. 4

Solve for y : $0.3y + 0.4 = 0.28y + 1.16$

Ans. $0.3y + 0.4 = 0.28y + 1.16$

$\Rightarrow 0.30y - 0.28y = 1.16 - 0.40$

$$\Rightarrow 0.02y = 0.76 \Rightarrow y=\frac{0.76}{0.02}=\frac{76}{2}=38$$

SE. 5

Solve $\dfrac{\frac{2}{3}x+1}{x+\frac{1}{4}}=\frac{5}{3}$ and check your solution.

Ans. By cross multiplying the given equation, we get

$$3\left(\frac{2}{3}x+1\right)=5\left(x+\frac{1}{4}\right)\Rightarrow 2x+3=5x+\frac{5}{4}$$

$$\Rightarrow 2x-5x=\frac{5}{4}-3\Rightarrow -3x=\frac{5-12}{4}$$

$$\Rightarrow -3x=\frac{-7}{4}\Rightarrow x=\frac{7}{4}\times\frac{1}{3}\Rightarrow x=\frac{7}{12}$$

Check : At $x=\frac{7}{12}$,

$$\text{L.H.S.}=\frac{\frac{2}{3}\left(\frac{7}{12}\right)+1}{\frac{7}{12}+\frac{1}{4}}=\frac{\frac{7}{18}+1}{\frac{7+3}{12}}=\frac{\frac{25}{18}}{\frac{10}{12}}$$

$$=\frac{25}{18}\times\frac{12}{10}=\frac{5}{3}=\text{R.H.S.}$$

SE. 6

Solve for x : $\dfrac{(x+2)(2x-3)-2x^2+6}{x-5}=2$ and check your solution.

Ans. $\dfrac{2x^2 - 3x + 4x - 6 - 2x^2 + 6}{x - 5} = 2$

$\Rightarrow \dfrac{x}{x-5} = \dfrac{2}{1}$ ....(i)

By cross multiplying, we get $x = 2(x - 5)$

$\Rightarrow x = 2x - 10 \Rightarrow x - 2x = -10 \Rightarrow x = 10$

Check : Putting x = 10 in equation (i), we get

L.H.S. $= \dfrac{10}{10-5} = \dfrac{10}{5} = 2$ = R.H.S.

SE. 7

Solve for y : $\dfrac{y-1}{3} - \dfrac{y-2}{4} = 1$

Ans. We have, $\dfrac{y-1}{3} - \dfrac{y-2}{4} = 1$

L.C.M. of denominators 3 and 4 on L.H.S. is 12.

So, $4(y - 1) - 3(y - 2) = 12$

$\Rightarrow 4y - 4 - 3y + 6 = 12$

$\Rightarrow 4y - 3y - 4 + 6 = 12$

$\Rightarrow y + 2 = 12 \Rightarrow y = 10$

Thus, y = 10 is the solution of the given equation.

SE. 8

Solve for x : $\dfrac{2}{5x} - \dfrac{5}{3x} = \dfrac{1}{15}$

Ans. We have, $\dfrac{2}{5x} - \dfrac{5}{3x} = \dfrac{1}{15}$

$\Rightarrow \dfrac{1}{x}\left[\dfrac{2}{5} - \dfrac{5}{3}\right] = \dfrac{1}{15} \Rightarrow \dfrac{6-25}{15} = \dfrac{x}{15}$

($\because$ L.C.M. of 5 and 3 is 15)

$\Rightarrow 6 - 25 = x \Rightarrow x = -19.$

Hence, x = – 19 is the solution of the given equation.

SE. 9

Solve for x : $\dfrac{17-3x}{5} - \dfrac{4x+2}{3} = 5 - 6x + \dfrac{7x+14}{3}$

Ans. $3(17 - 3x) - 5(4x + 2) = 15(5 - 6x) + 5(7x + 14)$

(Taking L.C.M. of 5 and 3)

$\Rightarrow 51 - 9x - 20x - 10 = 75 - 90x + 35x + 70$

$\Rightarrow 41 - 29x = 145 - 55x$

$\Rightarrow -29x + 55x = 145 - 41$

$\Rightarrow 26x = 104 \Rightarrow x = \dfrac{104}{26} \Rightarrow x = 4$

Thus, x = 4 is the solution of the given equation.

SE. 10

Solve for x : $\dfrac{3x+4}{x+1} = \dfrac{3x+2}{x-1}, x \neq 1, x \neq -1.$

Ans. $(x - 1)(3x + 4) = (x + 1)(3x + 2)$

(By cross multiplication)

$\Rightarrow 3x^2 - 3x + 4x - 4 = 3x^2 + 3x + 2x + 2$

$\Rightarrow 3x^2 + x - 4 = 3x^2 + 5x + 2$

$\Rightarrow 3x^2 + x - 3x^2 - 5x = 2 + 4 \Rightarrow -4x = 6$

$\Rightarrow x = \dfrac{-6}{4} \Rightarrow x = \dfrac{-3}{2}$

Hence, $x = \dfrac{-3}{2}$ is the solution of the given linear equation.

SE. 11

Solve for x : $2 + \dfrac{2x-3}{2x+3} = \dfrac{3x+4}{x+2}.$

Ans. $\dfrac{2(2x+3)+(2x-3)}{2x+3} = \dfrac{3x+4}{x+2}$

$\Rightarrow \dfrac{4x+6+2x-3}{2x+3} = \dfrac{3x+4}{x+2} \Rightarrow \dfrac{6x+3}{2x+3} = \dfrac{3x+4}{x+2}$

$\Rightarrow (6x + 3)(x + 2) = (3x + 4)(2x + 3)$

(By cross multiplication)

$\Rightarrow 6x^2 + 3x + 12x + 6 = 6x^2 + 8x + 9x + 12$

$\Rightarrow 6x^2 + 15x + 6 = 6x^2 + 17x + 12$

$\Rightarrow 6x^2 + 15x - 6x^2 - 17x = 12 - 6 \Rightarrow -2x = 6$

$\Rightarrow x = -3$

SE. 12

The difference between two positive integers is 50. The ratio of these integers is 1 : 3. Find these integers.

Ans. Let the one integers be x, so the greater integer is x + 50.

According to question, $\frac{x}{x+50} = \frac{1}{3}$

$\Rightarrow 3 \times x = (x + 50)$ (By cross multiplication)

$\Rightarrow 3x = x + 50 \Rightarrow 2x = 50 \Rightarrow x = 25$

Thus, two integers are x = 25 and x + 50 = 75.

SE. 13

The sum of the digits of a two-digit number is 15. If the number formed by reversing the digits is less than the original number by 27. Find the original number.

Ans. Let the digit at unit's place be x. Then, the digit at ten's place = (15 – x)

So, original number = 10 × (15 – x) + x = (150 – 9x). On reversing the digits, we have x at the ten's place and (15 – x) at the unit's palce.

So, the new number = 10x + (15 – x) = (9x + 15).

Now, (original number) – (new number) = 27

$\Rightarrow (150 - 9x) - (9x + 15) = 27$

$\Rightarrow 135 - 18x = 27 \Rightarrow 18x = (135 - 27)$

$\Rightarrow x = \left(\frac{108}{18}\right) = 6$.

$\therefore$ Unit's digit = x = 6 and ten's digit = (15 – x) = (15 – 6) = 9.

Hence, the original number is 96.

SE. 14

The length of a rectangle exceeds its breadth by 9 cm. If the length and breadth are each increased by 3 cm, the area of the new rectangle will be 84 cm² more than that of the given rectangle. Find the length and breadth of the given rectangle.

Ans. Let the breadth of the given rectangle be x cm.

Then its length = (x + 9) cm.

Area of the given rectangle = [x(x + 9)] cm².

Now, new breadth = (x + 3) cm and new length = [(x + 9) + 3] cm = (x + 12) cm.

$\therefore$ New area = (x + 3) (x + 12) cm².

Now, (Area of new rectangle) – (Area of given rectangle) = 84 cm²

$\Rightarrow (x + 12)(x + 3) - x(x + 9) = 84$

$\Rightarrow (x^2 + 15x + 36) - (x^2 + 9x) = 84$

$\Rightarrow 6x + 36 = 84 \Rightarrow 6x = 48 \Rightarrow x = \left(\frac{48}{6}\right) = 8$

Thus, breadth = 8 cm and length = (8 + 9) cm = 17 cm.

SE. 15

After 12 years I shall be 3 times as old as I was 4 years ago. Find my present age.

Ans. Let my present age be x years.

After 12 years, my age will be (x + 12) years.

4 years ago, my age was (x – 4) years.

It is given that after 12 years I shall be 3 times as old as I was 4 years ago.

$\therefore x + 12 = 3(x - 4) \Rightarrow x + 12 = 3x - 12$

$\Rightarrow x - 3x = -12 - 12 \Rightarrow -2x = -24 \Rightarrow x = 12$

Thus, my present age is 12 years.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. The value of x, if $13x + 7 = -9 - 3x$ is :

(A) 8 (B) $\frac{1}{5}$

(C) $\frac{1}{8}$ (D) – 1

2. If $-8 + n = \frac{5n}{4}$, then the value of n =

(A) – 40 (B) – 32

(C) – 16 (D) – 4

3. Given that $-0.3k + 2.1 = 0.4k$, the value of k =

(A) 21 (B) 7

(C) 3 (D) – 1

4. If $\frac{1}{7} + \frac{x}{7} = 3$, then the value of x =

(A) 20 (B) 7

(C) 3 (D) 1

5. Given that $\frac{-6p-9}{3} = \frac{2p+9}{5}$, the value of p is

(A) – 4 (B) – 2

(C) 3 (D) 5

6. When a certain number m is divided by 5 and added to 8, the result is equal to 3m subtracted from 4. The value of m is

(A) 2 (B) $\frac{4}{3}$

(C) $\frac{-1}{3}$ (D) $\frac{-5}{4}$

7. Kriti scored 80 marks in her Science test which is $\frac{5}{6}$ of the total marks. The total marks of the test are

(A) 96

(B) 100

(C) 104

(D) 108

8. If the total of four consecutive odd numbers is 40, then smallest number is

(A) 7 (B) 9

(C) 11 (D) 13

9. If p is added to – 5, then the result is 7. The value of 3p is

(A) 6 (B) 18

(C) 36 (D) 4

10. The denominator of a rational number is greater than its numerator by 3. If 3 is subtracted from the numerator and 2 is added to its denominator the new number becomes $\frac{1}{5}$. The original number is

(A) $\frac{-5}{8}$ (B) $\frac{5}{8}$

(C) $\frac{3}{8}$ (D) $\frac{-3}{8}$

11. Solve for x : $\frac{6x^2+13x-4}{2x+5} = \frac{12x^2+5x-2}{4x+3}$

(A) 3 (B) 2

(C) 1 (D) 4

12. Twenty years ago, Tina's age was one-third of what it is now. Tina's present age is

(A) 66 years (B) 30 years

(C) 33 years (D) 36 years

13. A number consists of two digits whose sum is 12. If 18 is added to the number, its digits get reversed. The number is

(A) 28 (B) 35

(C) 57 (D) 34

14. The number which should be added to twice the rational number $\frac{-7}{3}$ to get $\frac{3}{7}$ is

(A) $\frac{105}{21}$ (B) $\frac{100}{21}$

(C) $\frac{107}{21}$ (D) $-\frac{89}{21}$

15. Madhulika thought of a number, doubled it and added 20 to it. On dividing the resulting number by 25, she gets 4. What is the number ?
(A) 80 (B) 20
(C) 40 (D) 50

16. Two numbers are in the ratio 3 : 5. If the sum of the numbers is 184, find the difference between the numbers.
(A) 24 (B) 28
(C) 48 (D) 46

17. Divide the share 64 between Seeta and Geeta such that 3 times Seeta's share is greater than 4 times Geeta's share by 10. What is Geeta's share ?
(A) 26 (B) 38
(C) 36 (D) 28

18. Solve for x : $\frac{2x-(7-5x)}{9x-(3+4x)}=\frac{7}{6}$
(A) 3 (B) 2
(C) – 3 (D) $\frac{1}{3}$

19. Three-fourths of a number is 60 more than its one-third. The number is
(A) 108 (B) 84
(C) 144 (D) 116

20. If 20 is added to four times a certain number, the result is 5 less than five times the number. The number is
(A) 10
(B) 15
(C) 20
(D) 25

21. Solve for x : $\frac{2x-7}{4}-\left(\frac{1-x}{6}\right)=17$
(A) $\frac{8}{227}$ (B) $\frac{227}{8}$
(C) 227 (D) 8

22. In a two digit number, the number at ten's place is double of the number at unit's place. If we exchange the numbers mutually then the number decreases by 18, then the number is
(A) 24 (B) 36
(C) 39 (D) 42

23. Find the number whose fifth part increased by 5 is equal to its fourth part diminished by 5.
(A) 200 (B) 300
(C) 100 (D) 400

24. The father is 24 years older than his daughter. In 4 years, he will be thrice as old as his daughter. Find the present age of the daughter.
(A) 10 years (B) 7 years
(C) 9 years (D) 8 years

PARAGRAPH TYPE

PASSAGE # I

A rectangle has a perimeter of 60 cm.

1. If length and breadth of rectangle is (k + 4) cm and (3k – 2) cm respectively, then value of k is
(A) 7 (B) 9
(C) 12 (D) 29

2. The length and breadth of the rectangle is
(A) 11, 7 (B) 11, 19
(C) 19, 20 (D) 23, 11

3. If perimeter of rectangle is equal to perimeter of square, then side of square is
(A) 60 cm (B) 18 cm
(C) 225 cm (D) 15 cm

PASSAGE # II

Linear equation in one variable is an equation of the form ax + b = 0 or ax = – b, where a, b are the real numbers such that a ≠ 0. The value of variable which satisfies a given linear equation is known as its solution. If ax = – b is linear equation, then $x = \frac{-b}{a}$ is its solution. Geometrically, a linear equation is represented by a straight line. Based on above passage, answer the following questions:

4. ax + b = 0 represents an equation where a ≠ 0. If a = 0, then it is
(A) A linear equation
(B) An algebraic expression
(C) No more an equation
(D) None of these

5. The number of solutions of a linear equation in one variable is/are
(A) One (B) Three
(C) Two (D) Cannot be determined

6. Geometrical representation of a linear equation is
(A) Any curve (B) A straight line
(C) Circle (D) Triangle

TRUE / FALSE TYPE

1. An equation, in which the maximum degree of a term is one, is called a linear equation.
2. We cannot subtract the same number on both sides of the equation.
3. We can multiply both sides of the equation by same non zero number.
4. We can divide both sides of the equation by same non-zero number.
5. If we transpose any term of the equation from one side to other, its sign gets changed.
6. 8x – 3 = 25 + 174 then x is a rational number.
7. A linear equation in one variable has two solution.
8. In a linear equation the highest power of the variable appearing in the equation is one.
9. If $\frac{x}{11} = 15$ then $x = \frac{11}{15}$.
10. If 6x = 18 then 18x = 54.

MATCH THE COLUMN TYPE

1. Match the following :

List – I	List – II
(P) $\frac{2}{3}$ of a number is 20 less than the original number, then the number is	(i) 30
(Q) Four-fifth of a number is 10 more than two-third of that number, then the number is	(ii) 200
(R) A number whose double is 45 greater than its half, then the number is	(iii) 75
(S) A number whose fifth part increased by 5 is equal to its fourth part diminished by 5, then the number is	(iv) 60

(A) (P)→(iii), (Q)→(iv), (R)→(i), (S)→(ii)
(B) (P)→(iv), (Q)→(iii), (R)→(ii), (S)→(i)
(C) (P)→(iv), (Q)→(iii), (R)→(i), (S)→(ii)
(D) (P)→(i), (Q)→(ii), (R)→(iv), (S)→(iii)

2. Match the following :

List – I	List – II
(P) An expression in variable x is	(i) 2y = 0
(Q) An equation with negative root is	(ii) 7x + 9 = 23
(R) An equation with positive root is	(iii) $\frac{x}{5} + \frac{1}{2} = \frac{-2}{3}$
(S) An equation with zero root is	(iv) $\frac{x}{2} - 6$

(A) (P)→(iv), (Q)→(iii), (R)→(ii), (S)→(i)

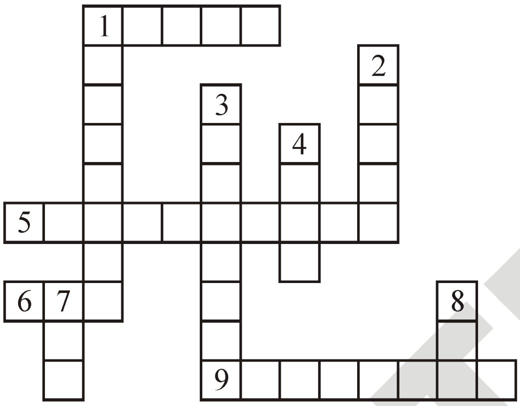
(B) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)

(C) (P)→(iii), (Q)→(i), (R)→(ii), (S)→(iv)

(D) (P)→(iv), (Q)→(ii), (R)→(iii), (S)→(i)

CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

1. Solving an equation means determining the ____ of the variable which satisfies it. [5]
5. While going upstream in a river by a boat, speed of the stream is ______ from the speed of the boat. [10]
6. In a linear equation, the maximum power of the variable is _______. [3]
9. A mathematical statement of equality which contains one or more variable is called an _____. [8]

Down

1. A linear equation in one _____ has one and only one solution. [8]
2. While going downstream in a river by a boat, the speed of the stream is ______ to the speed of the boat. [5]
3. When we transpose positive quantity from one side of equality to the other side, then the sign of the positive quantity changes to _______. [8]
4. The sign of equality will not change if we add the same quantity to ______ sides of the equation. [4]
7. $3x^3 + 5x^2 + 7x + 3 = 0$ is ______ a linear equation. [3]
8. $3x + 4y = 12$ contains ______ different variables. [3]

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Solve for x : $5x - 17 = 2x - 8$
2. Solve : $7(x + 1) - 9 = 16$.
3. Find the solution of $\frac{2(x+1)}{3} = 18(x-1)$.
4. Solve for x : $\frac{2x+4}{3x+6} = 4$
5. Solve : $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 7$
6. If 3 less than a number is 10. Find the number.
7. Find the length of a rectangle having breadth 2 metres and area 24 square metres.
8. Salim thought of a number, doubles it and subtracted 11 from it. The result was 59. Find the number.
9. Two numbers are in the ratio 5 : 8. If the sum of numbers is 182, then find the numbers.
10. Solve : $\frac{2x+3}{3x+4} = \frac{14}{20}$

SHORT ANSWER TYPE

1. Solve for x : $0.16(5x - 2) = 0.4x + 7$
2. Find the value of x if $\frac{x+b}{a-b} = \frac{x-b}{a+b}$.
3. Find the value of x if $\frac{x}{4} + \frac{x}{6} = x - 7$.
4. If $x = p - 2$ and $\frac{x}{p} - \frac{x+1}{p} = 1$, then find p.
5. Solve for x : $\frac{6x+7}{3x+2} = \frac{4x+5}{2x+3}$
6. Two years ago, Dilip was three times as old as his son and two years hence, twice his age will be equal to five times that of his son. Find their present ages.
7. A father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son. What are their present ages ?
8. Raj is three years older than Ravi. Six years ago, Raj's age was four times of Ravi's age. What are their present ages ?

LONG ANSWER TYPE

1. Three prizes are to be distributed in a quiz contest. The value of the second prize is five-sixths the value of the first prize and the value of the third prize is four-fifths that of the second prize. If the total value of three prizes is Rs. 150, then find the value of each prize.
2. Solve for x : $\frac{(4+x)(5-x)}{(2+x)(7-x)} = 1$
3. Solve for x : $\frac{1}{x+1} + \frac{1}{x+2} = \frac{2}{x+10}$
4. Solve for x :

   (i) $\frac{(x+2)(2x-3) - 2x^2 + 6}{x-5} = 2$

   (ii) $\frac{2-7x}{1-5x} = \frac{3+7x}{4+5x}$
5. A number is 56 greater than the average of its third, quarter and one-twelfth. Find the number.

NUMERICAL PROBLEMS

1. The sum of three consecutive integers is 33. The difference between largest and smallest integer is
2. The value of x if $5x - (3x - 1) = x - 4$ is $-k$. The value of k is
3. Twice number increased by 8 is 20. The number is

4. If $\frac{3x-4}{4x+3}=2$, then x is – a. The value of 6a is

5. The value of x, if $\frac{3}{x+8}=\frac{4}{6-x}$ is $-\frac{k}{2}$. The value of k is

6. The value of x if $\left(\frac{2x-3}{4}\right)-\left(\frac{2x-1}{3}\right)=1$ is $\frac{-k}{2}$. The value of k is

7. A is twice as old as B. Three years ago, A's age was three times as that of B. Find the age of B.

8. The difference between two positive integers is 36. The quotient, when one integer is divided by the other, is 4. The largest integer is

9. Find the value of s if 7 (s – 4) = – 3 + 2s.

10. Find the value of m if $\frac{6m+4}{4-m}=8$.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Solve for x : $\frac{2}{3}(4x-1)-\left(4x-\frac{1-3x}{2}\right)=\frac{x-7}{2}$

(A) 0 (B) 1
(C) 2 (D) 5

2. Solve for x : $\frac{2}{5}(4x-1)-\left[4x-\left(\frac{1-3x}{2}\right)\right]=\frac{x-7}{2}$

(A) 0 (B) $\frac{9}{11}$
(C) $\frac{2}{11}$ (D) – 1

3. The number '51' is divided into two parts such that $\left(\frac{4}{7}\right)^{th}$ of the first part is equal to $\left(\frac{5}{4}\right)^{th}$ of the second part. The numbers are

(A) 18, 33 (B) 29, 22
(C) 36, 15 (D) 35, 16

4. Vidhushi is 8 times as old as her grandson. Four years ago, Vidhushi was 12 times as old as her grandson. Find the difference between their present ages.

(A) 88 years (B) 77 years
(C) 11 years (D) 99 years

5. Sum of the digits of two digit number is 11. The number obtained by interchanging the digits is 27 more than the original number. The original number is :

(A) 74 (B) 47
(C) 56 (D) 65

6. State 'T' for true and 'F' for false.

(i) $y^2 + 9$ is a linear equation in one variable.
(ii) The terms linear equation and linear expression are same.
(iii) If x is an even number, then the next odd number is (2x + 1).
(iv) If both sides of an equation is to be divided by the same (non-zero) positive number, then there is a change in equality.
(v) 1 is the solution of $\frac{2x+1}{3x+5}=\frac{3}{8}$.

	(i)	(ii)	(iii)	(iv)	(v)
(A)	T	F	T	F	T
(B)	T	T	T	T	T
(C)	F	F	F	F	T
(D)	F	T	F	F	F

7. Sum of the digits of a two digit number is 9. The number obtained by interchanging the digits is 18 more than twice the original number. The original number is :

(A) 72 (B) 27
(C) 36 (D) 63

8. The tens digit of a two-digit number exceeds its unit digit by 6. The number itself is 10 times the sum of its digits. Find the number.

(A) 82 (B) 90

(C) 93 (D) 60

9. The numerator and the denominator of a rational number are in the ratio 5 : 7. When 6 is added to both the numerator and denominator, the ratio becomes 4 : 5. What is the rational number ?

(A) $\frac{7}{5}$ (B) $\frac{5}{7}$

(C) $\frac{2}{5}$ (D) $\frac{13}{14}$

10. A father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son. Father's present age is :

(A) 24 years (B) 28 years

(C) 30 years (D) 32 years

Space for Notes :

EXERCISE – III

PREVIOUS YEAR QUESTIONS (NTSE)

1. The sum of three numbers is 98. The ratio of the first to the second is $\frac{2}{3}$ and the ratio of the second to the third is $\frac{5}{8}$. The second number is :

(NSTSE – 2010)

(A) 15 (B) 20
(C) 30 (D) 32

2. What is the value of x in the given equation ?

$$\frac{(3x+1)}{16}+\frac{(2x-3)}{7}=\frac{(x+3)}{8}+\frac{(3x-1)}{14}$$

(IMO – 2010)

(A) 2 (B) 4
(C) 3 (D) 5

3. Jasmine is a much better tennis player than Reshma. They decide to have a contest. Every time Jasmine wins a game, she will earn 3 points and every time Reshma wins a game, she will earn 5 points. If they play 48 games and the final score is tied, how many games did Jasmine win ?

(IMO – 2010)

(A) 50 (B) 40
(C) 30 (D) 18

4. Madhuri is on the fourth floor of a building. Her car is in the parking garage three levels below the ground floor. She gets in the elevator and travels from the fourth floor above ground level to the third floor below ground level. How many floors did she travel ?

(IMO – 2010)

(A) 3 (B) 1
(C) 4 (D) 7

5. Mrs. Ravina needs to take a taxi to the doctor's clinic. The taxi ride costs Rs. 13.00 for the first km and Rs. 6 for each km thereafter. How much does Mrs. Ravina pay for a 2.3 km taxi ride ?

(IMO – 2010)

(A) Rs. 25 (B) Rs. 28
(C) Rs. 32 (D) Rs. 30

6. Of the three numbers, the first is twice the second and is half the third. If the average of three numbers is 56, the three numbers in order are

(Aryabhatta – 2011)

(A) 48, 24, 96 (B) 48, 36, 96
(C) 48, 12, 14 (D) 24, 12, 48

7. The sum of a two digit number and the number obtained by interchanging the digits of the number is 121. If the digits differ by 5, then find the number

(Aryabhatta – 2011)

(A) 38, 83 (B) 27, 72
(C) 39, 93 (D) 61, 16

8. The ages of Mira, Tina and Sania are in the ratio 6 : 4 : 7 respectively, if the sum of their ages is 34 years, what is Sania's age ?

(IMO – 2011)

(A) 12 years (B) 10 years
(C) 18 years (D) 14 years

9. $\left(\frac{3}{4}\right)^{th}$ of a number is 20 more than half of the same number. The required number is ________.

(IMO – 2011)

(A) 50 (B) 180
(C) 90 (D) 80

10. Mohan gets 3 marks for each correct answer and loses 2 marks for each wrong answer. He attempts 30 problems and obtains 40 marks. The number of problems solved correctly is.

(IMO – 2011)

(A) 10 (B) 15

(C) 20 (D) 25

11. Neeta's volvo bus takes 50 boys to a field trip. Some of them take Rs. 20 tickets while the rest take Rs. 45 tickets. If the total cost of tickets purchased is Rs. 2000, how many boys took the tickets of Rs. 20 each ?

(NSTSE – 2011)

(A) 7 (B) 10

(C) 12 (D) 15

12. For a journey the cost of a child ticket is 1/3rd of the cost of an adult ticket. If the cost of the tickets for 4 adults and 5 children is Rs. 85, the cost of a child ticket is

(IMO – 2012)

(A) Rs. 5

(B) Rs. 6

(C) Rs. 10

(D) Rs. 15

13. The ratio of present ages of Rahul and Deepesh is 3 : 5. 10 years later this ratio becomes 5 : 7. What is the present age of Deepesh ?

(IMO – 2012)

(A) 20 years

(B) 50 years

(C) 25 years

(D) 40 years

14. $\frac{x}{x-a}+\frac{x}{x-b}=2$ find x

(NSTSE – 2013)

(A) $\frac{a}{b}$ (B) ab

(C) $\frac{2ab}{a+b}$ (D) 2ab

15. If $x + y = 6$ and $3x - y = 4$, find the value of $x - y$.

(NSTSE – 2014)

(A) – 1 (B) 0

(C) 2 (D) 4

Answer Key

EXERCISE I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	B	C	A	B	D	A	A	C	B	C	B	C	C	C
16	17	18	19	20	21	22	23	24						
D	A	A	C	D	B	D	A	D						

PARAGRAPH

1. A **2.** B **3.** D **4.** C **5.** A **6.** B

TRUE / FALSE

1. T **2.** F **3.** T **4.** T **5.** T **6.** T **7.** F

8. T **9.** F **10.** T

MATCH THE COLUMN

1. C **2.** A

CROSSWORD PUZZLE

1. Value **2.** Added **3.** Negative **4.** Both **5.** Subtracted **6.** One

7. Not **8.** Two **9.** Equation

EXERCISE II

VERY SHORT ANSWER TYPE

1. 3 **2.** $\frac{18}{7}$ **3.** $\frac{14}{3}$ **4.** – 2 **5.** 12 **6.** 13 **7.** 12 m

8. 35 **9.** 70, 112 **10.** 2

SHORT ANSWER TYPE

1. 18.3 **2.** – a **3.** 12 **4.** – 1 **5.** $\frac{-11}{9}$ **6.** 14 yrs, 38 yrs

7. 4 yrs, 28 yrs **8.** Raj – 10 yrs, Ravi – 7 yrs

LONG ANSWER TYPE

1. Rs. 60, Rs. 50, Rs. 40 **2.** $\frac{3}{2}$ **3.** $\frac{-26}{17}$ **4.** (i)10,(ii) $\frac{1}{2}$ **5.** 72

NUMERICAL PROBLEMS

1. 2 **2.** 5 **3.** 6 **4.** 12 **5.** 4 **6.** 17 **7.** 6

8. 48 **9.** 5 **10.** 2

ANALYTICAL PROBLEMS & BRAIN TEASER

1. B **2.** B **3.** D **4.** B **5.** B **6.** C **7.** B

8. D **9.** B **10.** B

EXERCISE III

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	D	C	D	A	A	A	D	D	C	B	A	C	C	A

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : INTEGERS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

UNDERSTANDING QUADRILATERALS

3

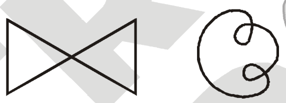
Concepts

INTRODUCTION

'Poly' means many and 'gon' means sides. So a polygon is a closed figure of many sides. A polygon of 'n' sides is also called n-gon. Polygon can be classified according to the number of sides like triangle (3 sides), Quadrilateral (4 sides). Pentagon (5 sides).

1. CURVE

A plane figure formed by joining a number of points without lifting a pencil from the paper and without retracing any portion of the drawing other than single points.

Open Curves	Closed Curves	Simple Closed Curves
A curve which does not cut itself is called an open curve. 	A figure which begins and ends at the same point is called a closed figure. 	A closed figure which does not intersect itself is called a simple closed figure. 

2. POLYGON

A simple closed curve which is made up of line segments only is called a polygon.

2.1 ELEMENTS OF A POLYGON

(i) Adjacent Sides : Any two sides with a common end-point (vertex) are called adjacent sides of the polygon.

(ii) Adjacent Vertices : The end-points of the same side of a polygon are known as adjacent vertices.

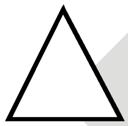
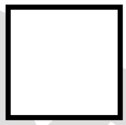
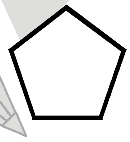
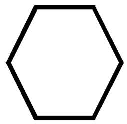
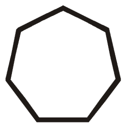
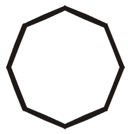
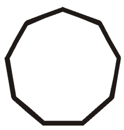
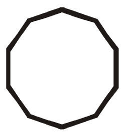
(iii) Diagonals : The line segments obtained by joining two non-adjacent vertices of a polygon are called the diagonals of the polygon.

Focus Point

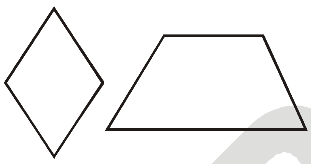
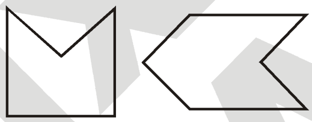
Note : The number of diagonals of a polygon with n sides is given by $\frac{n(n-3)}{2}$.

2.2 RECOGNISING AND NAMING POLYGON

We classify polygons according to the number of sides (or vertices) they have.

Number of sides	Name of polygon	Number of vertices	Number of diagonals	Figures
3	Triangle	3	0	
4	Quadri-lateral	4	2	
5	Pentagon	5	5	
6	Hexagon	6	9	
7	Heptagon	7	14	
8	Octagon	8	20	
9	Nonagon	9	27	
10	Decagon	10	35	

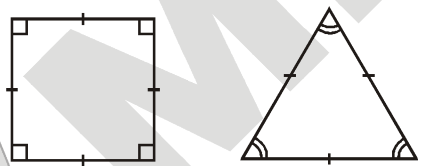
3. CONVEX AND CONCAVE POLYGON

Convex Polygon	Concave Polygon
A convex polygon is a polygon in which no line segment between the points on the boundary ever goes outside the polygon.  Convex polygons	Polygon that have portions of their atleast one diagonal lies in their exteriors is called concave polygon.  Concave polygons

4. REGULAR AND IRREGULAR POLYGONS

4.1 REGULAR POLYGON

A regular polygon is a polygon whose all sides and all angles are equal.

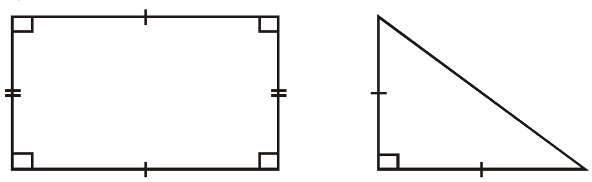


Example 1

A square, an equilateral triangle etc.

4.2 IRREGULAR POLYGON

A polygon which is not a regular polygon is called an irregular polygon.



Example 2

Rectangle, isosceles triangle etc.

Focus Point

(1) The sum of the interior angles of an n-sided polygon is $(2n - 4) \times 90°$.

(2) The sum of all the exterior angles of a polygon is 360°.

(3) Each interior angle of regular polygon of n-sided is $\left(\frac{(2n-4)90}{n}\right)^{\circ}$.

(4) Each exterior angle of a regular polygon n-sided is $\left(\frac{360}{n}\right)^{\circ}$.

(5) The number of sides n of a regular polygon whose exterior angle x° is = $\left(\frac{360°}{x}\right)$.

5. ANGLE SUM PROPERTY OF QUADRILATERAL

5.1 SUM OF THE ANGLES OF A QUADRILATERAL IS 360°

Statement : Sum of the angles of a quadrilateral is 360°.

Given : ABCD is a quadrilateral.

To prove : $\angle A + \angle B + \angle C + \angle D = 360°$

Proof : In ΔABC, $m\angle 4 + m\angle 5 + m\angle 6 = 180°$

[Using angle sum property of a triangle]

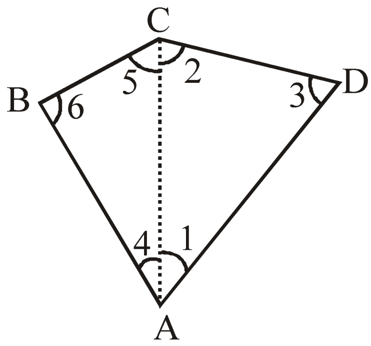
Also, in ΔADC, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Sum of the measures of ∠A, ∠B, ∠C and ∠D of quadrilateral

$m\angle 1 + m\angle 4 + m\angle 6 + m\angle 5 + m\angle 2 + m\angle 3 = 180° + 180°$

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

Thus, sum of the measures of four angles of a quadrilateral is 360°.



Example 3

Which of the following groups of angles can be the angle of a quadrilateral ?

(a) 70°, 80°, 90°, 120°

(b) 75°, 65°, 115°, 125°,

Solution :

(a) $\because$ Sum of the angles = 70° + 80° + 90° + 120° = 360°

It is a quadrilateral.

(b) $\because$ Sum of the angles = 70° + 65° + 115° + 125° = 380°, which is more than 360°

$\therefore$ It is not a quadrilateral.

Example 4

Three angles of a quadrilateral are 30°, 150° and 100°. Find the fourth angle.

Solution :

Sum of the angles of a quadrilateral = 360°

Sum of the given angles = 30° + 150° + 100° = 280°

$\therefore$ Fourth angle = 360° – 280° = 80°

Example 5

The angles of a quadrilateral are in the ratio 5 : 3 : 9 : 7. Find the angles.

Solution :

Let the angles be 5x, 3x, 9x and 7x.

$\therefore$ 5x + 3x + 9x + 7x = 360°

[Angle sum property of a quadrilateral]

$\Rightarrow$ 24x = 360°

$\Rightarrow x = \dfrac{360°}{24} = 15°$

$\therefore$ Angles are 5x = 5 × 15° = 75°, 3x = 3 × 15° = 45°

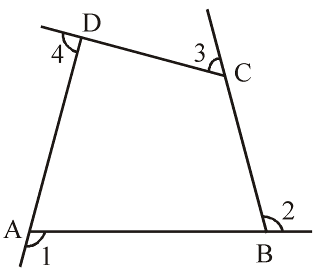
9x = 9 × 15° = 135° and 7x = 7 × 15° = 105°.

5.2 EXTERIOR ANGLE PROPERTY

Statement : If the sides of a quadrilateral are produced in order, the sum of four exterior angles so formed is 360°.

Proof : Let the sides of a quadrilateral ABCD be produced in order as shown in figure forming exterior angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$

Since $\angle 1$ and $\angle DAB$ forms a linear pair and the sum of the angles of a linear pair is 180°.



$\therefore \angle 1 + \angle DAB = 180°$ ... (i)

Similarly, we have

$\angle 2 + \angle CBA = 180°$ ... (ii)

$\angle 3 + \angle DCB = 180°$ ... (iii)

and $\angle 4 + \angle ADC = 180°$ ... (iv)

Adding (i), (ii), (iii) and (iv), we get

$(\angle 1 + \angle 2 + \angle 3 + \angle 4) + (\angle DAB + \angle CBA + \angle DCB + \angle ADC) = 180° + 180° + 180° + 180°$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 + 360° = 720°$ [$\therefore \angle DAB + \angle CBA + \angle DCB + \angle ADC = 360°$]

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 720° – 360° = 360°$

This is true whatever be the number of sides of polygon.

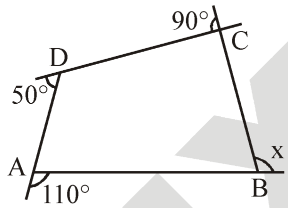
So, the statement can be generalised to :

The sum of the measures of the exterior angles of any polygon is 360°.

Remark : Each exterior angle of a regular polygon of n sides is equal to $\left(\frac{360°}{n}\right)$.

Example 6

Find the value of x in the given figure.



Solution :

We know that the sum of the measures of exterior angles of a polygon is 360°.

$\therefore$ x + 90° + 50° + 110° = 360°

$\Rightarrow$ x + 250° = 360° $\Rightarrow$ x = 360° – 250° $\Rightarrow$ x = 110°.

Example 7

How many sides does a regular polygon have, if the measure of an exterior angle is 24° ?

Solution :

Let there be n sides of the regular polygon. Then, the measure of each exterior angle is $\left(\frac{360°}{n}\right)$.

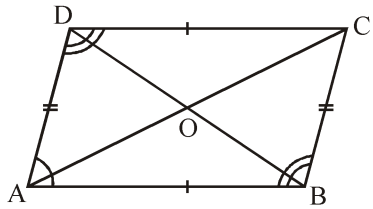
$\therefore \frac{360°}{n} = 24° \Rightarrow n = \left(\frac{360°}{24°}\right) = 15$. So, the polygon has 15 sides.

6. DIFFERENT TYPES OF QUADRILATERALS

6.1 PARALLELOGRAM

A quadrilateral is a parallelogram if its both pairs of opposite sides are equal and parallel.

Properties of a Parallelogram



In a parallelogram,

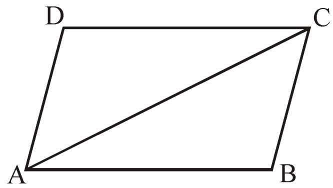
1. The opposite sides are equal i.e., AB = CD and BC = AD
2. The opposite angles are equal i.e., ∠A = ∠C and ∠B = ∠D
3. The diagonals bisect each other, i.e., AO = OC and BO = OD
4. Each diagonal divides a parallelogram into two congruent triangles.

i.e., ΔABC ≅ ΔCDA and ΔBCD ≅ ΔDAB.

Let us prove all these properties.

1. Opposite sides of a parallelogram are equal.

Let us consider a parallelogram ABCD in which AB || CD and BC || AD. Join AC.



Now, in ΔABC and ΔCDA

∠CAB = ∠ACD	[Alternate angles]
∠ACB = ∠CAD	[Alternate angles]
AC = CA	[Common]
∴ ΔABC ≅ ΔCDA	[ASA congruency]
Hence, AB = CD and BC = AD	[C.P.C.T.]

2. Opposite angles of a parallelogram are equal.

Let us consider a parallelogram ABCD in which AB || CD and BC || AD.

Now, AB || DC and AD is a transversal

∠A + ∠D = 180° ... (i)

[Sum of the measures of interior angles on the same side of a transversal is 180°]

Again, AD || BC and AB is a transversal.

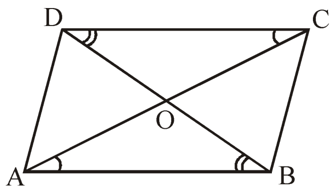
∴ ∠A + ∠B = 180° ... (ii)

From (i) and (ii), we have

∴ ∠D = ∠B

Similarly, we can prove that ∠A = ∠C.

3. Diagonals of a parallelogram bisect each other.



Let us consider a parallelogram ABCD in which AB || CD and AD || BC.

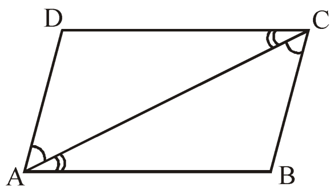
Join AC and BD to intersect at O.

In ΔAOB and ΔCOD, we have

∠DCO = ∠OAB	[Alternate angles]
∠CDO = ∠OBA	[Alternate angles]
AB = CD	[Opposite sides of a parallelogram]
∴ ΔAOB ≅ ΔCOD	[ASA congruency]
Hence, AO = OC and BO = OD	[C.P.C.T.]

4. Each diagonal of a parallelogram divides it into two congruent triangles.

Let us consider a parallelogram ABCD where AB || CD and AD || BC.



Join AC.

In ΔABC and ΔCDA, we have

∠DCA = ∠BAC	[Alternate angles]
∠DAC = ∠ACB	[Alternate angles]
AC = CA	[Common]
∴ ΔABC ≅ ΔCDA	[ASA congruency]

Similarly, by drawing the other diagonal BD, we can prove that ΔABD ≅ ΔCDB.

Example 8

If two adjacent angles of a parallelogram are equal. What is the measure of each angle ?

Solution :

Let the measure of two adjacent and equal angles be x. Since the sum of any two consecutive angles of a parallelogram is 180°.

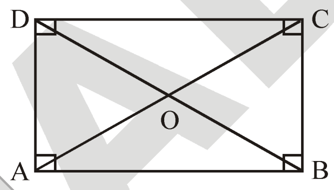
$\therefore\ x + x = 180° \Rightarrow 2x = 180° \ \Rightarrow x = \dfrac{180°}{2} = 90°$

Hence, the measure of each angle is 90°.

6.2 RECTANGLE

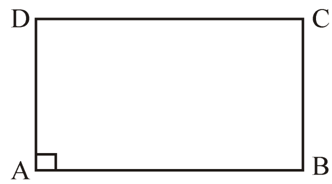
A parallelogram whose each angle is a right angle is called rectangle.

Properties of a Rectangle.



1. The opposite sides are equal i.e., AB = CD and BC = AD
2. The opposite angles are equal i.e., $\angle A = \angle C$ and $\angle B = \angle D$
3. The diagonals bisect each other, i.e., AO = OC and BO = OD
4. Each diagonal divides a parallelogram into two congruent triangles.
i.e., $\Delta ABC \cong \Delta CDA$ and $\Delta BCD \cong \Delta DAB$.
5. Each angle of a rectangle is a right angle.
6. The diagonals of a rectangle are equal.

1. Each angle of a rectangle is a right angle.



Let us consider a rectangle ABCD is which $\angle A = 90°$ ... (i)

Also, ABCD is a parallelogram.

$\therefore$ AB = DC, BC = AD and $\angle A = \angle C = \angle B = \angle D$

So, $\angle C = 90°$ [From (i)] ... (ii)

Now, AB || DC and AD is the transversal.

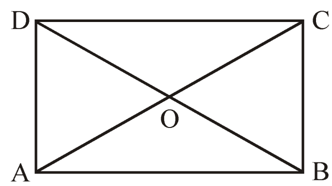
$\therefore\ \angle A + \angle D = 180° \ \Rightarrow\ \angle D = 90°$ ... (iii)

Also, $\angle B = \angle D \ \Rightarrow \angle B = 90°$ ... (iv)

Hence , $\angle A = \angle B = \angle C = \angle D = 90°$ [From (i),(ii), (iii) & (iv)]

2. The diagonals of a rectangle are equal.

Let us consider a rectangle ABCD with diagonals AC and BD.



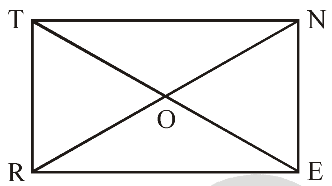
In ΔABC and ΔBAD, we have

$\angle ABC = \angle BAD$	[Right angle]
AB = AB	[Common]
AD = BC	[Opposite sides of rectangle]
So, $\Delta ABC \cong \Delta BAD$	[SAS congruency]
So, AC = BD	[C.P.C.T.]

Example 9

RENT is a rectangle. Its diagonals meets at O. Find x if OR = 2x + 3 and OT = 3x + 2.

Solution :



$OT = \frac{1}{2}$ TE [As diagonals of a rectangle bisect each other]

Also, $OR = \frac{1}{2}$ RN, Now, TE = RN

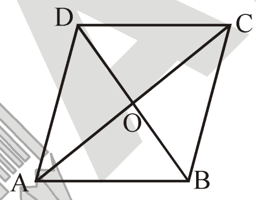
[Diagonals of a rectangle are of equal measure]

So, OT = OR $\Rightarrow 3x + 2 = 2x + 3 \Rightarrow x = 1$

6.3 RHOMBUS

A parallelogram having all sides equal is called a rhombus.

Properties of a Rhombus



1. All sides are equal i.e. AB = BC = CD = DA.
2. The opposite angles are equal i.e., $\angle A = \angle C$ and $\angle B = \angle D$
3. The diagonals bisect each other, i.e., AO = OC and BO = OD
4. Each diagonal divides a parallelogram into two congruent triangles.
i.e., $\Delta ABC \cong \Delta CDA$ and $\Delta BCD \cong \Delta DAB$.
5. Diagonals of a rhombus bisect each other at right angles.

1. Diagonals of a rhombus bisect each other at right angles.

Let us consider a rhombus ABCD with diagonals AC and BD.

In ΔAOB and ΔBOC, we have

AB = BC [Sides of a rhombus]

BO = OB [Common]

AO = OC [Diagonals bisect each other]

$\therefore \Delta AOB \cong \Delta COB$ [SSS congruency]

Hence, $\angle AOB = \angle BOC$ [C.P.C.T.]

But, $\angle AOB + \angle BOC = 180°$ [Linear pair]

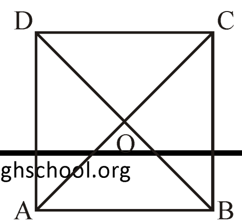
$2\angle AOB = 180°$

$\therefore \angle AOB = \angle BOC = 90°$.

6.4 SQUARE

A square is a rhombus with each of its angles equal to 90°.

Properties of a Square



1. All sides are equal i.e., AB = BC = CD = DA
2. All angles are equal i.e., $\angle A = \angle B = \angle C = \angle D$

3. The diagonals bisect each other, i.e., AO = OC and BO = OD

4. Each diagonal divides a square into two congruent triangles.

i.e., $\Delta ABC \cong \Delta CDA$ and $\Delta BCD \cong \Delta DAB$.

5. Diagonals of a rhombus bisect each other at right angles.

6. Diagonals of a square are equal.

1. Diagonals of a square are equal.

Let us consider a square ABCD with diagonals AC and BD.

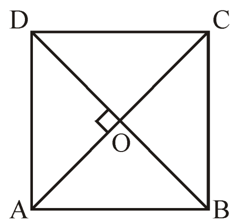
In ΔABD and ΔBAC, we have

AD = BC	[Sides of square]
AB = BA	[Common]
$\angle DAB = \angle ABC$	[Each 90°]
$\therefore \Delta ABD \cong \Delta BAC$	[SAS congruency]
AC = BD	[C.P.C.T.]

Example 10

If the length of a diagonal of a square is 6 cm. Find the length of the side of the square.

Solution :



If AC = 6 cm

$\Rightarrow$ AO = 3 cm and DO = 3 cm

As diagonals of square are perpendicular to each other. In right ΔAOD, we have

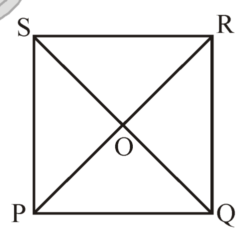
$AO^2 + DO^2 = AD^2$

[By Pythagoras Theorem]

$\Rightarrow 3^2 + 3^2 = AD^2$ $\Rightarrow 18 = AD^2$ $\Rightarrow \sqrt{18} = AD$ $\therefore AD = 3\sqrt{2}$ cm.

Example 11

PQRS is a square. PR and SQ interesect at O. State the measure of $\angle POQ$.



Solution :

Since the diagonals of a square intersect at right angles. Therefore, $\angle POQ = 90°$

6.5 KITE

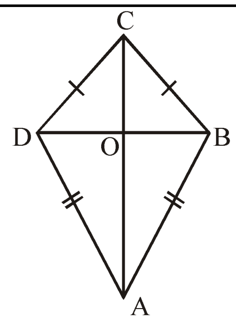
A quadrilateral in which two pairs of adjacent sides are equal is called a kite.

It has unequal opposite sides.

In the adjoining figure, ABCD is a kite.

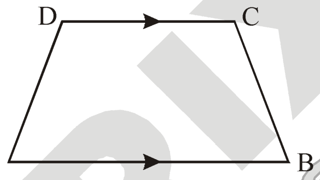
AB = AD and BC = CD

Diagonals AC and BD intersect each other at right angles.



6.6 TRAPEZIUM

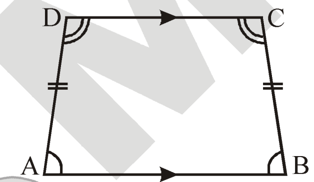
A quadrilateral in which one pair of opposite sides is parallel and the other pair of opposite sides is non-parallel is called trapezium.



In the given figure, ABCD is a trapezium in which AB || CD whereas AD and BC are its non-parallel sides. The trapezium is usually written as Trap.

6.7 ISOSCELES TRAPEZIUM

If the non-parallel sides of the trapezium (AD and BC as shown in figure) are equal, then it is called as isosceles trapezium. In the trapezium ABCD, AB || CD; AD and BC are its non-parallel sides such that AD = BC.



So, ABCD is an isosceles trapezium. In an isosceles trapezim, $\angle A = \angle B$ and $\angle C = \angle D$.

Example 12

PQRS is a trapezium in which SP || RQ, find the measures of $\angle P$ and $\angle R$.

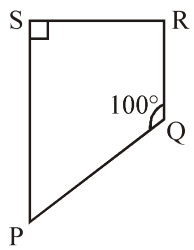
Solution :

Since PQRS is a trapezium in which SP || RQ.

$\therefore\ \angle P + \angle Q = 180°$ and $\angle S + \angle R = 180°$

$\Rightarrow \angle P + 100° = 180°$ and $90° + \angle R = 180°$

$\Rightarrow \angle P = 180° - 100° = 80°$ and $\angle R = 180° - 90° = 90°$



Example 13

ABCD is a trapezium in which AB || DC and ∠A = ∠B = 40°.

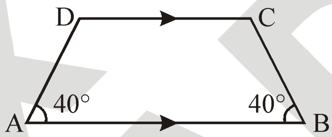
Find ∠C and ∠D. Are these angles equal ?

Solution :

AB || DC and transversal AD cuts them.

∴ ∠A + ∠D = 180°

⇒ ∠D = 180° – ∠A = 180° – 40° = 140°



Similarly, ∠B + ∠C = 180°

⇒ ∠C = 180° – ∠B = 180° – 40° = 140°

∴ ∠C = ∠D = 140°

Yes, these angles are equal.

∴ ABCD is an isosceles trapezium.

SOLVED EXAMPLES

SE. 1

The angles of a quadrilateral are 100°, 98° and 92° respectively. Find the fourth angle.

Ans. Let the measure of fourth angle be x. We know that the sum of the angles of a quadrilateral is 360°.

$\therefore$ 100° + 98° + 92° + x = 360°

$\Rightarrow$ 290° + x = 360° $\Rightarrow$ x = 360° – 290° = 70°

Hence, the measure of fourth angle is 70°.

SE. 2

The measures of two angles of a quadrilateral are 115° and 45°, and the other two angles are equal. Find the measure of each of the equal angles.

Ans. Let the measure of each of the equal angles be x. We know that the sum of all the angles of a quadrilateral is 360°

$\therefore$ 115° + 45° + x + x = 360°

$\Rightarrow$ 160° + 2x = 360°

$\Rightarrow$ 2x = (360° – 160°) = 200° $\Rightarrow$ x = 100°.

Hence, the measure of each of the equal angles is 100°.

SE. 3

One angle of a quadrilateral is 108° and the remaining three angles are equal. Find the three equal angles.

Ans. Let ABCD be a quadrilateral such that

$\angle A = 108°$ and $\angle B = \angle C = \angle D$

Further, let $\angle B = \angle C = \angle D = x$

Now, by angle sum property of a quadrilateral, we have

$\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow$ 108° + x + x + x = 360°

$\Rightarrow$ 108° + 3x = 360° $\Rightarrow$ 3x = 360° – 108°

$\Rightarrow 3x = 252° \Rightarrow x = \frac{252°}{3} = 84°$

Hence, the measure of each of the remaining three equal angles is 84°.

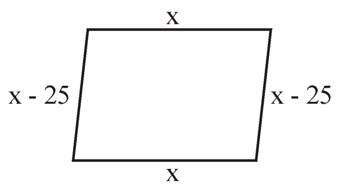
SE. 4

The perimeter of a parallelogram is 150 cm. One of its side is greater than the other by 25 cm. Find the lengths of all the sides of the parallelogram.

Ans. Let one pair of opposite sides of a parallelogram be x cm.

$\therefore$ Other pair of opposite sides be (x – 25) cm.

Perimeter of a parallelogram = 150 cm



$\Rightarrow$ x + (x – 25) + x + (x – 25) = 150

$\Rightarrow$ 4x – 50 = 150 $\Rightarrow$ 4x = 50 + 150

$\Rightarrow 4x = 200 \Rightarrow x = \frac{200}{4} \Rightarrow x = 50$

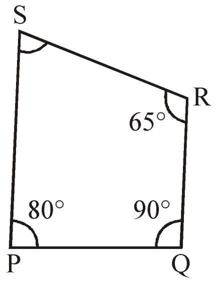
Therefore, x – 25 = 50 – 25 = 25

Hence, lengths of all sides of the parallelogram are 50 cm, 25 cm, 50 cm and 25 cm.

SE. 5

In a quadrilateral PQRS; $\angle P = 80°$, $\angle Q = 90°$; $\angle R = 65°$. Find the measure of $\angle S$. Is it a convex or concave quadrilateral ?

Ans. We know, sum of the interior angles of a quadrilateral = 360°



$\therefore \angle P + \angle Q + \angle R + \angle S = 360°$

$\Rightarrow 80° + 90° + 65° + \angle S = 360°$

$\Rightarrow 235° + \angle S = 360°$

$\Rightarrow \angle S = 360° - 235° = 125°$

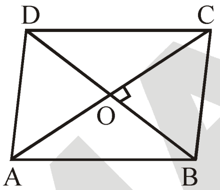
$\therefore$ PQRS is a convex quadrilateral.

SE. 6

The diagonals of a rhombus are 10 cm and 24 cm. Find the length of a side of the rhombus.

Ans. As AC = 24 cm, so CO = 12 cm

BD = 10 cm, so BO = 5 cm



Now, in right triangle BOC, we have

$BC^2 = BO^2 + OC^2$ [By Pythagoras Theorem]

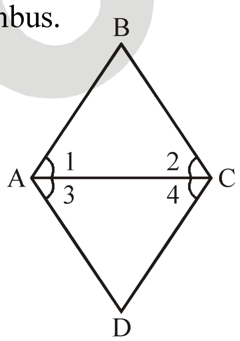
$\Rightarrow BC^2 = 5^2 + 12^2 \Rightarrow BC^2 = 25 + 144 = 169$

$\Rightarrow$ BC = 13 cm.

Hence, length of a side of the rhombus is 13cm.

SE. 7

Diagonal AC of a rhombus ABCD is equal to one of its sides BC (see fig.) Find all the angles of the rhombus.



Ans. ABCD is a rhombus

$\therefore$ AB = BC = CD = DA ....(i)

Also, given that diagonal

AC = side BC ....(ii)

From (i) and (ii), AB = BC = AC = CD = DA

Now, we have AB = BC = AC

and AC = CD = DA

Therefore ΔABC and ΔADC are equilateral triangles and in an equilateral triangle each angle is of measure 60°.

$\therefore \angle 1 = 60°, \angle 3 = 60°, \angle 2 = 60°$ and

$\angle 4 = 60°, \angle B = 60°$ and $\angle D = 60°$

Now, $\angle 1 + \angle 3 = 60° + 60° \Rightarrow \angle A = 120°$

And $\angle 2 + \angle 4 = 60° + 60° \Rightarrow \angle C = 120°$

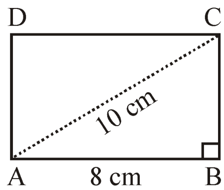
Hence, required respective angles of rhombus are 120°, 60°, 120° and 60°.

SE. 8

The length of a rectangle is 8 cm and each of its diagonals measures 10 cm. Find its breadth.

Ans. Let ABCD be the given rectangle in which length AB = 8 cm and diagonal AC = 10 cm.

Since, each angle of a rectangle is a right angle, we have $\angle ABC = 90°$.



$AC^2 = AB^2 + BC^2$ [Pythagoras theorem]

$\Rightarrow BC^2 = (AC^2 - AB^2) = \{(10)^2 - (8)^2\}$

$= (100 - 64) = 36$

$\Rightarrow BC = \sqrt{36} = 6$ cm

Hence, breadth = 6 cm.

SE. 9

Prove that the interior angle of a regular pentagon is three times the exterior angle of a regular decagon.

Ans. A pentagon has five sides.

$\therefore$ Each interior angle of a regular pentagon

$$= \left(\frac{2\times5-4}{5}\times90\right)^\circ = \left(\frac{6}{5}\times90\right)^\circ = 108^\circ$$

$$\left[\text{Putting } n = 5 \text{ in} \left(\frac{2n-4}{n}\times90\right)^\circ\right]$$

A decagon has 10 sides.
Exterior angle of a regular decagon

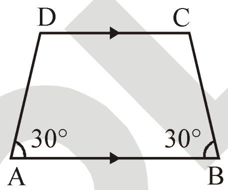
$$= \left(\frac{360}{10}\right)^\circ = 36^\circ$$

$$\left[\text{Putting } n = 10 \text{ in} \left(\frac{360}{n}\right)^\circ\right]$$

Clearly, each interior angle of a regular pentagon is three times the exterior angle of a regular decagon.

SE. 10

ABCD is a trapezium in which AB || DC and $\angle A = \angle B = 30°$. Find $\angle C$ and $\angle D$. Are these angles equal ?



Ans. AB || DC and a transversal AD cuts them.

$\therefore\ \angle A + \angle D = 180°$

$\Rightarrow \angle D = 180° - \angle A = 180° - 30° = 150°$

Similarly, $\angle B + \angle C = 180°$

$\Rightarrow \angle C = 180° - \angle B = 180° - 30° = 150°$

$\therefore\ \angle C = \angle D = 150°$

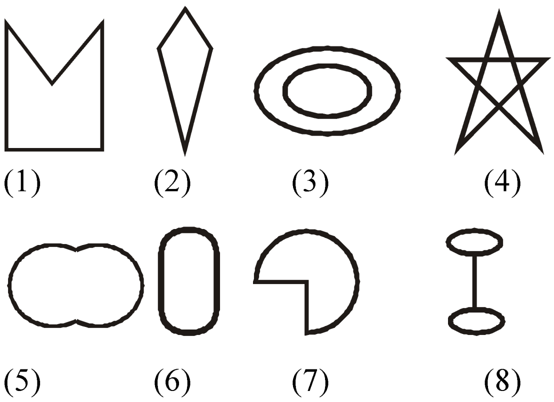
Yes, they are equal.

Space for Notes :

EXERCISE – 3.1

NS. 1

Given here are some figures.



Classify each of them on the basis of the following.

(A) Simple curve

(B) Simple closed curve

(C) Polygon

(D) Convex polygon

(E) Concave polygon

Ans. **(A)Simple curve :** It is a curve that does not intersect itself.

(1), (2), (5), (6) and (7) are simple curve.

(B) Simple closed curve : A closed curve if it does not pass through one point more than once

(1), (2), (5), (6) and (7) are simple closed curve.

(C) Polygon : A simple closed curve made up of only line segments is called a polygon.

(1) and (2) are polygons.

(D) Convex polygon : A polygon that has all its interior angles less than 180º.

(2) is a convex polygon.

(E) Concave polygon – A polygon that has at least one interior angle greater than 180º.

(1) is a concave polygon.

NS. 2

How many diagonals does each of the following have ?

(A) A convex quadrilateral

(B) A regular hexagon

(C) A triangle

Ans. A diagonal is a line segment joining two non consecutive vertices.

(A) Convex quadrilateral : Convex quadrilateral has 2 diagonals.

(B) A regular hexagon : A regular hexagon has 9 diagonals.

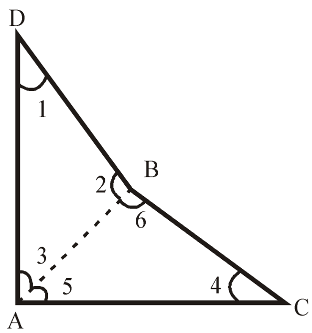
(C) A triangle : It has 0 diagonal, i. e., no diagonal.

NS. 3

What is the sum of the measures of the angles of a convex quadrilateral ? Will this property hold if the quadrilateral is not convex ? (Make a non–convex quadrilateral and try !)

Ans. Sum of measures of four angles of a convex quadrilateral is 360º.

Draw a figure given below and divide it into two triangles by joining AB, named ΔCAB and ΔDBA. Now in ΔDBA, we have m∠1 + m∠2 + m∠3 = 180º...(i)



[Using angle sum property of a triangle]

Also in ΔCAB : m∠4 + m∠6 + m∠5 = 180º ... (ii)

[Angle sum property]

Adding (i) and (ii), we get

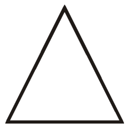
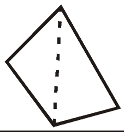
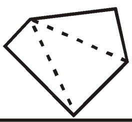
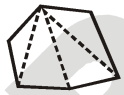
$\underbrace{m\angle 1}+\underbrace{m\angle 2+m\angle 6}+\underbrace{m\angle 5+m\angle 3}+\underbrace{m\angle 4}=180^\circ + 180^\circ$

$\angle D + \angle B + \angle A + \angle C = 360^\circ$

Which shows that a quadrilateral which is not convex also have sum of measure of its angles is 360º

NS. 4

Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that).

Figure				
Side	3	4	5	6
Angle sum	180º	2×180º= (4-2×180º)	3×180º= (5-2×180º)	4×180º= (6-2×180º)

What can you say about the angle sum of a convex polygon with number of sides ?

(A) 7 (B) 8

(C) 10 (D) *n*

Ans. (A) If a convex polygon has 7 sides, then angle sum = (7 – 2) × 180º = 5 × 180º = 900º.

(B) If a convex polygon has 8 sides, then angle sum = (8 – 2) × 180º = 6 × 180º = 1080º.

(C) If a convex polygon has 10 sides, then angle sum = (10 – 2) × 180º = 8 × 1440º.

(D) If a convex polygon has *n* sides, then angle sum = (*n* – 2) × 180º.

NS. 5

What is a regular polygon ? State the name of a regular polygon of

(i) 3 sides (ii) 4 sides (iii) 6 sides

Ans. Regular polygon – A polygon, which has all sides of equal length and angles of equal measure is called a regular polygon.

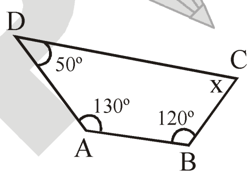
(i) An equilateral triangle, as all 3 sides are equal and all 3 angles are also equal (= 60º).

(ii) A square, as it has all 4 sides equal and all 4 angles are also equal (= 90º).

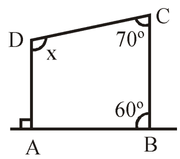
(iii) A regular hexagon, as it has all 6 sides equal and all 6 angles equal (= 120º)

NS. 6

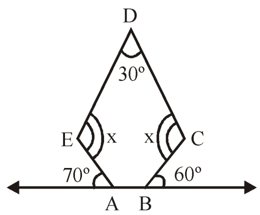
Find the angle measure *x* in the folowing figures.



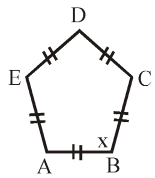
(A)



(B)

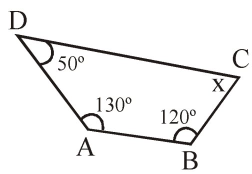


(C)



(D)

Ans. (A) Let ABCD be a given quadrilateral.



∵ Sum of all angles of a quadrilateral.

∴ $m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$.

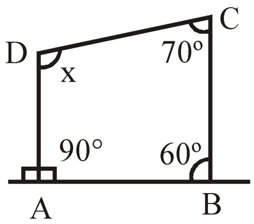
⇒ $130^\circ + 120^\circ + x + 50^\circ = 360^\circ$

⇒ 300° + x = 360°

⇒ x = 360° – 300°

⇒ x = 60°

(B) Let ABCD be a given quadrilateral.



Sum of all angles of a quadrilateral is 360°

∴ m∠A + m∠B + m∠C + m∠D = 360°.

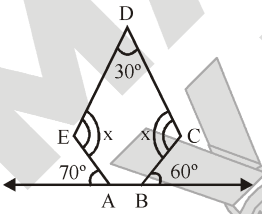
⇒ 90° + 60° + 70° + x = 360°

⇒ 220° + x = 360°

⇒ x = 360° – 220°

⇒ x = 140°.

(C) Let ABCDE be a given polygon, which has 5 sides. Now, sum of angles = (5 – 2) × 180° = 3 × 180° = 540°



Also, m∠A =180° – 70°

=110° [By linear pair]

and m∠B = 180° – 60°

= 120° [By linear pair]

Thus, m∠A + m∠B + m∠C + m∠D + m∠E = 540°

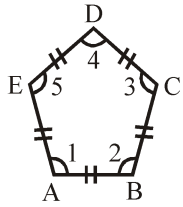
⇒ 110° + 120° + x + 30° + x = 540°

⇒ 260° + 2x = 540°

⇒ 2x = 540° – 260°

⇒ 2x = 280° ∴ x = 140°.

(D) Let ABCDE be a given pentagon.



∴ Sum of angles of a given pentagon ABCDE = (5 – 2) × 180° = 540°.

∴ m∠A + m∠B + m∠C + m∠D + m∠E = 540°

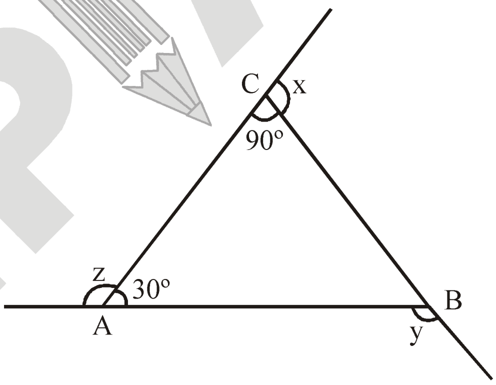
⇒ x + x + x + x + x + = 540°

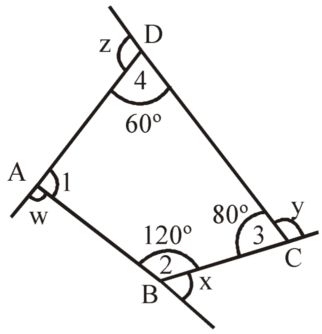
⇒ 5x = 540°

⇒ x = 108°.

NS. 7

(A) Find x + y + z. (B) Find x + y + z+ *w*.





Ans. (A) Let ABC be a given triangle .

Sum of angles of a triangle is 180°

∴ m∠A + m∠B + m∠C + = 180°

⇒ 30° + m∠B + 90° = 180°

⇒ m∠B + 120° = 180°

$\Rightarrow m\angle B = 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow m\angle B = 60^\circ$ ...(i)

Clearly, $x + 90^\circ = 180^\circ$ [By linear pair]

$\Rightarrow x = 180^\circ - 90^\circ \Rightarrow x = 90^\circ$ ...(ii)

Also, $z + 30^\circ = 180^\circ$ [By linear pair]

$\Rightarrow z = 180^\circ - 30^\circ \Rightarrow z = 150^\circ$ ...(iii)

and $y + 60^\circ = 180^\circ$ [By linear pair]

$\Rightarrow y = 180^\circ - 60^\circ \Rightarrow y = 120^\circ$ ...(iv)

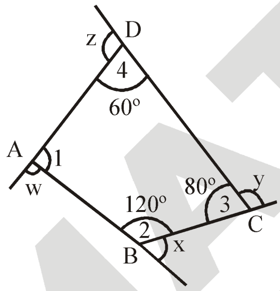
$\therefore$ by using (ii), (iii) and (iv), we get

$x + y + z = 90^\circ + 120^\circ + 150^\circ$

$\Rightarrow x + y + z = 360^\circ$

(B) Let ABCD be a given quadrilateral.

Sum of angles of a quadrilateral is 360°



$\therefore m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$

$\Rightarrow m\angle 1 + 120^\circ + 80^\circ + 60^\circ = 360^\circ$

$\Rightarrow m\angle 1 + 260^\circ = 360^\circ$

$\Rightarrow m\angle 1 = 360^\circ - 260^\circ \Rightarrow m\angle 1 = 100^\circ$

Clearly, $w + 100^\circ = 180^\circ$ [By linear pair]

$\Rightarrow w = 180^\circ - 100^\circ = 80^\circ$ ...(i)

$x + 120^\circ = 180^\circ$ [By linear pair]

$\Rightarrow x = 180^\circ - 120^\circ = 60^\circ$ ...(ii)

$y + 80^\circ = 180^\circ$ [By linear pair]

$\Rightarrow y = 180^\circ - 80^\circ = 100^\circ$ ...(iii)

Also, $z + 60^\circ = 180^\circ$ [By linear pair]

$\Rightarrow z = 180^\circ - 60^\circ \Rightarrow z = 120^\circ$

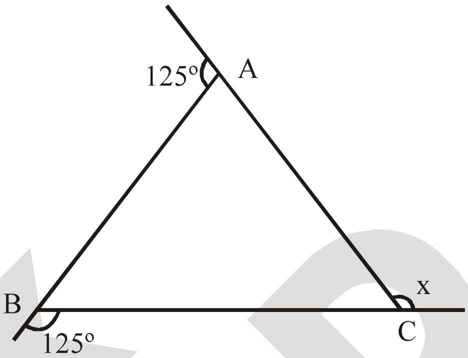
Thus, by (i), (ii), (iii) & (iv), we get

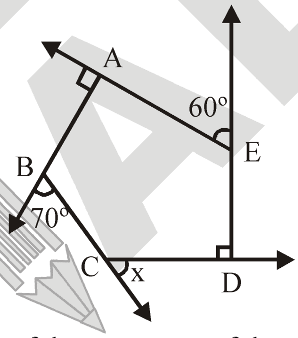
$w + x + y + z = 80^\circ + 60^\circ + 100^\circ + 120^\circ = 360^\circ$

EXERCISE – 3.2

NS. 1

Find x in the following figures.





Ans. (A) Sum of the measures of the exterior angles of any polygon is 360°

$\therefore 125^\circ + 125^\circ + x = 360^\circ \Rightarrow 250^\circ + x = 360^\circ$

$\Rightarrow x = 360^\circ = 250^\circ \Rightarrow x = 110^\circ$.

(B) Since, sum of the exterior angles of any polygon is 360°

$\therefore x + 90^\circ + 60^\circ + 90^\circ + 70^\circ = 360^\circ$

$\Rightarrow x + 310^\circ = 360^\circ \Rightarrow x = 360^\circ - 310^\circ$ $x = 50^\circ$.

NS. 2

Find the measure of each exterior angle of a regular polygon of

(i) 9 sides (ii) 15 sides.

Ans. (i) Let each exterior angle of a regular polygon who has 9 sides is equal to x.

Sum of exterior angles of any polygon is 360°.

$$\Rightarrow 9x = 360^\circ \Rightarrow x = \frac{360^\circ}{9} \Rightarrow x = 40^\circ$$

Thus, each exterior angle of a regular polygon of 9 sides is 40°

(ii) Let each exterior angle of a regular polygon which has 15 sides is x.

Sum of all exterior angles of a polygon is 360°.

$\therefore\ 15x = 360° \Rightarrow x = \frac{360°}{15} \Rightarrow x = 24°$

Thus, each exterior angle of a regular polygon of 15 sides is 24°.

NS. 3

How many sides does a regular polygon have if the measure of an exterior angle is 24° ?

Ans. Total measure of all exterior angles = 360°

Measure of each exterior angle = 24°

Therefore, the number of sides = $\frac{360°}{24°} = 15$

The polygon has 15 sides.

NS. 4

How many sides does a regular polygon have if each of its interior angles is 165° ?

Ans. Total measure of all exterior angles = 360°

Measure of each interior angle = 165°

Measure of each exterior angle = 180° – 165° = 15°

Therefore, number of sides = $\frac{360°}{15°} = 24$

NS. 5

(A) Is it possible to have a regular polygon with measure of each exterior angle as 22° ?

(B) Can it be an interior angle of a regular polygon ? Why ?

Ans. (A) No, because 22° is not a divisor of 360°

(B) No, because each exterior angle is 180° – 22° = 158°, which is not a divisor of 360°.

NS. 6

(A) What is the minimum interior angle possible for a regular polygon ? Why ?

(B) What is the maximum exterior angle possible for a regular polygon ?

Ans. (A) Since each angle of an equilateral triangle is 60°.

And equilateral triangle is a regular polygon.

$\therefore$ Minimum interior angle is 60°, for a regular polygon.

(B) Equilateral triangle is a regular polygon with each interior angle is 60°.

$\therefore$ Possible maximum exterior angle of a regular polygon is 120°

EXERCISE – 3.3

NS. 1

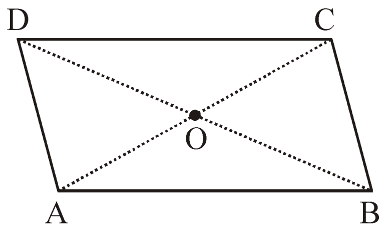
Given a parallelogram ABCD. complete each statement along with the definition or property used.

(i) AD = ...

(ii) ∠DCB = ...

(iii) OC = ...

(iv) m∠DAB + m∠CDA = ...



Ans. Given that ABCD is a parallelogram.

(i) AD = BC

[∵ In a parallelogram opposite sides are equal]

(ii) ∠DCB = ∠DAB

[∵ In a parallelogram opposite angles are equal]

(iii) OC = OA

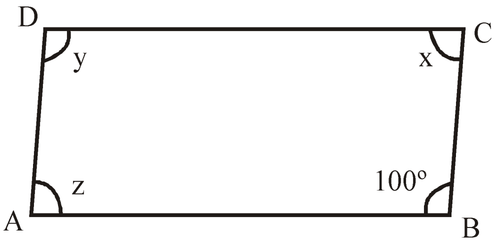
[∵ In a parallelogram diagonals bisect each other]

(iv) m∠DAB + m∠CDA = 180°

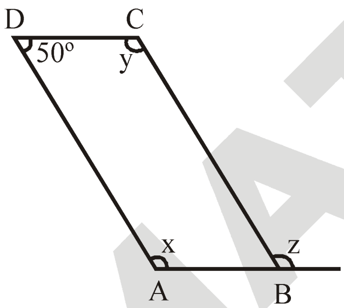
[∵ Adjacent angles in a parallelogram are supplementary]

NS. 2

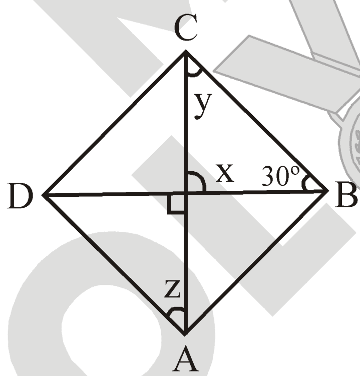
Consider the following parallelograms. Find the value of the unknowns x, y, z.



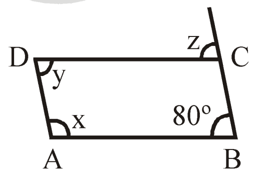
(i)



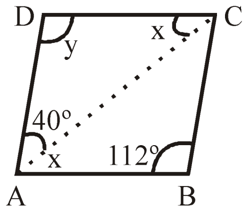
(ii)



(iii)



(iv)



(v)

Ans. ABCD is a parallelogram in which ∠B = 100° (given)

∴ ∠A + ∠B = 180°

[∵ Sum of adjecent angles is 180°]

⇒ ∠z + 100° = 180°

⇒ ∠z = 180° – 100° = 80°

Also, ∠B = ∠D and ∠A = ∠C

[∵ Opposite angles are equal]

∴ ∠B = 100° = ∠D = y and ∠A = 80° = ∠C = x

∴ x = 80°, y = 100°, z = 80°.

(ii) y + 50° = 180°

[∵ Sum of adjacent angles is 180°]

⇒ y = 180° – 50° = 130°

Also, y = x = 130°

[∵ Opposite angles are equal in a parallelogram]

And 180° – z = 50° [Linear pair]

⇒ 180° – 50° = z ⇒ z = 130°.

(iii) Clearly, x = 90°

[∵ Vertically opposite angles are equal]

Also, x + y + 30° = 180°

[By using angle sum property of a triangle]

⇒ 90° + 30° + y = 180°

⇒ 120° + y = 180° ⇒ y = 180° – 120° = 60°

Since, alternate angles are equal in a parallelogram

∴ y = z = 60°

Thus x = 90°, y = 60° and z = 60°

NS. 3

Can a quadrilateral ABCD be a parallelogram if

(i) $\angle D + \angle B = 180^\circ$

(ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm ?

(iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$

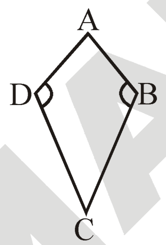
Ans. (i) Yes, but need not be true.

(ii) No, because in a parallelogram opposite sides are equal but here $AD \neq BC$.

(iii) No, because in a parallelogram opposite angles are equal but here $\angle A \neq \angle C$.

NS. 4

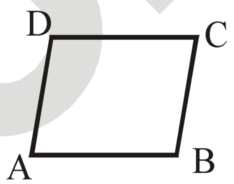
Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.



Ans. We can draw a figure of a kite in which exactly two opposite angles are equal. Hence $\angle D = \angle B$.

NS. 5

The measures of two adjacent angles of a parallogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.



Ans. Let ABCD be a given parallelogram.

Let $\angle A = 3x$ and $\angle B = 2x$.

Since the sum of adjacent angles in parallelogram is 180°

$\therefore\ m\angle A + m\angle B = 180^\circ$

$3x + 2x = 180^\circ \Rightarrow 5x = 180^\circ \Rightarrow x = 36^\circ$

$\therefore\ m\angle A = 3 \times 36^\circ = 108^\circ$

and $m\angle B = 2 \times 36^\circ = 72^\circ$.

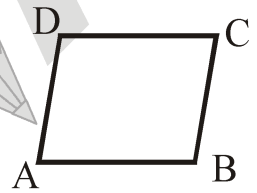
Also, $m\angle A = m\angle C = 108^\circ$

and $m\angle B = m\angle D = 72^\circ$

[$\because$ In a parallelogram opposite angles are equal]

NS. 6

Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.



Ans. Let ABCD be a given parallelogram.

Let $\angle A = \angle B = x$.

Since the sum of adjacent angles in a parallelogram is 180°

$\therefore\ m\angle A + m\angle A = 180^\circ$

$\Rightarrow x + x = 180^\circ \Rightarrow 2x = 180^\circ \Rightarrow x = 90^\circ$

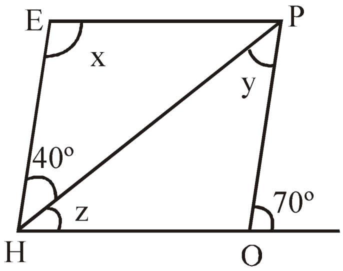
Also $m\angle A\ m\angle C = 90^\circ$

[$\because$ In a parallelogram opposite angles are equal]

and $m\angle B = m\angle D = 90^\circ$.

NS. 7

The given figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.



Ans. y = 40°

[Since PO || HE ∴ alternate angles are equal]

70° = y + z

[Exterior angle property of a triangle]

⇒ z = 70° – 40° = 30°

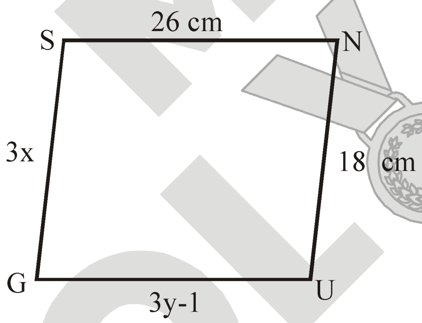
∠POH = 180° – 70° = 110°

∠POH = x = 110° [Opposite angles are equal]

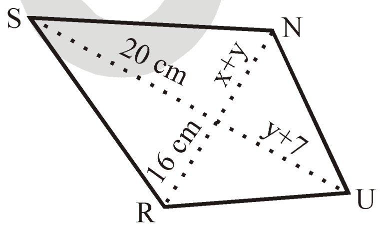
Thus, x = 110°, y = 40°, z = 30°

NS. 8

The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm).



(i)



(ii)

Ans. (i) Since, GUNS is a parallelogram.

∴ GS = UN and GU = SN

[∵ In a parallelogram opposite sides are equal]

⇒ 3x = 18 ⇒ x = 6

and 3y – 1 = 26 ⇒ 3y = 1 + 26 = 27 ⇒ y = 9

Thus x = 6 cm and y = 9 cm.

(ii) Since diagonals bisects each other in a parallelogram.

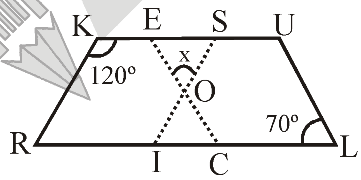
So, 20 = y + 7 ⇒ y = 13

Also, x + y = 16 ⇒ x = 16 – 13 = 3

Thus, x = 3 cm and y = 13 cm.

NS. 9

In the given figure both RISK and CLUE are parallelograms. Find the value of x.



Ans. Since RISK and CLUE are parallelograms.

∴∠SKR = ∠RIS = 120°

[∵ Opposite angles are equal]

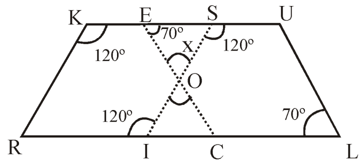
Also, ∠ULC = ∠CEU = 70°

[Opposite angles are equal]

∠RIS + ∠ISK = 180°

[Adjacent angles are supplementary]

⇒ ∠ISK = 180° – 120° = 60°



In ΔOES, we have

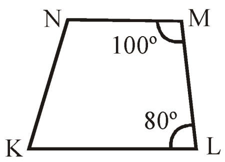
70° + x + 60° = 180° [Angle sum property]

⇒ x + 130° = 180°

⇒ x = 180° – 130° = 50°

NS. 10

Explain how this figure is a trapezium. Which of its two sides are parallel ?



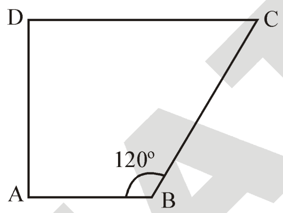
Ans. In a trapezium, only one pair of opposite sides are parallel, whereas other pair of opposite sidea are non–parallel.

∴ KLMN is a trapeium becaus MN || KL.

[∵ Sum of two adjacent interior angles is 180° = (80° + 100°)].

NS. 11

Find m∠C in given figure if $\overline{AB} \parallel \overline{DC}$.



Ans. We are given, $\overline{AB} \parallel \overline{DC}$

∴ Sum of two adjacent interior angles is 180°

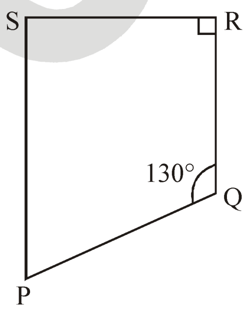
i.e., ∠B + ∠C = 180° ⇒ 120° + ∠C = 180°

⇒ ∠C = 180° – 120° = 60°.

Thus ∠C = 60°.

NS. 12

Find the measure of ∠P and ∠S if $\overline{SP} \parallel \overline{QR}$ in figure. (If you find m∠R , is there more than one method to find m∠P ?)



Ans. Since $\overline{SP} \parallel \overline{QR}$

Thus, ∠R + ∠S = 180°

[∵ Sum of two adjacent interior angles is 180°]

⇒ 90° + ∠S = 180° ⇒ ∠S = 180° – 90° = 90°

∠P + ∠Q + ∠R + ∠S = 360°

[∵ Sum of all angles of a quadrilateral is 360°]

∠P + 310° = 360° ⇒ ∠P = 360° – 310°

⇒ ∠P = 50°

Thus, ∠P = 50° and ∠S = 90°

Also, m∠P can be found as

∠P + ∠Q = 180°

[Adjacent angles are supplementary]

∠P + 130° = 180°

∠P = 180° – 130° ⇒ ∠P = 50°

EXERCISE – 3.4

NS. 1

State whether true or False.

(A) All rectangles are squares.
(B) All rhombuses are parallelograms.
(C) All squares are rhombuses and also rectangles.
(D) All squares are not parallelograms.
(E) All kites are rhombuses.
(F) All rhombuses are kites.
(G) All parallelograms are trapeziums.
(H) All squares are trapeziums.

Ans. (A) No, because in a square all sides are equal, but it is not true in case of rectangle.

(B) Yes, because in both diagonals bisects each other and opposite sides are equal.

(C) Yes, because in square all sides are equal.

(D) No, all squares are parallelograms. Because all squares satisfies the conditions of a parallelogram.

(E) No, because only two pair of consecutive sides are equal in kites whereas in rhombus all sides are of equal length.

(F) Yes, all rhombuses are kites.

(G) Yes, since both has atleast one pair of parallel sides.

(H) Yes, both has atleast one pair of parallel sides.

NS. 2

Identify all the quadrilaterals that have

(A) four sides of equal length.

(B) four right angles.

Ans. (A) The quadrilaterals, those have four sides of equal length are square and rhombus.

(B) The quadrilaterals, those have four right angles, are square and rectangle.

NS. 3

Explain how a square is

(i) a quadrilateral (ii) a parallelogram

(iii) a rhombus (iv) a rectangle

Ans. (i) A square is four sided, so it is a quadrilateral.

(ii) Since a square has opposite sides parallel and diagonals bisect each other, so it is a parallelogram.

(iii) Since square is a parallelogram with all 4 sides equal, so it is a rhombus.

(iv) Since square is a parallelogram with each angle a right, so it is a rectangle.

NS. 4

Name the quadrilaterals whose diagonals

(i) bisect each other.

(ii) are perpendicular bisectors of each other.

(iii) are equal.

Ans. (i) The quadrilaterals in which diagonals bisect each other are rhombus, rectangle, square and parallelogram.

(ii) The quadrilaterals in which diagonals are perpendicular bisectors of each other are rhombus and square.

(iii) The quadrilaterals in which diagonals are equal are square and rectangle.

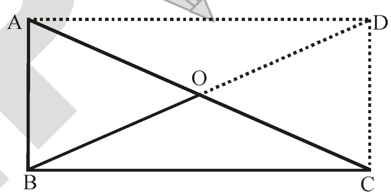
NS. 5

Explain why a rectangle is a convex quadrilateral.

Ans. When we draw the diagonals joining the end points of a rectangle it lies in its interior. So, it is a convex quadrilateral.

NS. 6

ABC is a right–angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



Ans. Since, ΔABC is right angled at B. So $\angle B = 90^\circ$,

$\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$

$\Rightarrow$ ABCD is a rectangle where AB = CD and AD = BC, AC and BD are the diagonals which bisects each other. Also AC = BD

Thus, AO = OC and BO = OD, also BO = OC which shows that O is equidistant from A, B, C.

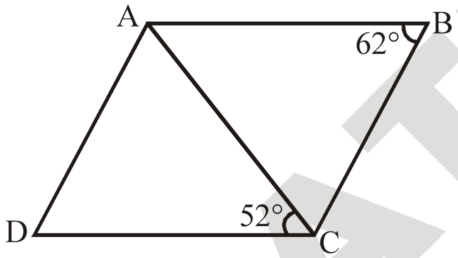
EXERCISE – I

ONLY ONE CORRECT TYPE

1. Two adjacent angles of a parallelogram are $(2x + 25)°$ and $(3x – 5)°$. The value of x is
(A) 28 (B) 32
(C) 36 (D) 42

2. The diagonals do not necessarily intersect at right angles in a
(A) Parallelogram (B) Square
(C) Rhombus (D) Kite

3. In the given quadrilateral ABCD (not drawn to scale), BC = AC = AD. Find the sum of $\angle DAC$ and $\angle ACB$.



(A) 76° (B) 132°
(C) 56° (D) 112°

4. If an angle of a parallelogram is two-third of its adjacent angle, then the smallest angle of the parallelogram is
(A) 54° (B) 72°
(C) 81° (D) 108°

5. A quadrilateral can have
(A) 4 acute angles
(B) 4 obtuse angles
(C) 3 obtuse angles
(D) 2 right angles and 2 obtuse angles

6. A quadrilateral, in which only one pair of opposite sides is parallel is a
(A) rectangle (B) kite
(C) trapezium (D) rhombus

7. A quadrilateral, which is both a rectangle and a rhombus is a
(A) square (B) parallelogram
(C) Kite (D) trapezium

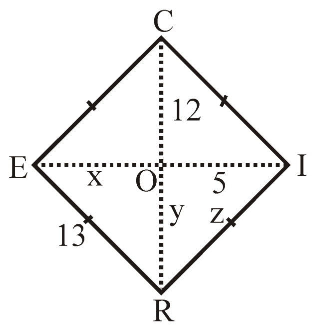
8. A regular polygon is
(A) equilateral (B) equiangular
(C) both (A) & (B) (D) none of these

9. Three angles of a quadrilateral are in the ratio 1 : 5 : 6. The mean of these angles is 64°. Find the fourth angle.
(A) 168° (B) 162°
(C) 120° (D) 90°

10. If PQRS is a parallelogram, then $\angle P – \angle R$ is
(A) 0° (B) 90°
(C) 180° (D) 360°

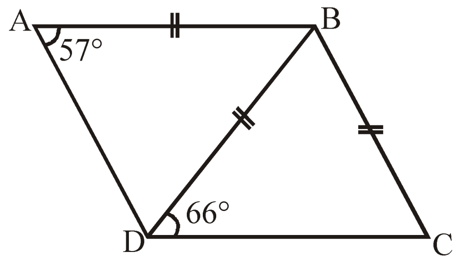
11. Whch of the following is a false statement for a parallelogram ?
(A) Opposite sides are equal
(B) Opposite angles are equal
(C) Diagonals bisect each other
(D) Diagonals bisect each other at right angles.

12. In the given figure, RICE is a rhomubs.
Find x + y + z.



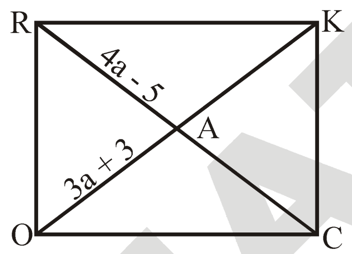
(A) 30 (B) 35
(C) 25 (D) 20

13. In the given figure, ABD and BCD are isosceles triangles, where AB = BC = BD. The special name that is given to quadrilateral ABCD is



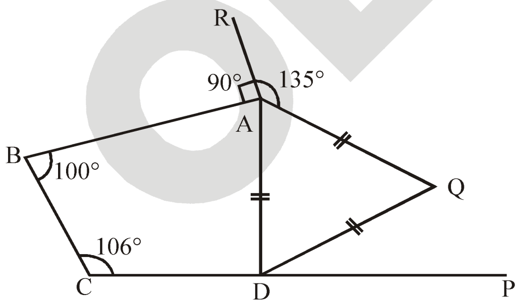
(A) Rectangle (B) Rhombus
(C) Parallelogram (D) Trapezium

14. ROCK is a rectangle. Its diagonals meet at A. Find 'a', if RA = 4a – 5 and OA = 3a + 3.



(A) 3 (B) 5
(C) 8 (D) 4

15. In the given figure, CDP is a straight line, ΔAQD is an equilateral triangle, ∠BAR = 90°, ∠QAR = 135°, ∠BCD = 106° and ∠ABC = 100°. Then, ∠PDQ equals.

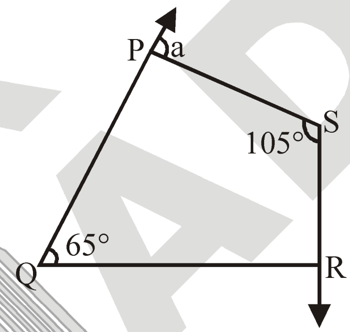


(A) 39° (B) 21°
(C) 41° (D) 53°

16. If one angle of a parallelogram is 24° less than twice the smallest angle, then the largest angle of the parallelogram is

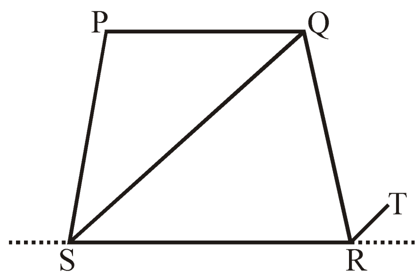
(A) 68° (B) 102°
(C) 112° (D) 176°

17. In the adjacent figure, angle P and angle R are in the ratio of 3 : 7, then the value of a is



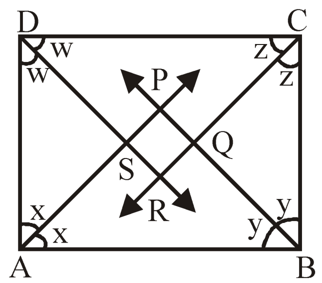
(A) 47° (B) 57°
(C) 123° (D) none of these

18. In the given figure, line RT is drawn parallel to SQ. If ∠QPS = 100°, ∠PQS = 40°, ∠PSR = 85° and ∠QRS = 70°, then ∠QRT =



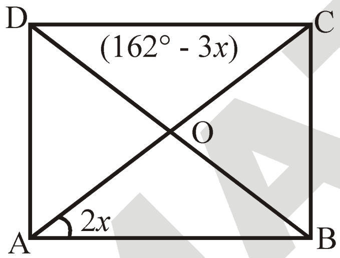
(A) 45° (B) 65°
(C) 85° (D) 90°

19. In the given figure, ABCD is a parallelogram. The quadrilateral PQRS is exactly a



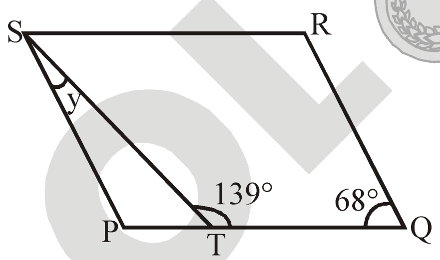
(A) Square
(B) Parallelogram
(C) Rectangle
(D) Rhombus

20. ABCD is a rectangle. Find the value of x.



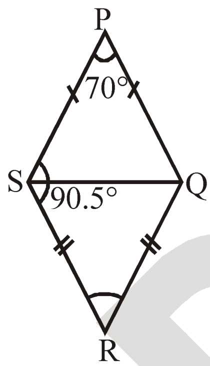
(A) 54° (B) 36°
(C) 24° (D) 18°

21. If PQRS is a parallelogram, then y equals



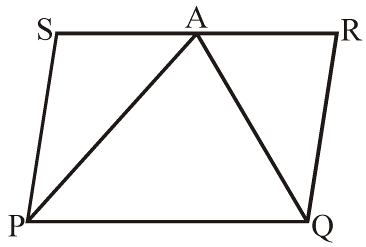
(A) 27° (B) 61°
(C) 41° (D) 28°

22. PQRS is a kite. If ∠P = 70° and ∠S= 90.5°, then ∠R equals



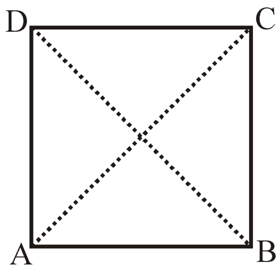
(A) 99° (B) 91°
(C) 111° (D) 109°

23. In the given figure, PQRS is parallelogram and ∠SPQ = 60°. If the bisectors of ∠P and ∠Q meet at A on RS, then which of the following is not correct ?



(A) AS = SP (B) AS = AR
(C) AR = SP (D) AQ = PQ

24. ABCD is a square of area of 4 square units which is divided into 4 non-overlapping triangles as shown in figure, then sum of perimeters of the triangles so formed is



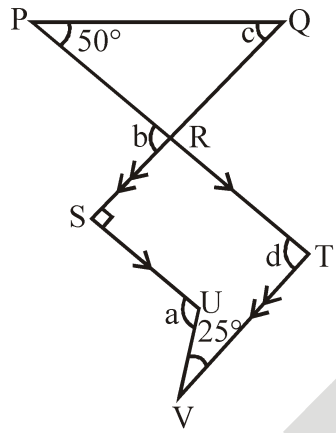
(A) $8(2+\sqrt{2})$ units

(B) $8(1+\sqrt{2})$ units

(C) $4(1+\sqrt{2})$ units

(D) $4(2+\sqrt{2})$ units

25. In the given figure (not drawn to scale), find the value of (b + d) – (a + c).



(A) 115° (B) 40°

(C) 90° (D) 25°

PARAGRAPH TYPE

Passage # I

The angles of a quadrilateral are in the ratio 3 : 5 : 7 : 9.

26. If measure of angle be (3x), (5x), (7x) and (9x), then the value of x is :

(A) 20° (B) 15°

(C) 25° (D) 10°

27. The measure of all angles are :

(A) 45°, 70°, 105°, 140°

(B) 40°, 80°, 105°, 135°

(C) 45°, 75°, 110°, 130°

(D) 45°, 75°, 105°, 135°

28. The sum of the least and greatest angle is :

(A) 175° (B) 180°

(C) 170° (D) 185°

Passage # II

Measure of each exterior angle of a regular polygon of n sides = $\left(\frac{360}{n}\right)^\circ$

29. The measure of each exterior angle of a 10 sided regular polygon is :

(A) 36° (B) 30°

(C) 18° (D) 35°

30. If measure of an exterior angle is 45°, the number of sides in a regular polygon is :

(A) 7 (B) 11

(C) 10 (D) 8

31. The polygon with nine sides is called

(A) Octagon (B) Quadrilateral

(C) Nonagon (D) Hexagon

MATCH THE COLUMN TYPE

In this, section, each question has two matching lists. Choices for the correct combination of elements from Column - I and Column - II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the columns :

Column – I	Column – II
(P) Diagonals of a rectangle	(1) Bisect each other at right angles
(Q) Diagonals of a square	(2) Bisect each other
(R) Diagonals of a rhombus	(3) Equal and bisect each other
(S) Diagonals of a parallelogram	(4) Equal and bisect each other at right angles

(A) P – 1, Q – 2, R – 3, S – 4
(B) P – 3, Q – 4, R – 1, S – 2
(C) P – 4, Q – 2, R – 3, S – 1
(D) P – 4, Q – 3, R – 2, S – 1

33. Match the columns :

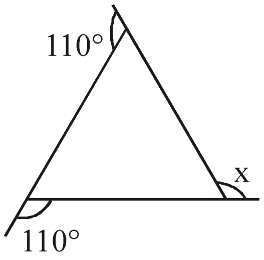
Column – I	**Column – II**
(P) ABCDE is a regular pentagon. The bisector of ∠A of the pentagon meets the side CD at M. Then m∠AMC is	(1) 70°
(Q) The angles of a quadrilateral are respectively equal to 110°, 50° and 40°. The fourth angle is	(2) 80°
(R) In a quadrilateral ABCD, the bisector of ∠A and ∠B meet at point P. If ∠C = 100°, ∠D = 60°, then m ∠APB is	(3) 90°
(S) Three angles of a quadrilateral are equal. Fourth angle is of measure 150°. The measure of equal angles is	(4) 160°

(A) P – 1, Q – 2, R – 3, S – 4
(B) P – 2, Q – 4, R – 3, S – 1
(C) P – 3, Q – 4, R – 2, S – 1
(D) P – 4, Q – 3, R – 2, S – 1

Space for Notes :

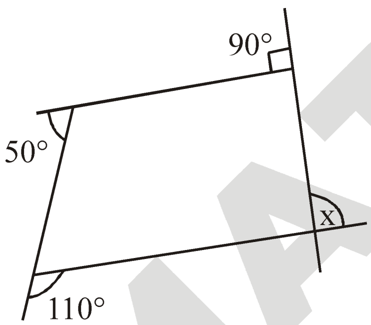
VERY SHORT ANSWER TYPE

1. Find the measure of x in the given figure.



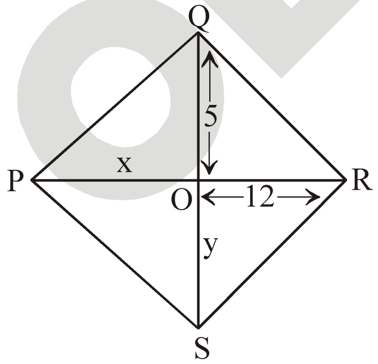
2. In a quadrilateral PQRS ; ∠P = 70°, ∠Q = 90°; ∠R = 55°. Find the measure of ∠S. What kind of quadrilateral is it, convex of concave ?

3. Find the measure of x



4. Which of the following groups of angles can be the angles of a quadrilateral ?
   (i) 120°, 90°, 75°, 30°
   (ii) 100°, 100°, 70°, 90°

5. Let PQRS be a rhombus, find x, y



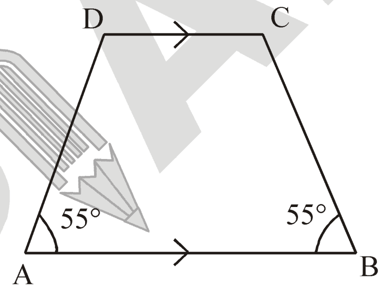
6. In a quadrilateral PQRS, ∠P = 40°, ∠Q = 60°, ∠R = 60°. Find ∠S. Is this quadrilateral convex or concave ?

7. Adjacent angles of a parallelogram are in the ratio of 2 : 7. Find their values.

8. In a parallelogram, show that any two adjacent angles are supplementary.

9. The diagonals of a quadrilateral are 8 cm and 6 cm. If the diagonals bisect each other at right angles, find the length of the sides of the quadrilateral.

10. In the given figure, find the measure of ∠C.



SHORT ANSWER TYPE

1. Three angles of a quadrilateral are in the ratio 1 : 2 : 3. The mean of these angles is 32°. Find all the four angles.

2. Prove that in a rhombus with an angle of 60°, the shorter diagonal divides it into two equilateral triangles.

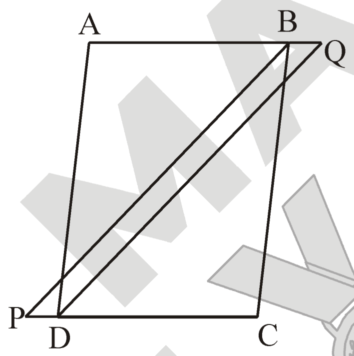
3. ABCD is a rhombus whose diagonals AC and BD intersect at a point O. If side AB = 10 cm and diagonal BD = 16 cm, find the length of diagonal AC.

4. The ratio of two sides of a rectangle is 3 : 5 and its perimeter is 48 m. Find the sides of the rectangle.

5. The interior angle of a regular polygon is 156°. Find the number of sides of the polygon

LONG ANSWER TYPE

1. Two regular polygons are such that the ratio between their number of sides is 2 : 1 and the ratio of measures of their interior angles is 4 : 3. Find the number of sides of each polygon.
2. The exterior angle of a regular polygon is one-third of its interior angle. How many sides the polygon has ?
3. In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$.
4. In the given figure, bisectors of $\angle B$ and $\angle D$ of quadrilateral ABCD meet CD and AB produced at P and Q respectively. Prove that $\angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$



5. The diagonals of a rectangle ABCD meet at O. If $\angle BOC = 44°$, then find $\angle OAD$.

TRUE / FALSE TYPE

1. If all the angles of a quadrilateral are equal, it is a rectangle.
2. The adjacent angles of a parallelogram are equal.
3. The diagonals of a parallelogram bisect each other.
4. In a parallelogram, the diagonals are equal.
5. The diagonals of a rectangle are of equal length.

NUMERICAL PROBLEMS

1. The number of sides of a regular polygon whose sum of each exterior and interior angles are in the ratio 1 : 5 is k. The value of k is.
2. The ratio of two sides of a rectangle is 4 : 3 and its perimeter is 56 m. If the sides of rectangle be a and b. Then, a + b (in m) is :
3. Find the number of sides of a regular polygon when each of its angle has a measure of 135° is
4. Find the number of sides of a regular polygon when each of its angle has a measure of 90°.
5. The measure of angles of a quadrilateral are x, 2x, 3x and 4x. Find the value of $\frac{x}{12°}$

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	A	B	B	C	C	A	C	A	A	D	A	D	C	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	C	B	C	D	A	D	D	B	D	B	D	B	A	D
31	32	33												
C	B	C												

EXERCISE II

VERY SHORT ANSWER TYPE

1. 140° **2.** 145°, Convex **3.** 110° **4.** (i) No, (ii) Yes

5. x = 12, y = 5 **6.** 200°, Concave **7.** 40°, 140° **9.** AB = AD = DC = BC = 5 cm

10. 125°

SHORT ANSWER TYPE

1. 16°, 32°, 48°, 264° **3.** 12 cm **4.** 9 m, 15 m **5.** 15

LONG ANSWER TYPE

1. 5, 10 **2.** 8 **5.** 68°

TRUE / FALSE

1. T **2.** F **3.** T **4.** F **5.** T

NUMERICAL PROBLEMS

1. 12 **2.** 28 **3.** 8 **4.** 4 **5.** 3

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : UNDERSTANDING QUADRILATERALS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Solutions			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

DATA HANDLING 4

Concepts

INTRODUCTION

In our day-to-day life we might have come across various informations, such as :

(i) Number of runs made by a batsman in the last 15 test matches.

(ii) Marks obtained by the students of class VIII in the Mathematics unit test.

(iii) Number of cars sold by a car manufacturing company in the months of a particular year.

(iv) The number of novels read by some friends in a particular year.

All the information which is collected in the above cases is called data.

In this chapter, we shall discuss about the data and its representation in different forms.

1. DATA

The word 'data' means the collection of information in the form of numerical figures or set of given facts.

For example :

(i) The marks obtained by 10 students of a class in a test out of 100 marks are

78, 85, 97, 59, 100, 80, 75, 68, 47, 55

(ii) The following data gives the information regarding the favourite game of 110 students of a school.

Sports	Football	Cricket	Tennis	Carrom Board
No.of students	35	45	20	10

When some information is collected and presented in an unorganised manner, then it is called raw data.

And when the data is classified into groups, then it is called grouped data.

Data in raw form can be presented in pictorial form and this makes the data attractive and easy to understand. Also, we can compare it with other information.

Some commonly known diagrams in which numerical data can be represented are as follows:

(i) Pictograph

(ii) Bar graph

(iii) Double bar graph

(iv) Circle graph or Pie-chart or Pie- diagram

1.1 PICTOGRAPH

It is represents data through appropriate pictures. Generally, same type of symbols or pictures are used to represent data. Each picture or symbol is used to represent a certain value and it is clearly mentioned in the graph. For example, the given pictograph represents the number of tractors produced in the month of April, May and June.

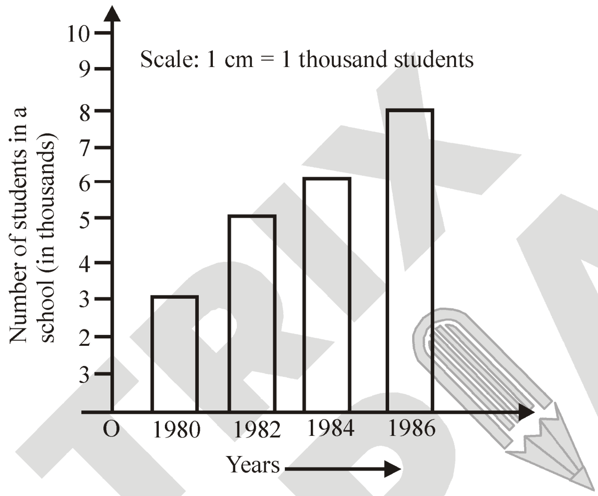
1.2 BAR GRAPH

The representation of data in the form of bars or rectangles in a diagram is called a bar graph or a bar diagram.

Here, each bar represents only one value of the data and so, there are as many bars as the number of values in the data.

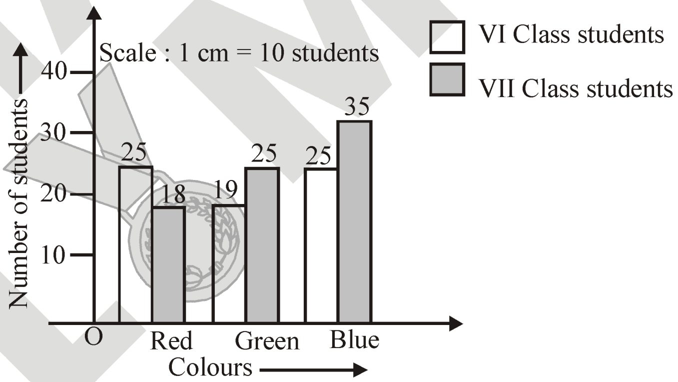
The length of the bar represents the value of the item. The width of the bars should be uniform.

The given bar graph represents the number of students in a school in different years (in thousands).



1.3 DOUBLE BAR GGRAPH

It is a bar graph which shows two sets of data on the same graph. It is useful in the comparison of data.



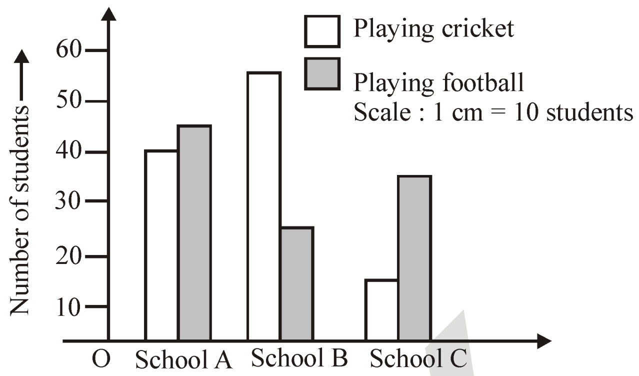
Bar graph above represents liking of different colours by class VI and VII students.

Example 1

Draw a double bar graph to represent the information given below :

Children who prefer	School A	School B	School C
Playing cricket	40	55	15
Playing football	45	25	35

Solution :



2. ORGANISING DATA

Generally, the data which we get is in unorganised form (called as raw data). To understand it meaningfully, we must organise it systematically.

For example, a group of children was asked about their favourite game. The results were as given below.

Cricket, Football, Volleyball, Tennis, Cricket, Football, Cricket, Volleyball, Football, Cricket, Football, Cricket, Tennis.

By looking directly to the data, we cannot answer the number of students and their favourite games.

In this case, we arrange data in a table by using tally marks as shown:

Favourite Games	Tally Marks	No. of Students
Cricket	~~IIII~~	5
Football	IIII	4
Volleyball	II	2
Tennis	III	3

Note: We generally use ~~IIII~~ Tally marks to represent value 5. To represent 7, we use ~~IIII~~ II Tally marks.

3. GROUPING DATA

Something organising large data becomes a tedious process. In such cases, we make groups of the raw data and write the groups as intervals. Each such groups is called class-interval. The class intervals have a lower class limit and an upper class limit. The difference between the upper and lower class interval is called class size or width of the class interval. The number of times a particular item appears within a particular class interval is called frequency.

We will now fill up the rows with tally marks and count the total number of tally marks in each group.

The total number of tally marks of each group is listed in frequency column. The table so obtained is called frequency distribution table.

Class Interval	Tally Marks	Frequency (No. of Students)
0 - 10	|||	3
10 - 20	~~||||~~	5
20 -30	~~||||~~ ||	7
30 - 40	~~||||~~ ||||	9
40 - 50	~~||||~~ ~~||||~~ ||	12

Frequency Distribution Table

Note: In class ineral 0 - 10, 10 is not included. Similarly, it is true for other intervals also. Now we shall discuss the above mention points in more details.

4. FREQUENCY DISTRIBUTION

Frequency table or Frequency distribution is a method to present raw data in the form from which one can easily understand the information contained in it. Frequency distributions are of two types :

(i) Discrete frequency distribution

(ii) Continuous or grouped frequency distribution

4.1 DISCRETE FREQUENCY DISTRIBUTION

The process of preparing this type of distribution is very simple. The construction of a discrete frequency distribution from the given raw data is done by the method of tally marks. In the first column of the frequency table we write all possible values of the variable from the lowest to the highest.

We now look at the first value in the given raw data and put a bar (vertical line) in the second column opposite to it. Now, we see the second value in the given raw data and put a bar opposite to it in the second column. This process is repeated till all observations in the given data are exhausted. To facilitate counting blocks of five, ~~||||~~ are prepared and some space is left in between each block. We finally count the number of bars corresponding to each value of the variable and place it in the third column of frequency. The process will be clear from the following example of the number of children in 20 families

1, 1, 2, 3, 4, 3, 2, 1, 1, 4, 5, 2, 4, 2, 2, 1, 3, 3, 2, 5

The data may be put in the form of discrete frequency distribution as follows :

Number of Children	Tally Marks	Frequency
1.	~~||||~~	5
2.	~~||||~~ |	6
3.	||||	4
4.	|||	3
5.	||	2

4.2 CONTINUOUS OR GROUPED FREQUENCY DISTRIBUTION

The above method of condensing the raw data is convenient only where the values in the raw data are less repeating and the difference between the greatest and the smallest observation is not very large. If the number of observations in data is large and the difference between the greatest and the smallest observations is large, then we condense the data into classes or groups. For example, let the marks obtained by 30 students of a class in a test be 39, 25, 5, 33, 19, 21, 12, 48, 13, 21, 9, 1, 10, 8, 12, 17, 19, 17, 41, 40, 12, 46, 37, 17, 27, 30, 6, 2, 23, 19.

We can arrange these marks as follows :

Marks (Class intervals)	Tally Marks	Number of students (Frequency)
0 - 10	~~||||~~ |	6
10 - 20	~~||||~~ ~~||||~~ |	11
20 - 30	~~||||~~	5
30 - 40	||||	4
40 - 50	||||	4

Such a presentation of data is known as the grouped frequency distribution.

In the above example, 30 observations have been divided into 5 groups. These groups are called classes. The class 0 - 10 means the marks obtained between 0 and 10 including 0 and excluding 10. The number of observations falling in a particular class is called the frequency of that class or class frequency. Thus, the class 0 -10 has frequency 6 and the class 10 - 20 has 11 as class frequency. In the class 0 - 10, we say that 0 is the lower limit and 10 is the upper limit of the class. Similarly, in the class 10 - 20, 10 is the lower limit and 20 is the upper limit. The span of the class i.e., the difference between the upper limit and the lower limit, is known as the class size. For example, in the class 10 - 20, the class size is 20 -10 = 10.

5. METHODS OF CLASSIFYING DATA ACCORDING TO CLASS INTERVALS

There are two methods of classifiying the data according to the class intervals, viz. (i) exclusive method, and (ii) inclusive method.

5.1 EXLUSIVE METHOD

When the class intervals are so formed that the upper limit of one class is the lower limit of the next class, it is known as the exclusive method of classification. In this method, the upper limit of a class is not include in the class. Thus, in the class 0 - 10 of marks obtained by students, a student who has obtained 10 marks is not included in this class. It is counted in the next class 10 - 20.

5.2 INCLUSIVE METHOD

In this method, the classes are so formed that the upper limit of a class is included in that class. Following example illustrates the method.

By Inclusive Method :

Wages (in Rs.)	No. of workers
1000-1099	125
1100-1199	150
1200-1299	200
1300-1399	250
1400-1499	175
1500-1599	100
	Total = 1000

In the class 1000 - 1099, we include workers having from Rs. 1000 to Rs. 1099 is included. If the income of worker is exactly Rs. 1100, it is included in the next class 1100 - 1199.

By Exclusive Method :

Wages (in Rs.)	No. of workers	Wages (in Rs.)	No. of workers
1000-1100	125	1000-1099	125
1100-1200	150	1100-1199	150
1200-1300	200	1200-1299	200
1300-1400	250	1300-1399	250
1400-1500	175	1400-1499	175
1500-1600	100	1500-1599	100
	Total = 1000		Total = 1000

It is evident from the above example that both the inclusive and exclusive methods give us the same class frequency, although the class intervals are apparently different in the two cases. In the above example, in case of exclusive method the class size is 100. However, 99 is not the correct class size in case of inclusive method. Whenever inclusive method is used it is necessary to make an adjustment to determine the correct class intervals and to have continuity. If a - b is a class in inclusive method, then in exclusive method it becomes.

$\left(a-\frac{h}{2}\right)-\left(b+\frac{h}{2}\right)$, where h = (lower limit of a class) - (upper limit of previous class)

In the above example, in inclusive method, the difference beween the lower limit of a class and the upper limit of the preceding class i.e., h = 1. Therefore, we subtract 1/2 from the lower limit of each class and add 1/2 in the upper limit of each class to make it continuous. The adjusted classes would then be as follows.

Wages (in Rs.)	No. of workers
999.5 - 1099.5	125
1099.5-1199.5	150
1199.5-1299.5	200
1299.5- 1399.5	250
1399.5 - 1499.5	175
1499.5 - 1599.5	100

The mid-value of a class is called the class mark. For example, the class-mark or mid - value of the class 1000-1100 is 1050. In fact

$$\text{Class mark} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

or, Class mark = Lower limit + $\frac{1}{2}$ (Difference between the upper and lower limits).

6. CONSTRUCTION OF A DISCRETE FREQUENCY DISTRIBUTION

To prepare a discrete fequency distribution from the given raw data, we use the following algorithm.

Step I : Obtain the given raw data.

Step II : Prepare a table with three columns : first for variable under study such as marks, weight height etc., second for 'Tally marks' and third for representing corresponding frequency to each value or size of the variable.

Step III : Place all the values of variable in the first column in ascending order.

Step IV : Take the first observation in the raw data and put a bar in the second column opposite to it. Then take the second observation in the given raw data and put a bar opposite to it. Continue this process till all the observations in the given raw data are exhausted. For the sake of convenience, record tally marks in blocks of five, the fifth, is shown by crossing diagonally the other four. Leave some space between each block of bars.

Step V : Count the number of bars (tally marks) in respect of each value of the variable and place it in the third column.

Step VI : Give suitable title to the frequency distribution table so that it conveys exactly what the table is about.

Example 2

Given below are the ages of 25 students of class VIII in a school. Prepare a discrete frequency distribution.

15, 16, 16, 14, 17, 17, 16, 15, 15, 16, 16, 17, 15, 16, 16, 14, 16, 15, 14, 15, 16, 16, 15, 14, 15.

Solution :

Frequency distribution of ages of 25 students.

Age	Tally	Frequency
14	||||	4
15	~~||||~~ |||	8
16	~~||||~~ ~~||||~~	10
17	|||	3
	Total	25

7. CONSTRUCTION OF A GROUPED FREQUENCY DISTRIBUTION

Following algorihm is used for the construction of grouped frequency distribution.

Step I : Determine the maximum and minimum value of the variate occuring in the data.

Step II : Decide the number of classes to be formed. Note that the number of classes should be in range of 5 to 15.

Step III : Find the difference between the maximum value and minimum value and divide this difference by the number of classes to be formed to determine the class interval. The difference between the maximum value and minimum value in data is called range.

Step IV : Be sure that there must be classes with us to include minimum and maximum value occuring in the data.

Step V : Take each item from the data, one at time and put a tally mark (|) against the class to which the items belongs. If tally marks are more then 4, then record them in the block of five, the fifth one is marked by crossing diagonally the first four.

Step VI : By counting,.determine the total number of tally marks in each class, which gives us the frequency of the class.

Step VII : Check that the total of all frequencies is same as the total number of observations.

Step VIII : Give a suitable title to the frequency table so that it conveys exactly what the table is about.

Example 3

The water tax bills (in rupees) of 30 houses in a locality are given below. Construct a grouped frequency distribution with class size of 10.

30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88, 40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 95, 102, 110, 88, 74, 112, 14, 34, 44.

Solution :

Here, the maximum and minimum values of the variate are 112 and 14 respectively.

$\therefore$ Range = 112 – 14 = 98

It is given that the class size is 10, and $\frac{\text{Range}}{\text{Class size}} = \frac{98}{10} = 9.8$

So, we should have 10 classes each of size 10.

The minimum and maximum values of the variage are 14 and 112 respectively. So, we have to make the classes in such a way that first class includes the minimum value and last class includes the maximum value. If we take the first class as 14-24 it includes the minimum value 14. If the last class is taken as 104-114, then it includes the maximum value 112. Here, we form classes by exlusive method. In the class 14-24, 14 is included but 24 is exluded. Similarly, in other classes, the lower limit is included and the upper limit is excluded.

In the view of above discussion, we construct the frequency distribution table as follows :

Bill (in rupees)	Tally marks	Frequency
14 - 24	||||	4
24 - 34	||	2
34 - 44	|||	3
44 - 54	|||	3
54 - 64	|	1
64 - 74	||	2
74 - 84	~~||||~~	5
84 - 94	|||	3
94 - 104	|||	3
104 - 114	||||	4
	Total	30

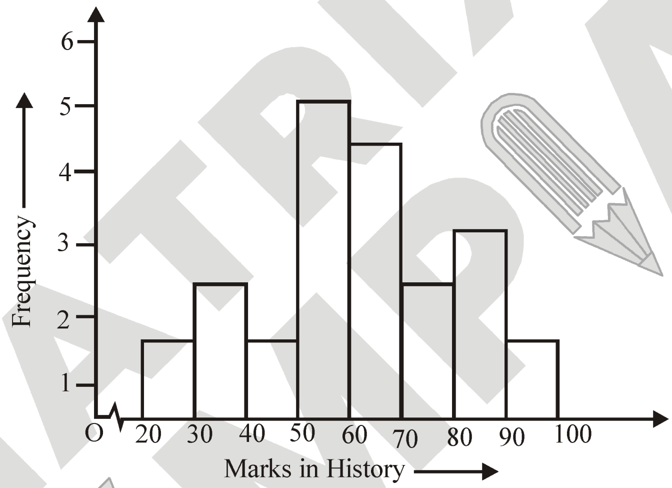
8. GRAPHICAL REPRESENTATION OF FREQUENCY DISTRIBUTION AND HISTOGRAM

In earlier classes, we have learnt how to represent a given infomation by means of bar graphs etc. Now, we will learn how to represent a grouped frequency distribution graphically.

This can be done in a number of ways. Among the commonly used graphical methods, histogram is one. It is a vertical bar graph with no space between bars. To make a histogram, we draw two perpendicular axes. We mark class boundaries of the grouped data on the horizontal axis and the respective class frequencies on the vertical axis, using suitable scale on the axis. In this way, we construct rectangles with respective class intervals as the bases and the corresponding class as heights.

Example 4

The figure shows the histogram for the frequency distribution of marks of 21 students in a History test (Out of 100).



Study the histogram and answer the following questions:

(i) What does the longest rectangle depict ?

(ii) What is the class size ?

(iii) How many students got marks between 80 and 90 ?

Solution :

(i) The longest rectangle depicts the marks obtained by the students in the interval 50-60 in history is maximum.

(ii) The class intervals are 20 - 30, 30 - 40 etc.

$\therefore$ Class size = 10

(iii) Marks between 80 and 90 means marks in the class interval, the corresponding height of the rectangle is 3 units.

$\therefore$ Number of students getting marks between 80 and 90 = 3

9. CIRCLE GRAPH OR PIE CHART

A circle graph or Pie chart or Pie diagram is a circle which is divided into several sectors. Circle represents the total value of the given data and the sectors represent the proportion of the components of the total.

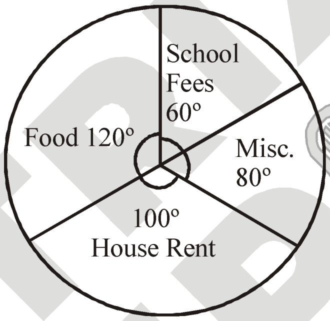
Another name of this diagram is an angular diagram or a circular diagram.

For example, the monthly expenditure on various items of a family is given below:

Item	Food	House Rent	Misc.	School Fees
Amount spent	Rs. 3000	Rs. 2500	Rs. 2000	Rs. 1500

We can represent this infomation on a pie chart as shown below:

Total Expnditure = Rs. 9000



A pie chart

Item	Amount Spent	In Fraction	Central Angle
Food	Rs. 3000	$\frac{3000}{9000} = \frac{1}{3}$	$\frac{1}{3} \times 360° = 120°$
House Rent	Rs. 2500	$\frac{2500}{9000} = \frac{5}{18}$	$\frac{5}{18} \times 360° = 100°$
Misc.	Rs. 2000	$\frac{2000}{9000} = \frac{2}{9}$	$\frac{2}{9} \times 360° = 80°$
School Fees	Rs. 1500	$\frac{1500}{9000} = \frac{1}{6}$	$\frac{2}{6} \times 360° = 60°$

10. DRAWING PIE CHARTS

Let us understand to draw a pie chart with the help of an example.

The favourite games liked by students of a school is given in the percentage form are as follows:

Games	Percentage students who like the games
Cricket	60%
Football	20%
Tennis	20%

Let us represents this data using pie chart.

We know that total angle at the centre of a circle is 360º.

The central angle of the different sectors are fractions of 360º.

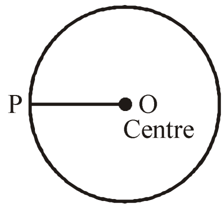
Firstly, we will find the central angle for each game.

Games	Students favouring the games (in %)	In fractions	Central
Cricket	60%	$\frac{60}{100}$	$\frac{60}{100} \times 360^\circ = 216^\circ$
Football	20%	$\frac{20}{100}$	$\frac{20}{100} \times 360^\circ = 72^\circ$
Tennis	20%	$\frac{20}{100}$	$\frac{20}{100} \times 360^\circ = 72^\circ$

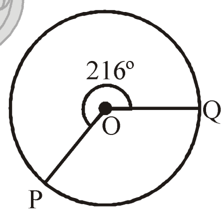
Steps to Draw a Pie Chart :

Step I : Draw a circle of any convenient radius.

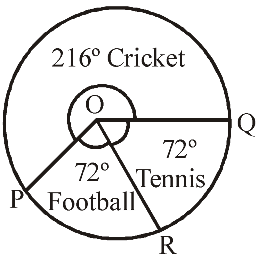
Centre O and radius = OP



Step II : Make the angle of sector for cricket as 216º.



Step III : In the same way, mark the remaining sectors.



Example 5

In a motor factory, five varieties of vehicles was manufactured in one year whose data is given below.

Motor bikes	Cars	Buses	Vans	Mini trucks
6000	3600	1200	800	400

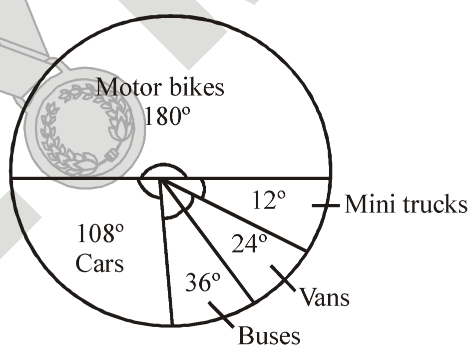
Represent this data as pie chart.

Solution :

Total number of vehicles = 12000

Vehicles	Number	In Fraction	Central Angle
Motor bikes	6000	$\frac{6000}{12000} = \frac{1}{2}$	$\frac{1}{2} \times 360 = 180^\circ$
Cars	3600	$\frac{3600}{12000} = \frac{3}{10}$	$\frac{3}{10} \times 360 = 108^\circ$
Buses	1200	$\frac{1200}{12000} = \frac{1}{10}$	$\frac{1}{10} \times 360 = 36^\circ$
Vans	800	$\frac{800}{12000} = \frac{1}{15}$	$\frac{1}{15} \times 360 = 24^\circ$
Mini trucks	400	$\frac{400}{12000} = \frac{1}{30}$	$\frac{1}{30} \times 360 = 12^\circ$

Now, we will represent the above information on a pie chart.



Chance and Probability

(i) Most probably it will rain today.

(ii) Chances are high that the prices of petrol will go up.

(iii) I doubt that he will win the race.

The word 'most probably', 'chances', 'doubt', etc., shows the probablity of occurence of an event. In our fast going life, we come across words like probably, like, chance, may be, hope etc. All these have the same meaning as probablity.

Probability is defined as the numerical method of measuring uncertainty involved in any situation.

It is used very widely in the field of mathematics, statistics, gambling, weather forecasting, physical science, biological science, etc. to draw any conclusion.

11. SOME EXPERIMENTS AND THEIR OUTCOMES

11.1 TOSSING A COIN

Suppose we toss a coin and left it falls flat on the ground. Its upper face will show either Head (H) or Tail (T).

(i) Whatever comes up, is called an outcome.

(ii) Possible outcomes are Head (H) and Tail (T).

11.2 THROWING A DICE

A dice is a solid cube having 6 faces, marked as 1, 2, 3, 4, 5, 6 respectively.

Suppose we throw a dice and let it fall flat on the ground. Its upper face will show one of the numbers 1, 2, 3, 4, 5, 6.

The act if tossing coin or throwing a dice is called an experiment.

Whatever comes up, is called an outcome.

In an experiment, all possible outcomes (i.e., 1, 2, 3, 4, 5, 6) are known.

12. SOME TERMS RELATED TO PROBABILITY

12.1 EXPERIMENT

An operation which can produce some well-defined outcomes is called an experiment. Each outcome is called an event.

12.2 RANDOM EXPERIMENT

An experiment in which all possible outcome are known and the exact outcome cannot be predicted in advance, is called a random experiment. Thus, when we throw a coin we know that all possible outcomes are Head and Tail. But, if we throw a coin at random, we cannot predict in advance whether its upper face will show a head or a tail.

So, tossing a coin is a random experiment.

Similarly, throwing a dice is a random experiment.

12.3 TRIAL

By a trial, we mean performing a random experiment.

Probability of occurrence of an event:

$$\text{P(occurrence of an event)} = \frac{\text{Number of trials in which event occured}}{\text{Total number of trials}}.$$

Competition window :

A pack of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each.

Each suit consists of one ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are named as spades (♠), hearts (♥), diamonds (♦), and clubs (♣).

P (not occuring an event) = 1 – Probability of occuring an event.

Example 6

An unbised die is thrown. What is the probability of getting :

(i) an even number

(ii) a multiple of 3

(iii) an even number or a multiple of 3

(iv) an even number and a multiple of 3

(v) a number 3 or 4

(vi) an odd number

(vii) a number less than 5

(viii) a number greater than 3

(ix) a number between 3 and 6.

Solution :

In a single throw of a die, we can get any one of the six numbers 1, 2,.....,6 marked on its six faces. Therefore, the total number of elementary events associated with the random experiment of throwing a die is 6.

(i) Let A denotes the event 'Getting an even number'.

Clearly event A occurs if we obtain any one of 2, 4, 6 as an outcome.

∴ Favourable number of outcomes = 3

Hence, $P(A) = \frac{3}{6} = \frac{1}{2}$

(ii) Let A denotes the event 'Getting a multiple of 3'.

We observe that the event A occurs if we obtain either 3 or 6 as an outcome.

∴ Favourable number of outcomes = 2

Hence, P(A) = $\frac{2}{6} = \frac{1}{3}$

(iii) An even number or a multiple of 3 is obtained if we obtain one of the numbers 2, 3, 4, 6 as an outcome.

$\therefore$ Favourable number of outcomes = 4

Hence, required probability = $\frac{4}{6} = \frac{2}{3}$

(iv) Let A denotes the event 'Getting an even number and multiple 3'.

Clearly, even A happens if we get 6 as an outcome.

$\therefore$ Favourable number of outcomes = 1

Hence, P(A) = $\frac{2}{6}$

(v) Let A deontes the event 'Getting 3 or 4'

Clearly, A occurs when we get either 3 or 4 as an outcome.

$\therefore$ Favourable number of outcomes = 2

Hence, P(A) = $\frac{2}{6} = \frac{1}{3}$

(vi) Let A deontes the event 'Getting an odd number'

We observe that the event A occurs when we get 1 or 3 or 5 as outcome.

$\therefore$ Favourable number of outcomes = 3

Hence, P(A) = $\frac{3}{6} = \frac{1}{2}$

(vii) The event 'Getting a number less than 5' will occur if we get one of the numbers 1,2,3,4 as an outcome.

$\therefore$ Favourable number of outcomes = 4

Hence, P(A) = $\frac{4}{6} = \frac{2}{3}$

(viii) Then event 'Getting a number between 3 will occurs if we obtain one of the number 4,5,6 as an outcome.

$\therefore$ Favourable number of outcomes = 3

Hence, required probability = $\frac{3}{6} = \frac{1}{2}$

(ix) The event 'Getting a number 3 and 6' occurs if we obtain either 4 or 5 as an outcome.

$\therefore$ Favourable number of outcomes = 2

Hence, required probability = $\frac{2}{6} = \frac{1}{3}$

Example 7

What is the probability that a number selected from the numbers 1,2,3, ... , 25 is a prime number, when each of the given numbers is equally likely to be selected ?

Solution :

Out of 25 numbers 1,2,3,......25, one number can be chosen in 25 ways.

$\therefore$ Total number of elementary events = 25

The number selected will be a prime number if it is chosen from the numbers 2,3,5,7,11,13, 17, 19, 23.

$\therefore$ Favourable number of outcomes = 9

Hence, required probability = $\frac{9}{25}$.

SE. 1

The given data shows the marks obtained by 25 students in a class test 74, 64, 70, 31, 41, 53, 61, 64, 70, 56, 28, 28, 88, 53, 56, 31, 53, 64, 56, 32, 53, 56, 61, 53, 74, 33 Find:

(i) the maximum marks obtained

(ii) the minimum marks obtained

(iii) the range

(iv) the mean marks

Ans. Arranging the given data in an ascending order, we get marks as 28, 31, 31, 32, 33, 41, 53, 53, 53, 53, 53, 56, 56, 56, 56, 61, 61, 64, 64, 64, 70, 70, 74, 74, 88

Clearly,

(i) the maximum marks obtained is 88.

(ii) the minimum marks obtained is 28.

(iii) Range = (88 – 28) = 60.

(iv) mean marks = $\dfrac{\text{sum of observations}}{\text{number of observations}}$

$= \left(\dfrac{1375}{25}\right) = 55$

SE. 2

A selection test was given to a group of 50 students. The test was completed by them in the following times (in minutes):

38, 40, 42, 41, 39, 27, 28, 26, 30, 42, 41, 43, 45, 46, 37, 37, 43, 44, 49, 36, 31, 32, 33, 35, 48, 43, 39, 36, 29, 31, 32, 34, 44, 43, 36, 37, 38, 40, 49, 41, 42, 45, 47, 48, 45, 39, 38, 37, 40, 29.

Prepare a grouped frequency distribution table taking class intervals 25-30, 30-35 etc.

Ans.

Class interval	Tally	Frequency
25-30	卌	5
30-35	卌 ||	7
35-40	卌 卌 ||||	14
40-45	卌 卌 卌	15
45-50	卌 ||||	9
	Total	50

SE. 3

Form a discrete frequency distribution from the following scores :

15, 18, 16, 20, 25, 24, 25, 20, 16, 15, 18, 18, 16, 24, 15, 20, 28, 30, 27, 16, 24, 25, 20, 18, 28, 27, 25, 24, 24, 18, 18, 25, 20, 16, 15, 20, 27, 28, 29, 16.

Ans. Frequency distribution of scores.

Variate	Tally Marks	Frequency
15	||||	4
16	卌 |	6
18	卌 |	6
20	卌 |	6
24	卌	5
25	卌	5
27	|||	3
28	|||	3
29	|	1
30		1
	Total	40

SE. 4

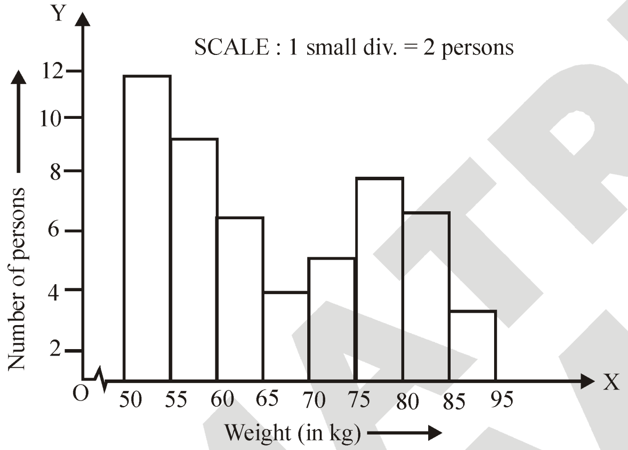
The following is the distribution of weights (in kg) of 50 persons:

Weight (in kg)	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
No. of persons	12	8	5	4	5	7	6	3

Draw a histogram for the above data.

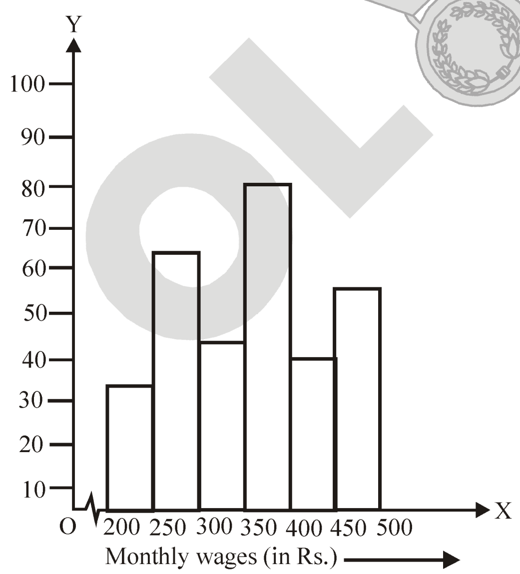
Ans. We represent the class limits along X-axis using a suitable scale and the frequencies along Y-axis using a suitable scale.

Since the scale on X -axis starts at 50, a kink (break) is indicated near the origin to signify that the graph is drawn to scale beginning at 50, and not at the origin.



SE. 5

The following histogram shows the monthly wages (In Rs.) of workers in a factory:



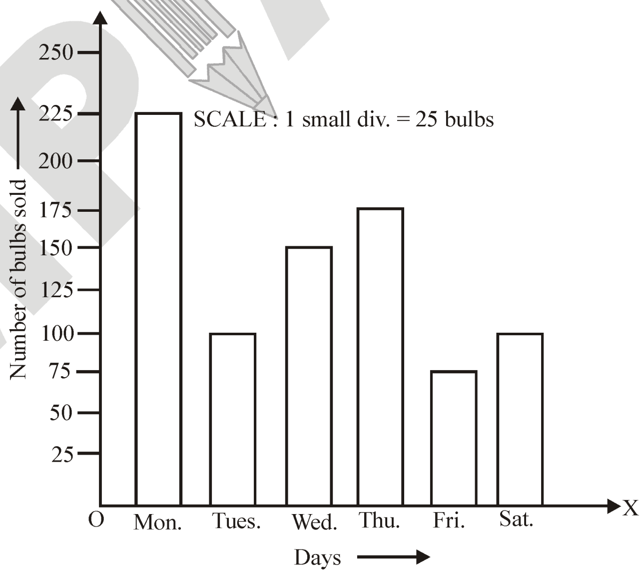
Which groups have the largest and least number or works ? Also find their number.

Ans. In the above histogen, we see that the highest rectangle corresponds to the maximum number of workers, that is 100, and the corresponding group is from Rs. 350 - 400.

The rectangle of minimum height corresponds to the least number of workers, that is 15 and the corresponding group is Rs. 400 -450.

SE. 6

Given below is a graph which shows the number of electric bulbs sold in a shop during a week.



Read the the bar graph carefully and answer the questions given below:

(i) On which day of the week was the sale minimum?

(ii) On which day of the week was the sale maximum ?

(iii) What was the total sale during the week ?

(iv) What is the ratio between the minimum sale and the maximum sale ?

Ans. (i) It is clear from the bar graph that the bar of minimum height corresponds to the sale on Friday.

∴ The sale was minimum of Friday.

(ii) From the bar graph, we find that the bar of maximum height corresponds to the sale on Monday.

∴ The sale was maximum of Monday.

(iii) The total sale during the week = (225 + 100 + 150 + 200 + 75 + 100) bulbs = 850 bulbs.

(iv) The minimum sale during the week = 75 bulbs.

The maximum sale during the week = 225 bulbs.

∴ Minimum sale : Maximum sale = 75 : 225

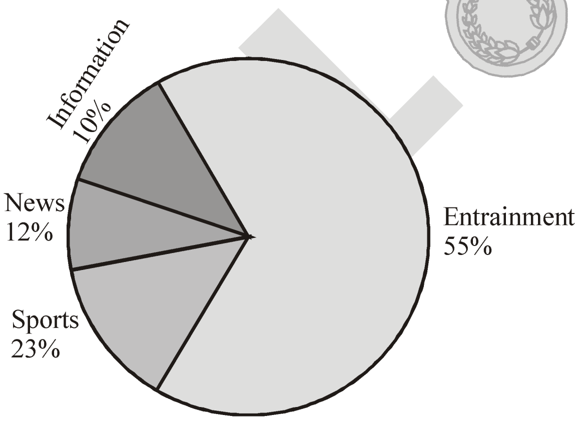
= 1 : 3.

SE. 7

Answer the following questions based on the pie chart given below :

(i) Which type of programmes are viewed the most ?

(ii) Which two types of programmes together have viewers almost equal to those watching sports channels ?



Viewers waching different types of channels on TV.

Ans. (i) The entertainment programmers are viewed the most.

(ii) Information and News channels.

SE. 8

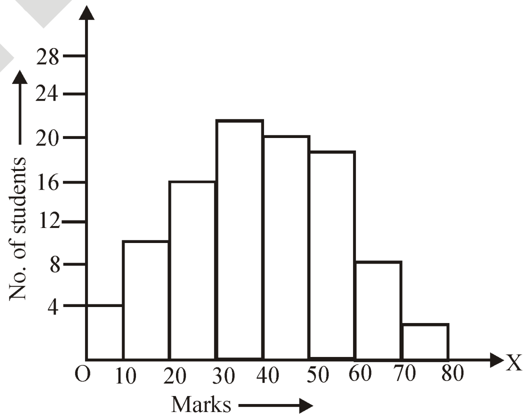
The following table gives the marks scored by 100 students in an entrance examination.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students (Frequency)	4	10	16	22	20	18	8	2

Represent this data in the form of a histogram.

Ans. We represent the class intervals along X-axis using a suitable scale and the frequencies along Y-axis using a suitable scale.

Taking class-intervals as bases and the corresponding frequencies as heights, we construct rectangles to obtain the histogram of the given frequency distribution as shown.



SE. 9

The data on the mode of transport used by 720 students are given below:

Mode of Transport	Bus	Cycle	Train	Car	Scooter
No. of students	120	180	240	80	100

Represent the above data by a pie chart.

Ans. Total number of students = 720

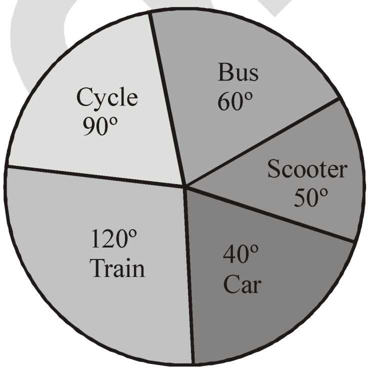
Central angle for a mode of transport

$$= \left(\frac{\text{number of students using that mode}}{\text{total number of students}} \times 360\right)^\circ$$

Mode of transport	Number of students	Central angle
Bus	120	$\left(\frac{120}{720}\times 360\right)^\circ = 60^\circ$
Cycle	180	$\left(\frac{180}{720}\times 360\right)^\circ = 90^\circ$
Train	240	$\left(\frac{240}{720}\times 360\right)^\circ = 120^\circ$
Car	80	$\left(\frac{80}{720}\times 360\right)^\circ = 40^\circ$
Scooter	100	$\left(\frac{100}{720}\times 360\right)^\circ = 50^\circ$

Steps of construction of pie chart:

1. Draw a circle of any convenient radius.
2. Draw a radius of this circle.
3. Starting with this radius, draw sectors whose central angles are 60°, 90°, 120°, 40°, and 50° respectively.
4. Shade the sectors so obtained different and label each one of them.



Thus, we obtain the required pie chart, as shown in the given figure.

SE. 10

Pooja spends different hours of a working day as follows :

Activity	School	Home assignment	Play	Sleep	Other
No. of hours	8	3	2	8	3

Draw a pie chart to represent the above data.

Ans. We know that central angle for a variable

$$= \frac{\text{Frequency of the variable}}{\text{Total frequencies}} \times 360^\circ$$

Total frequencies = 8 + 3 + 2 + 8 + 3 = 24

$\therefore$ Central angle for school = $\frac{8}{24} \times 360^\circ = 120^\circ$

Central angle for home assignments = $\frac{3}{24} \times 360^\circ$

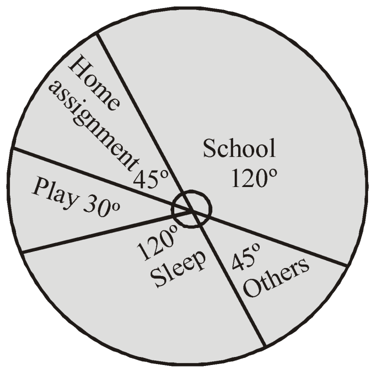
= 45°

Central angle for play = $\frac{2}{24} \times 360^\circ = 30^\circ$

Central angle for sleep = $\frac{8}{24} \times 360^\circ = 120^\circ$

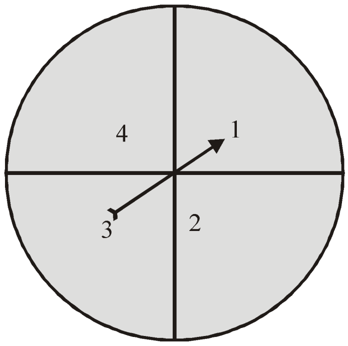
Central angle for other activities = $\frac{3}{24} \times 360^\circ$

= 45°



SE. 11

A spinner as shown in figure has spun 80 times. How many times do you expect the number '3' ?



Ans. When this spinner spun one time, the possible outcomes are 1, 2, 3, 4.

$$= \frac{\text{Number of favourable outcome}}{\text{Total number of outcome}}$$

$\therefore$ Probability of getting '3' = $\frac{1}{4}$

In 80 times, one can expect '3' to come

$= \frac{1}{4} \times 80 = 20$ times.

SE. 12

A survey of 250 families shows the results given below:

Number of girls in the family	2	1	0
Number of families	45	165	40

Out of these families one is chosen at random. What is the probability that the chosen family has 1 girl?

Ans. Total number of families = 250

Number of families having 1 girl = 165.

Probability of getting a family having 1 girl

$$= \frac{\text{Number of families having 1 girl}}{\text{Total number of families}} = \frac{165}{250} = \frac{33}{50}$$

SE. 13

Tickers numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is multiple of 3 or 7?

Ans. Out of 20 tickets numbered from 1 to 20, one can be chosen in 20 ways. So, total number of elementary events associated with the given random experiment is 20. Out of 20 tickets numbered 1 to 20, tickets bearing numbers which are multiple 3 or 7 bear numbers 3, 6, 7, 9, 12, 14, 15 and 18.

Favourable number of elementry events = 8

Hence, required probability $= \frac{8}{20} = \frac{2}{5}$.

SE. 14

A bag has 4 red balls and 2 yellow balls. (The balls are identical in all respects other than colour). A ball is drawn from the bag without looking into the bag. What is the probability of getting red ball ? Is it more or less than getting a yellow ball ?

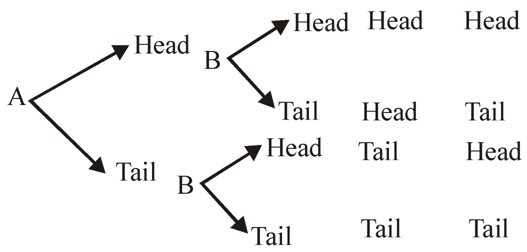
Ans. There are in all (4 + 2 = 6) outcomes of the event. Getting a red ball consists of 4 outcomes. Therefore, the probability of getting red ball is $\frac{4}{6} = \frac{2}{3}$.

In the same way, probability of getting a yellow ball $= \frac{2}{6} = \frac{1}{3}$. Therefore, the probability of getting a red ball is more than that of getting a yellow ball.

SE. 15

If two coins are tossed together, what are the chances of getting 'head, head' ?

Ans. Let us suppose the coins are marked as A and B. Each event of A will be matched with both events of B and we get



∴ All possible outcomes = Head Head, Head Tail, Tail Head, Tail Tail

We can see that out of 4 outcomes, 'Head Head' occurs only once.

∴ Hence, required probability = $\frac{1}{4}$.

Space for Notes :

EXERCISE – 5.1

NS. 1

For which of these would you use a histogram to show the data ?

(a) The number of letters for different area in postman's bag.

(b) The height of competitors in an athletics meet.

(c) The number of passengers boarding trains from 7: 00 a.m. to 7 : 00 p.m. at a station.

Ans. (a) We use pictograph for the given statement. Since, letters can be represented through appropriate picture or symbols, we will not use histogram.

(b) We use histogram for the given statement since the height of competitors in an athletics meet can be divided into class intervals.

(c) We will not use histogram. We use pictograph for the given statement since the cassettes can be represented through appropriate picture or symbols.

(d) We use histogram for the given statement since the time of boarding trains from 7 :00 a.m. to 7: 00 p.m. can be divided into class intervals.

NS. 2

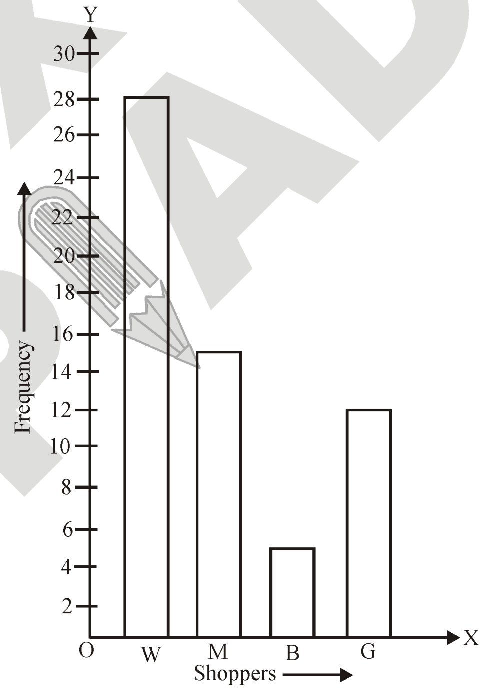
The shoppers who come to a departmental store are marked as : man (M), woman (W), boy (B) or girl (G). The following list gives the shoppers who came during the first hour in the morning :

W W W G B W W M G G M M W W W W G B M W B G G M W W M M W W W M W B W G M W W W W G W M M W W M W G W M G W M M B G G W

Make a frequency distribution table using tally marks. Draw a bar graph to illustrate it.

Ans. Fristly by using a given information, we make a frequency distribution table by using tally marks.

Shopper	Tally marks	Frequency
W	卌 卌 卌 卌 卌 |||	28
M	卌 卌 卌	15
B	卌	5
G	卌 卌 ||	12



NS. 3

The wekkly gages (in Rs.) of workers in a factory are:

830, 835, 810, 835, 836, 869, 845, 898, 890, 820, 860, 832, 833, 855, 845, 804, 808, 812, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840.

Using tally marks, make a frequency table with intervals as 800 – 810, 810 – 820 and so on.

Ans. By using a given information we have to make a frequency table

Class Interval	Tall marks	Frequency
800-810	III	3
810-820	II	2
820-830	I	1
830-840	𝍸 IIII	9
840-850	𝍸	5
850-860	I	1
860-870	III	3
870-880	I	1
880-890	I	1
890-900	IIII	4
	Total	30

NS. 4

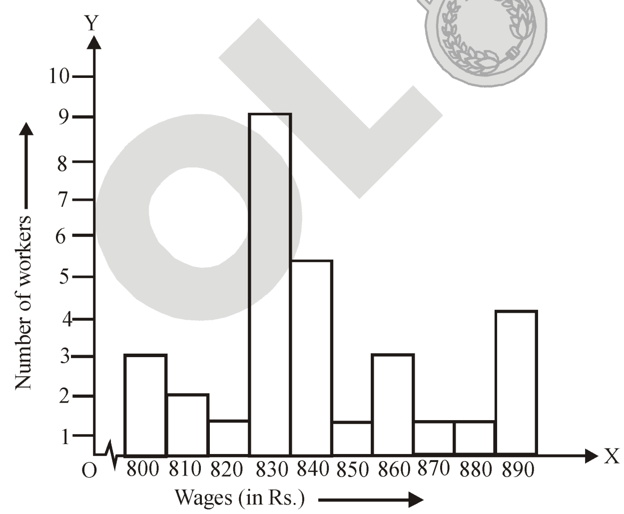
Draw a histogram for the frequency table made for data in Question 3, and answer the following questions.

(i) Which group has the maximum number of workers ?

(ii) How many workers earn Rs. 850 and more ?

(iii) How many workers earh has less than Rs. 850 ?

Ans. We have to draw a histogram :



(i) 830-840 is a group which has the maximum number of workers.

(ii) 10 workers earn Rs. 850 and more.

(iii) 20 workers earn less than Rs. 850.

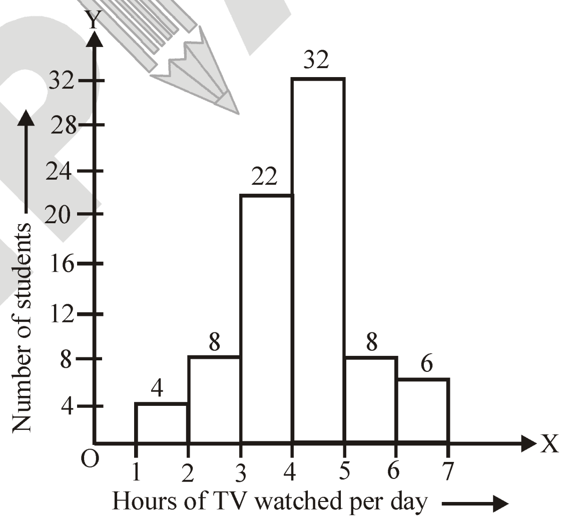
NS. 5

The number of hours for which students of a particular class watched television during holidays is shown through the given graph.

(i) For how many hours did the maximum number of students watch TV ?

(ii) How many students watches TV for less than 4 hours ?

(iii) How many students spent more than 5 hours in watching TV ?



Ans. (i) The above given graph shows that in between 4 - 5 hours, maximum number of students used to watch T.V.

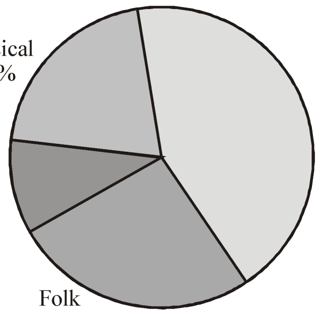
(ii) Number of students watched T.V. for less than 4 hours = 4 + 8 + 22 = 34.

(iii) Number of students spent more than 5 hours in watching T.V. = 8 + 6 = 14.

EXERCISE – 5.2

NS. 1

A survey was made to find the type of music that a certain group of young people liked in a city. Adjoining pie chart shows the finding of this survey.



From this pie chart answer the following:

(i) If 20 people liked classical music, how many young people were surveyed ?

(ii) Which type of music is liked by the maximum number of people ?

(iii) If a cassette company were to make 1000 CD's, how many of each type would they make ?

Ans. (i) Let total number of x young people were surveyed.

$\therefore$ According to question, $x \times \frac{10}{100} = 20$

$\Rightarrow x = 20 \times \frac{100}{10} = 200.$

Thus, 200 young people were surveyed.

(ii) Pie chart shows that maximum number of people like light music.

(iii) Cassette company has to make 1000 CD's.

$\therefore$ Number of CD's of semi classical music

$= 1000 \times \frac{20}{100} = 200$

Number of CD's of classical music

$= 1000 \times \frac{10}{100} = 100$

Number of CD's of folk music

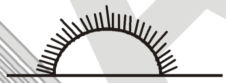
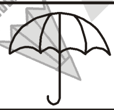
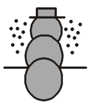
$= 1000 \times \frac{30}{100} = 300$

Number of CD's of light music

$= 1000 \times \frac{40}{100} = 400.$

NS. 2

A group of 360 people were asked to vote for their favourite season from the three seasons rainy, winter and summer.

Season	Number votes
Summer 	90
Rainy 	120
Winter 	150

(i) Which season got the most votes ?

(ii) Find the central angle of each sector.

(iii) Draw a pie chart to show this information.

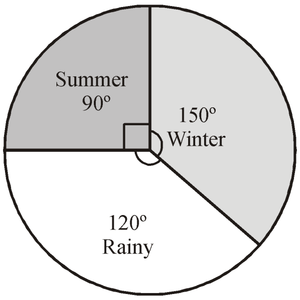
Ans. (i) Winter season got the most votes.

(ii) Total number of votes = 90 + 120 + 150 = 360

Total angle of one revolution of a circle = 360°

Seasons	Votes	In fraction	Central Angle
Summer	90	$\frac{90}{360}$	$\frac{90}{360} \times 360^\circ = 90^\circ$
Rainy	120	$\frac{120}{360}$	$\frac{120}{360} \times 360^\circ = 120^\circ$
Winter	150	$\frac{150}{360}$	$\frac{150}{360} \times 360^\circ = 150^\circ$

(iii) Now we have to draw a pie chart to show the given information.



NS. 3

Draw a pie chart showing the following information. The table shows the colours preferred by a group of people.

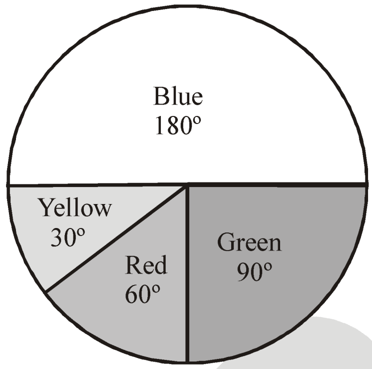
Colours	Number of people
Blue	18
Green	9
Red	6
Yellow	3
Total	36

Find the proportion of each sector. For example, Blue is $\frac{18}{36}=\frac{1}{2}$; Green $\frac{9}{36}=\frac{1}{4}$ and so on. Use this to find the corresponding angles.

Ans.

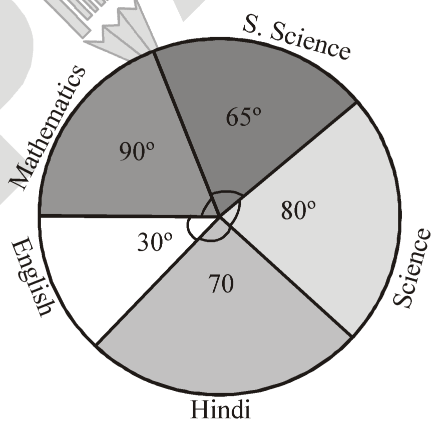
Colours	No. of people	Fraction	Central Angle
Blue	18	$\frac{18}{36}$	$\frac{18}{36}\times 360=180^\circ$
Green	9	$\frac{9}{36}$	$\frac{9}{36}\times 360=90^\circ$
Red	6	$\frac{6}{36}$	$\frac{6}{36}\times 360=60^\circ$
Yellow	3	$\frac{3}{36}$	$\frac{3}{36}\times 360=30^\circ$
Total	36		

Pie Chart :



NS. 4

The given pie chart gives the makes scored in an examination by a student in Hindi, English, Mathematics, Social Science. If the total marks obtained by the students were 540, answer the following questions.



(i) In which subject did the student score 105 marks ?

(ii) How many more marks were obtained by the students in Mathematics than in Hindi ?

(iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.

Ans. (i) Total marks scored by students = 540

Central angle

$$=\frac{\text{Marks scored in a subject}}{\text{Total marks}}\times 360^\circ$$

$= \frac{105}{540} \times 360 = 70^\circ$

Since central angle is 70°, thus we found that in Hindi, student scored 105 marks.

(ii) Marks scored in Mathematics

$= \text{Central angle} \times \frac{\text{Total marks}}{360^\circ} = 90^\circ \times$

$\frac{540}{360^\circ} = 135$

Thus marks obtained by the student in Mathematics more than in Hindi = 135 – 105 = 30.

(iii) Marks obtained in Social Science

$= \text{Central angle} \times \frac{\text{Total marks}}{360^\circ}$

$= 65^\circ \times \frac{540}{360^\circ} = 97.5$

Marks obtained in Science

$= \text{Central angle} \times \frac{\text{Total marks}}{360^\circ}$

$= 80^\circ \times \frac{540}{360^\circ} = 120$

So, marks obtained in Social Science and Mathematics = 97.5 + 135 = 232.5

And marks obtained in Science and Hindi = 120 + 105 = 225

Thus (i) and (ii) shows that marks obtained in Mathematics and Social Science is more than that in Science and Hindi.

NS. 5

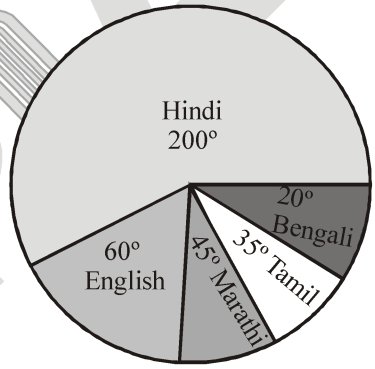
The number of students in a hostel, speaking different languages is given below. Display the data in a pie chart.

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of students	40	12	9	7	4	72

Ans.

Language	No. of Students	Fraction	Central Angle
Hindi	40	$\frac{40}{72}$	$\frac{40}{72} \times 360^\circ = 200^\circ$
English	12	$\frac{12}{72}$	$\frac{12}{72} \times 360^\circ = 60^\circ$
Marathi	9	$\frac{9}{72}$	$\frac{9}{72} \times 360^\circ = 45^\circ$
Tamil	7	$\frac{7}{72}$	$\frac{7}{72} \times 360^\circ = 35^\circ$
Bengali	4	$\frac{4}{72}$	$\frac{4}{72} \times 360^\circ = 20^\circ$
Total	72		

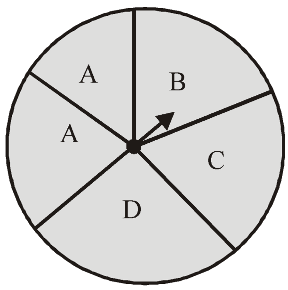
The pie chart is shown below:



EXERCISE – 5.3

NS. 1

List the outcomes you can see in these experiments.



(a) Spinning a wheel

(b) Tossing two coins together.

Ans. (a) After spinning a wheel, we obtain outcomes: A, B, C and D.

(b) After tossing two coins together, we get the following outcomes:

(i) HT i.e., Head on first coin and Tail on the second coin.

(ii) HH i.e., Head on both the coins.

(iii) TH i.e., Tail on first coin and Head on second coin.

(iv) TT i.e., Tail on both the coins.

NS. 2

When a die is thrown, list the outcomes of an event of getting.

(i) (a) a ptime number.

(b) not a prime number.

(ii) (a) a number greater than 5.

(b) a number not greater than 5.

Ans. Since numbers on a die are 1, 2, 3, 4, 5 and 6.

After throwing a die, we get

(i) (a) 2, 3, 5 as prime numbers.

(b) 1, 4, 6 are not prime numbers.

(ii) (a) 6 which is a number greater than 5.

(b) 1,2, 3, 4, 5 are not greater than 5.

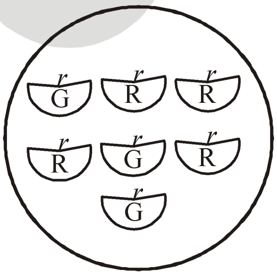
NS. 3

Find the

(a) Probability of the pointer stopping D in (Question 1(a)).

(b) Probability of getting an ace from a well shuffled deck of 52 playing cards.

(c) Probability of getting a red apple. (See figure below).



Ans. (a) Since outcomes are A, A, B, C, D, i.e., total 5 outcomes.

D occurs only once in a spinning wheel Then,

Probability of the pointer stopping on D

$$= \frac{\text{Favourable outcome}}{\text{Total number of outcome}} = \frac{1}{5}$$

(b) Number of aces = 4.

Total number of cards = 52

$\therefore$ Probability of getting an ace from a well shuffled deck of 52 playing cards.

$$= \frac{\text{Favourable outcome}}{\text{Total number of outcome}} = \frac{4}{52} = \frac{1}{13}.$$

(c) Number of red apples = 4

Total number of apples = 7

$\therefore$ Probability of getting a red apple

$$= \frac{\text{Favourable outcome}}{\text{Total number of outcome}} = \frac{4}{7}.$$

NS. 4

Numbers 1 to 10 are written on ten separate slips (on number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of

(i) getting a number 6?

(ii) getting a number less than 6?

(iii) getting a number greater than 6?

(iv) getting a 1-digit number ?

Ans. Total number of outcomes = 10.

(i) Favourable outcomes of getting a number 6 = 1.

Probability of getting a number 6

$$= \frac{\text{Favourable outcomes of getting a number 6}}{\text{Total number of outcomes}} = \frac{1}{10}.$$

(ii) Favourable outcomes of getting number less than 6 = 5 i.e., 1,2, 3, 4, 5

∴ Probability of getting a number less than 6

$$= \frac{\text{Favourable outcomes of getting a number less than 6}}{\text{Total number of outcomes}} = \frac{5}{10} = \frac{1}{2}.$$

(iii) Favourable outcomes of getting a number geater than 6 = 4 i.e., 7, 8, 9, 10

∴ Probability of getting a number greater than 6

$$= \frac{\text{Favourable outcomes of getting a number greater than 6}}{\text{Total number of outcomes}} = \frac{4}{10} = \frac{2}{5}.$$

(iv) Favourable outcomes of getting a 1-digit number = 9 i.e., 1, 2, 3, 4, 5, 6, 7, 8, 9

∴ Probability of getting a 1-digit number

$$= \frac{\text{Favourable outcomes of getting a 1 – digit number}}{\text{Total number of outcomes}} = \frac{9}{10}.$$

NS. 5

If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probabiliy of getting a green sector ? What is the probability of getting a non blue sector ?

Ans. Total number of outcomes = 5.

Favourable number of outcomes of getting a green sector = 3.

Favourable number of outcomes of getting a non-blue sector = 4.

Thus, probability of getting a green sector

$$= \frac{\text{Favourable outcomes of getting a green sector}}{\text{Total number of outcomes}} = \frac{3}{5}.$$

Probability of getting a non-blue sector

$$= \frac{\text{Favourable outcomes of getting a non – blue sector}}{\text{Total number of outcomes}} = \frac{4}{5}.$$

NS. 6

Find the probability of events given in Question 2.

Ans. Total number of outcomes = 6.

(i) (a) probability of getting a primer number

$$= \frac{\text{Favourable outcomes of getting a prime number}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}.$$

(b) Probability of not getting a prime number

$$= \frac{\text{Favourable outcomes of not getting a prime number}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}.$$

(ii) (a) Probability of getting a number greater than 5

$$= \frac{\text{Favourable outcomes of getting a number greater than 5}}{\text{Total number of outcomes}} = \frac{1}{6}.$$

(b) Probability of getting a number not greater than 5

$$= \frac{\text{Favourable outcomes of getting a number not greater than 5}}{\text{Total number of outcomes}} = \frac{5}{6}.$$

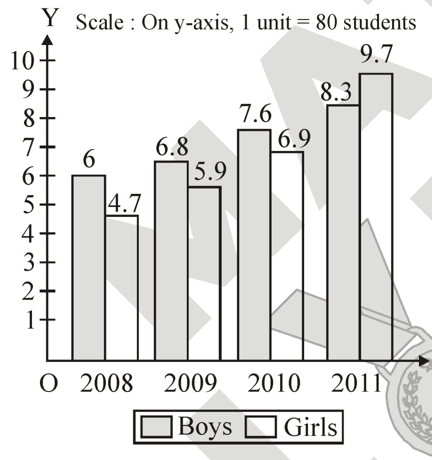
EXERCISE – I

ONLY ONE CORRECT TYPE

1. The range of x, 32, 41, 62, 64 and 71 is 45. Which of the following can be the value of x ?
(A) 32 (B) 47
(C) 48 (D) 26

2. In a pie chart representing the percentages of students having interest in reading various kinds of books, the central angle of the sector representing students reading novels is 81°. What is the percentage of students interested in reading novels ?
(A) 15 % (B) 18 %
(C) $22\frac{1}{2}\%$ (D) $27\frac{1}{2}\%$

Directions (3 – 7) : Read the following bar graph and answer the questions.



Bar graph of number of boys and number of girls in a school from 2008 to 2011.

3. In which year the difference between the number of boys and the number of girls is maximum ?
(A) 2008 (B) 2009
(C) 2010 (D) 2011

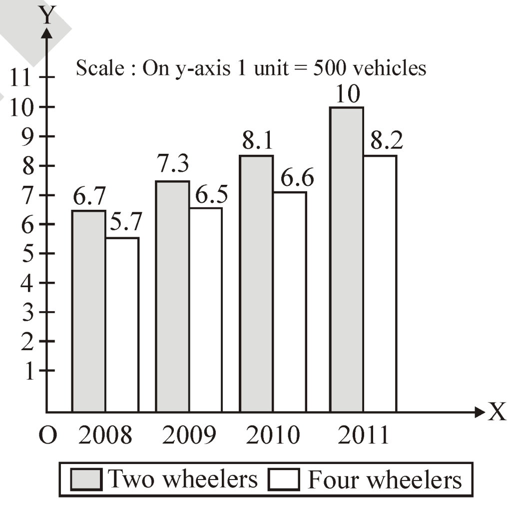
4. Total number of students in the year 2009 is
(A) 1016 (B) 1270
(C) 1380 (D) 1490

5. Find the minimum difference between the number of boys and girls in any year in the given period.
(A) 90 (B) 70
(C) 56 (D) 30

6. In which year the number of girls is more than the number of boys ?
(A) 2008 (B) 2009
(C) 2010 (D) 2011

7. Find the ratio between the number of students in the year 2008 and in 2009.
(A) 107 : 145
(B) 127 : 145
(C) 29 : 36
(D) 107 : 127

Directions (8 – 12) : Read the bar graph and answer the following questions.



Bar graph represents sale of two wheelers and four wheelers in a city from 2008 to 2011.

8. In which year the difference between sales of two wheelers and four wheelers is minimum ?
(A) 2008 (B) 2009
(C) 2010 (D) 2011

9. Total number of vehicles (two wheelers and four wheelers) sold in the year 2008 and 2009 is
(A) 26100 (B) 28500
(C) 13100 (D) 27500

10. Find the maximum difference between sale of two wheelers and that of four wheelers, in any year, in the given period.
(A) 900 (B) 1700
(C) 1800 (D) 2000

11. Find the total number of two wheelers sold in four years.
(A) 16050
(B) 27000
(C) 31000
(D) 32000

12. Find the ratio between number of vehicles sold in the year 2009 and in the year 2011.
(A) 41 : 46 (B) 69 : 91
(C) 147 : 182 (D) 46 : 49

13. Probability of drawing a consonant randomly from English alphabets is
(A) $\frac{21}{26}$ (B) $\frac{1}{26}$
(C) 0 (D) $\frac{1}{5}$

14. Three coins are tossed simultaneously. What is the probability that head and tail show alternately (i.e., HTH or THT) ?
(A) $\frac{3}{8}$ (B) $\frac{1}{4}$
(C) $\frac{1}{8}$ (D) $\frac{1}{2}$

15. A card is drawn from a well shuffled pack of 52 cards. The probability of drawing a king is
(A) $\frac{1}{52}$ (B) $\frac{2}{13}$
(C) $\frac{1}{26}$ (D) $\frac{1}{13}$

16. A bag contains 5 red balls, 3 blue balls and 1 white ball. A ball is drawn at random without looking into the bag. What is the probability of getting a blue ball ?
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{5}{9}$ (D) $\frac{1}{9}$

17. From the frequency table, the percentage of families with less than 3 children is

Number of children	0	1	2	3	4
Number of families	4	6	12	15	7

(A) 45 % (B) 50 %
(C) 11 % (D) 18 %

18. The difference between upper limit and lower limit is known as
(A) Class size (B) Class mark
(C) Class interval (D) None of these

19. The number of times a particular observation occurs is called its
(A) Range (B) Frequency
(C) Observation (D) None of these

20. The mid-value of a class interval is called its
(A) Class mark (B) Class size
(C) Frequency (D) Range

21. In a single throw of two dice, find the probability of getting a total of 3 or 5.

(A) $\frac{1}{3}$ (B) $\frac{5}{6}$

(C) $\frac{1}{9}$ (D) $\frac{1}{6}$

22. A bag contains x red balls, (x + 5) blue balls and (3x + 10) white balls. If the probability of drawing a white ball is $\frac{11}{18}$, what is the number of blue balls ?

(A) 15

(B) 20

(C) 35

(D) 55

23. The number of students in a hostel speaking different languages is given. The central angle for Tamil is

Language	Number of students
Hindi	40
English	12
Marathi	9
Tamil	7
Bengali	4
Total	72

(A) 75° (B) 45°

(C) 35° (D) 32°

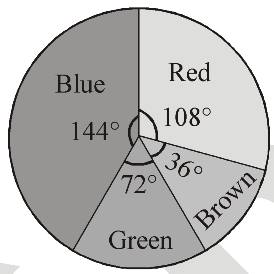
24. Monthly savings (in rupees) of 14 students of class VIII are as follows :

53, 80, 43, 90, 64, 20, 24, 30, 53, 56, 60, 64, 53, 40. The frequency of the observation 53 is

(A) 2 (B) 4

(C) 1 (D) 3

25. Hridya asked 20 of his friends about their favourite colour. The pie chart below shows the data he collected. What percentage of his friends picked blue ?



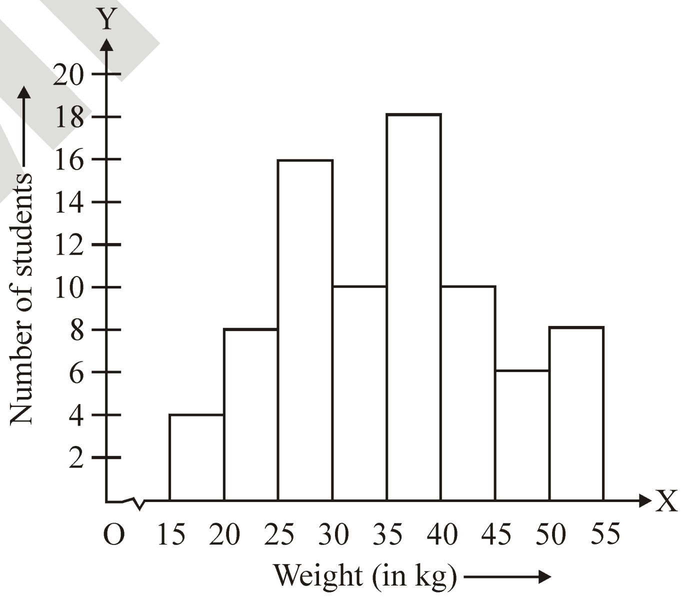
(A) 30 % (B) 10 %

(C) 40 % (D) 20 %

PARAGRAPH TYPE

PASSAGE # I

Given below is the histogram showing weight (in kg) of the students of class VIII in a school.



Study the histogram and answer the following questions :

26. How many students have been observed ?

(A) 20 (B) 55

(C) 40 (D) 80

27. What is the class size ?

(A) 15 (B) 10

(C) 5 (D) 55

28. How many students weigh less than 35 kg ?

(A) 38 (B) 24

(C) 16 (D) 18

PASSAGE # II

In a simultaneous throw of a pair of dice, find the probability of getting :

29. A doublet of prime numbers.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{5}{12}$ (D) $\frac{1}{12}$

30. An even number on one and a multiple of 3 on the other.

(A) $\frac{11}{36}$ (B) $\frac{12}{35}$

(C) $\frac{1}{6}$ (D) $\frac{5}{12}$

31. Neither 9 nor 11 as the sum of the numbers on the faces.

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$

(C) $\frac{3}{5}$ (D) $\frac{1}{2}$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column – I and Column – II are given as options (A), (B), (C) and (D) out of which one is correct.

32. If three coins are tossed together. Find the probability of getting

Column – I	Column – II
(P) Exactly two heads	(i) $\frac{1}{2}$
(Q) Atleast two heads	(ii) $\frac{1}{8}$
(R) At least one head	(iii) $\frac{3}{8}$
(S) No tail	(iv) $\frac{7}{8}$

(A) (P) → (i), (Q) → (iii), (R) → (iv), (S) → (ii)

(B) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)

(C) (P) → (ii), (Q) → (i), (R) → (iv), (S) → (iii)

(D) (P) → (ii), (Q) → (iv), (R) → (i), (S) → (iii)

33. Match Column – I with Column – II.

Column – I	Column – II
(P) The collection of observation is called	(i) Organising data
(Q) Entry in data is called	(ii) Raw Data
(R) The collection of observations collected initially is called	(iii) Data
(S) Data handling means	(iv) Observation

(A) (P) → (iv), (Q) → (iii), (R) → (ii), (S) → (i)

(B) (P) → (iii), (Q) → (ii), (R) → (iv), (S) → (i)

(C) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)

(D) (P) → (i), (Q) → (ii), (R) → (iv), (S) → (iii)

VERY SHORT ANSWER TYPE

1. The rainfall (in mm) in Delhi on 7 days of August 2003 is given below :

Day	Rainfall (in mm)
11th	2.2
12th	21.3
13th	25.6
14th	4.9
15th	0.0
16th	25.6
17th	0.0

(i) What is the highest value of rainfall ?

(ii) What is the range of rainfall in the above data?

2. Given below are the heights (in cm) of 11 boys of a class : 146, 143, 148, 132, 128, 139, 140, 152, 154, 142, 149

Arrange the above data in ascending order and find

(i) The height of the tallest boy.

(ii) The height of the shortest boy.

(iii) The range of the given data.

3. In a study of number of accidents per day, the observations for 10 days were obtained as follows : 0, 1, 3, 4, 0, 3, 2, 2, 3, 0

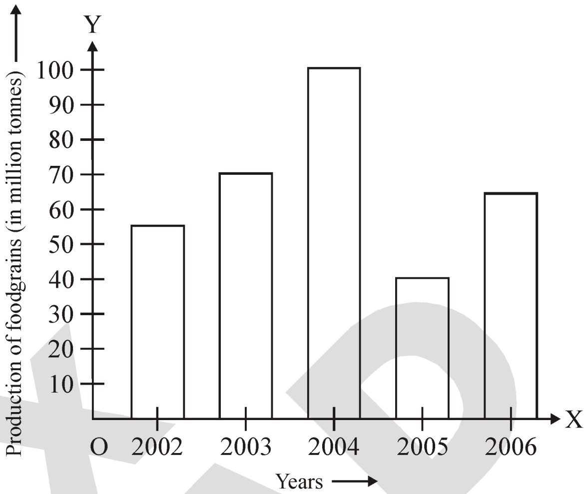
Prepare a frequency table.

4. The marks obtained by Kunal in his annual examination are shown below :

Subjects	Hindi	English	Maths	Sci.	Soc.Stu.
Marks Obtained	60	75	90	70	55

Draw a bar graph to represent the above data.

5. A bar graph is given below :



Production of foodgrains in an Indian state during five consecutive years

Read the bar graph carefully and answer the questions given below :

(i) What information is given by the bar graph ?

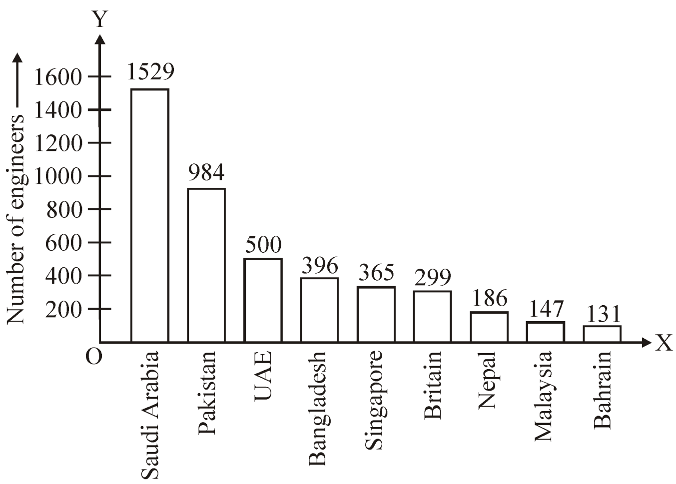
(ii) In which year was the production maximum ?

(iii) After which year was there a sudden fall in the production ?

6. Read the given bar graph and answer the following questions :

(i) What information is given by the bar graph ?

(ii) Which countries have more than 500 engineers ?

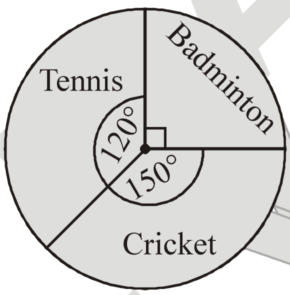


7. A bag contains 3 red and 2 blue marbles. A marble is drawn at random. What is the probability of drawing a blue marble ?

8. The letters of the word 'SWORD' are written on similar cardboards. These cardboards are then mixed and put upside down so that no one can see the letters. Rajni takes one of the cardboards. What is the probability that it has the letter 'R' on it ?

9. It is known that a box of 600 electric bulbs contains 12 defective bulbs. One bulb is taken out at random from this box. What is the probability that it is a non-defective bulb ?

10. A pie chart is shown, represents the games liked by the students of class VIII. A boy is selected at random. What is the probability that he likes cricket ?



SHORT ANSWER TYPE

1. The frequency distribution of weights (in kg) of 40 persons of a locality is given below :

Weights (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
Freq.	4	12	13	6	5

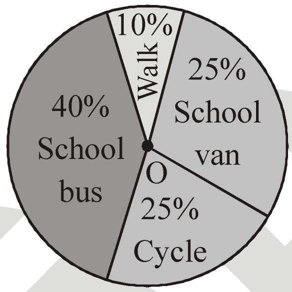
(i) What is the upper limit of the fourth class interval ?

(ii) Find the class marks of all the classes.

(iii) What is the class size of each class interval ?

(iv) Which class interval has the highest frequency?

2. The given pie chart depicts the modes of transport used by the students to commute to school. Now, answer the following questions.



(i) Which is the most preferred mode of transport ?

(ii) Find out the least preferred mode of transport ?

(iii) If the total number of students are 3600, then find out the number of students who come by cycle ?

3. A coin is tossed 130 times and head is obtained 75 times. Now, if a coin is tossed at random, what is the probability of getting a tail ?

4. A dice is thrown 65 times and 4 appeared 21 times. Now, in a random throw of a dice, what is the probability of getting a 4 ?

5. A bag contains 5 red balls, 8 white balls, 4 green balls and 7 black balls. If one ball is drawn at random, find the probability that it is :

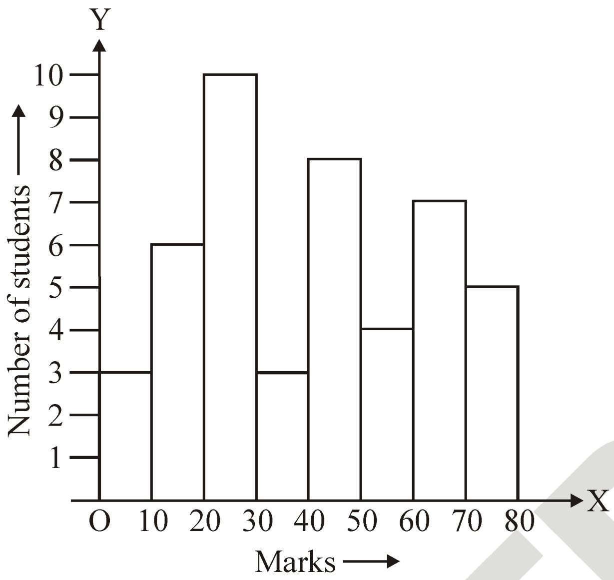
(i) black

(ii) red

(iii) not green

LONG ANSWER TYPE

1. The following histogram depicts the marks obtained by 45 students of a class :



Look at the histogram and answer the following questions :

(i) What is the class size ?

(ii) How many students obtained less than 10 marks?

(iii) How many students obtained more than 30 marks but less than 40 marks ?

(iv) What is the interval of highest marks and how many students are there in this interval ?

(v) If passing marks are 30, what is the number of failures ?

2. Two dice are thrown simultaneously. Find the probability of getting :

(i) an even number as the sum.

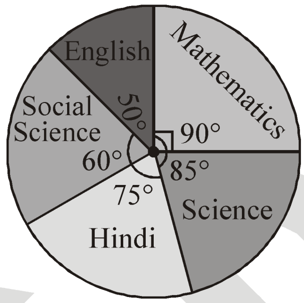
(ii) the sum as a prime number.

(iii) a total of atleast 10.

(iv) a doublet of even number.

(v) a multiple of 2 on one die and a multiple of 3 on the other.

3. The following pie-chart gives the marks scored in an examination by a student in various subjects. If the total marks obtained by the student were 540, answer the following questions :



(i) In which subject did the student score 75 marks?

(ii) How many more marks were obtained by the student in Mathematics than in Hindi ?

(iii) Examine whether the sum of marks obtained in Social Science and Mathematics is more/less than that in Science and Hindi.

4. Cards marked with the number 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is :

(i) an even number.

(ii) a number less than 14.

(iii) a number which is a perfect square.

(iv) a prime number less than 20.

5. The following data relates to the expenditure of the families A, B and C per month :

Items of Expenditure	Family A	Family B	Family C
Food	400	600	1600
Rent	200	400	1500
Clothing	200	300	1000
Education	100	400	800
Litigation	50	100	300
Miscellaneous	50	200	800

Represent this data by pie-diagrams.

TRUE / FALSE TYPE

1. The mean of the observations 7, 8, 9, 11 and 15 is 10.
2. If the range of a data is 9 and its highest value is 81, then its least value is 73.
3. The observation 20 is included in interval 20-30 and not in 10-20.
4. The upper limit of one class interval is always equal to the lower limit of next class interval.
5. The bars in bar graph are always drawn vertically.

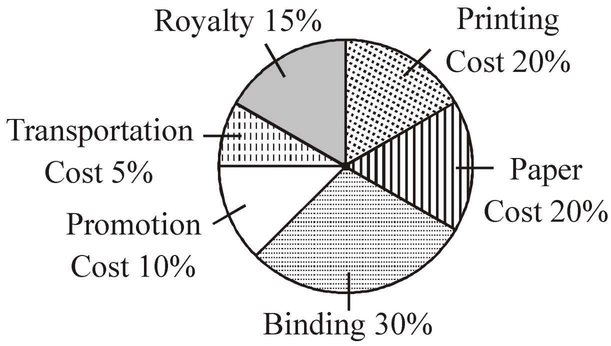
NUMERICAL PROBLEMS

1. Find the range of the data 143, 147, 135, 151, 128, 138, 144, 146, 151 and 153.
2. The probability that a number selected from the numbers 1, 2, 3, ... 15 is a multiple of 4, is.
3. A die is thrown 24 times, 4 is obtained 12 times. The probability of number '4' to come up is k. The value of k is.
4. A pentagonal spinner having numbers 1, 2, 3, 4 and 5 is spun. The probability that the pointer will be at 3 is $\frac{1}{x}$. The value of x is.
5. The probability of selecting a letter 'P' from the letters of the word 'PARKER' is $\frac{k}{m}$. The value of m – k is.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Study the pie-chart and answer the question based on it.

   VARIOUS EXPENDITURES (IN PERCENTAGE INCURRED IN PUBLISHING A BOOK)



   Which of the following two expenditures together have a central angle of 108° ?

   (A) Binding and Transportation
   (B) Printing and Paper
   (C) Royalty and Promotion
   (D) Promotion and Paper

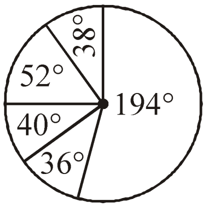
2. Two dice are tossed. The probability that the total score is a prime number is.

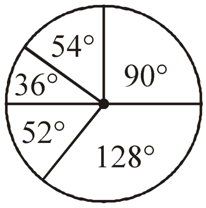
   (A) $\frac{1}{6}$ (B) $\frac{5}{12}$
   (C) $\frac{1}{2}$ (D) $\frac{7}{9}$

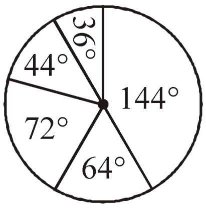
5. The expenditure incurred on various things during the construction of a house are given below.

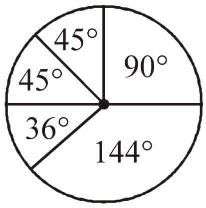
Items	Expenditure (In thousands Rs.)
Bricks	180
Cement	80
Timber	90
Steel	55
Wood	45
Total	450

   Which of the following pie charts exhibits the given information ?

(A) 

(B) 

(C) 

(D) 

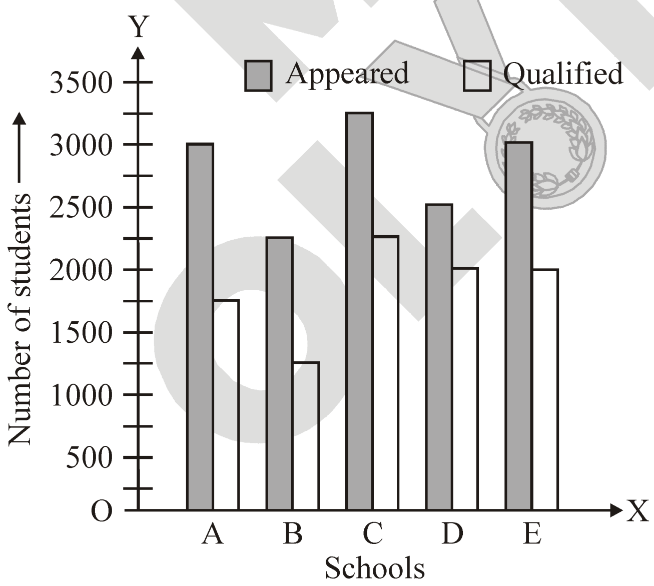
6. One card is drawn from a well-shuffled deck of 52 cards. Find the probability that the number on the card drawn is a multiple of 5.

(A) $\frac{4}{52}$ (B) $\frac{4}{13}$

(C) $\frac{7}{52}$ (D) $\frac{2}{13}$

Direction (7 – 8) : Study the following graph carefully and answer the questions given below :

Total number of Students Appeared and Qualified from Various Schools at a Scholarship Exam



7. The average number of students qualified in the examination from schools C and D is what percent of the average number of students appeared for the examination from the same schools ? (Rounded off to 2 digits after decimal).

(A) 58.62 % (B) 73.91 %

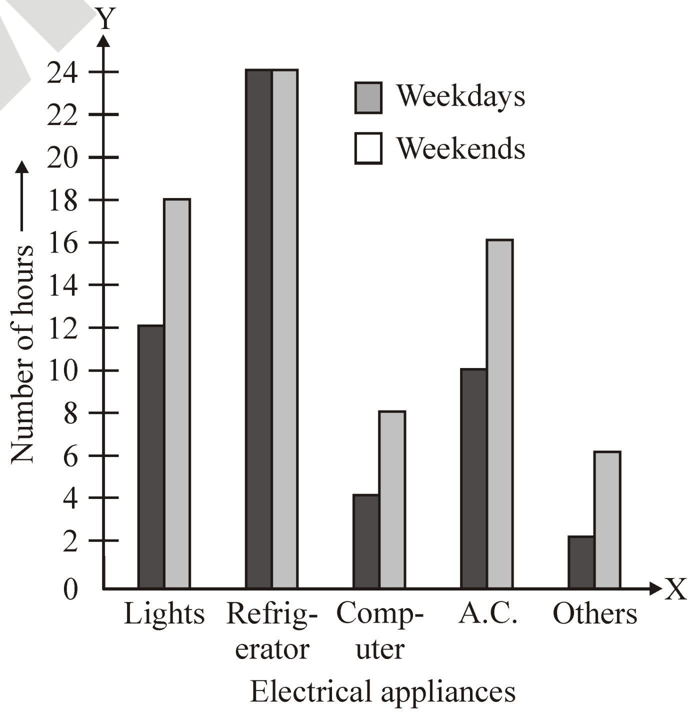
(C) 62.58 % (D) 58.96 %

8. What is the ratio of the number of students qualified in the scholarship examination from school A to the number of students qualified in the examination from school B ?

(A) 8 : 3 (B) 5 : 7

(C) 7 : 3 (D) 7 : 5

Direction (9 – 10) : The given double column graph shows the average number of hours for which an electrical appliance is used on weekdays and wekends. Study the graph carefully and answer the questions that follow :



9. Which appliance is used half as many hours on the weekdays as it is used on the weekends ?

(A) Lights

(B) Refrigerator

(C) Computer

(D) A.C.

10. Average number of hours, all appliances were used on weekdays is what percent of the average number of hours they were used on weekends ?

(A) 82.35 %

(B) 72.22 %

(C) 154 %

(D) None of these

Space for Notes :

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	C	D	A	C	D	D	B	C	A	A	A	B	B	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	B	C	B	A	D	D	C	D	C	D	C	A	D	A
31	32	33												
B	B	C												

EXERCISE II

VERY SHORT ANSWER TYPE

1. (i) 25.6 mm, (ii) 25.6 **2.** (i) 154 cm, (ii) 128 cm, (iii) 26 cm **5.** (ii) 2004, (iii) 2004

6. (ii) June 30, 1994, (iii) Saudi Arabia and Pakisthan **7.** $\frac{2}{5}$ **8.** $\frac{1}{5}$ **9.** $\frac{49}{50}$

10. $\frac{5}{12}$

SHORT ANSWER TYPE

1. (i) 60, (iii) 5, (iv) 50-55 **2.** (i) School bus, (ii) Walking, (iii) 900

3. $\frac{11}{26}$ **4.** $\frac{21}{65}$ 5. (i) $\frac{7}{24}$ (ii) $\frac{5}{24}$ (iii) $\frac{5}{6}$ **10.** (i) $\frac{7}{24}$, (ii) $\frac{5}{24}$, (iii) $\frac{5}{6}$

LONG ANSWER TYPE

1. (i) 10, (ii) 3, (iii) 3, (iv) 70-80, 5, (v) 19 **2.** (i) $\frac{1}{2}$, (ii) $\frac{5}{12}$, (iii) $\frac{1}{6}$, (iv) $\frac{1}{12}$, (v) $\frac{11}{36}$

3. (i) English, (ii) 22.5, (iii) Less **4.** (i) $\frac{1}{2}$, (ii) $\frac{3}{25}$, (iii) $\frac{9}{100}$, (iv) $\frac{2}{25}$

NUMERICAL PROBLEMS

1. 25 **2.** 0.2 **3.** 0.5 **4.** x = 5 **5.** $\frac{5}{6}$m **6.** 0

7. 0.5 **8.** 5 **9.** 0.5 **10.** 5

ANALYTICAL PROBLEMS & BRAIN TEASER

1. D **2.** B **3.** D **4.** D **5.** C **6.** D **7.** B

8. D **9.** C **10.** B

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : DATA HANDLING)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Solutions			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

SQUARES AND SQUARE ROOTS 5

Concepts

INTRODUCTION

We know that the area of a square = side × side (where 'side' means 'the length of a side'). Study the following table.

Side of a square (in cm)	Area of the square (in cm^2)
1	$1 \times 1 = 1 = 1^2$
2	$2 \times 2 = 4 = 2^2$
3	$3 \times 3 = 9 = 3^2$
5	$5 \times 5 = 25 = 5^2$
8	$8 \times 8 = 64 = 8^2$
a	$a \times a = a^2$

What is special about the numbers 4, 9, 25, 64 and other such numbers ? Since, 4 can be expressed as $2 \times 2 = 2^2$, 9 can be expressed as $3 \times 3 = 3^2$, all such numbers can be expressed as the product of the number with itself. Such numbers like 1, 4, 9, 16, 25, ... are known as square numbers.

1. SQUARE OF A NUMBER AND A PERFECT SQUARE

The square of a number is that number raised to the power 2. Thus, if 'a' is a number, then the square of a is written

For example : $2^2 = 2 \times 2 = 4$, so we say that the square of 2 is 4.

$3^2 = 3 \times 3 = 9$, so we say that the square of 3 is 9.

$12^2 = 12 \times 12 = 144$, so we say that the square of 12 is 144.

The following table contains the squares of first thirty natural numbers.

Number	**Square**	**Number**	**Square**
1	$1^2 = 1$	16	$16^2 = 256$
2	$2^2 = 4$	17	$17^2 = 289$
3	$3^2 = 9$	18	$18^2 = 324$
4	$4^2 = 16$	19	$19^2 = 361$
5	$5^2 = 25$	20	$20^2 = 400$
6	$6^2 = 36$	21	$21^2 = 441$
7	$7^2 = 49$	22	$22^2 = 484$
8	$8^2 = 64$	23	$23^2 = 529$
9	$9^2 = 81$	24	$24^2 = 576$
10	$10^2 = 100$	25	$25^2 = 625$
11	$11^2 = 121$	26	$26^2 = 676$
12	$12^2 = 144$	27	$27^2 = 729$
13	$13^2 = 169$	28	$28^2 = 784$
14	$14^2 = 196$	29	$29^2 = 841$
15	$15^2 = 225$	30	$30^2 = 900$

2. PERFECT SQUARE

A natural number is called a perfect square number if it is the square of a natural number i.e., if $n^2 = m$, when n and m are natural numbers, then m is a perfect square.

How to Check a Perfect Square : In order to check whether a given natural number is a perfect square or not, we follow the following steps :

Step – 1 : Obtain the given number.

Step – 2 : Write the number as a product of its prime factors.

Step – 3 : If the prime factors can be grouped into pairs then the number is a perfect square otherwise, the number is not a perfect square.

3. PROPERTIES OF PERFECT SQUARE NUMBERS

(1) A natural number having 2, 3, 7 or 8 at the one's place is never a perfect square.

For example : 12, 23, 37 and 48 are not perfect squares.

(2) A natural number having 0, 1, 4, 5, 6 or 9 at the one's place may or may not be a perfect square.

For example : 100, 121, 144, 256 and 169 are perfect squares but 10, 221, 224, 326 and 389 are not perfect squares.

(3) A number ending with odd number of zeroes is never a perfect square.

For example : 40, 5000, 100000 are not perfect squares.

(4) The square of an odd number is always odd.

For example : $5^2 = 25$ and $9^2 = 81$.

Here, 25 and 81 are odd numbers.

(5) The square of an even number is always even.

For example : 14 is an even number and $14^2 = 196$ is also an even number.

(6) Factors of every square number can be grouped into pairs of equal numbers.

For example : $729 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

(7) (a) The square of a number ending with 1 or 9 ends with 1.

For example : $21^2 = 441$ and $9^2 = 81$.

(b) The square of a number ending with 2 or 8 ends with 4.

For example : $2^2 = 4$, $18^2 = 324$.

(c) The square of a number ending with 3 or 7 ends with 9.

For example : $7^2 = 49$, $13^2 = 169$.

(d) The square of a number ending with 4 or 6 ends with 6.

For example : $4^2 = 16$, $6^2 = 36$.

(e) The square of a number ending with 5 ends with 5.

For example : $5^2 = 25$, $15^2 = 225$.

(f) The square of a number ending with 0 ends with even number of zeroes.

For example : $10^2 = 100$, $20^2 = 400$.

Example 1

Which of the following are not perfect squares ?

(i) 256 (ii) 289 (iii) 418 (iv) 563

Solution :

(i) $256 = 16^2$, so it is a perfect square.

(ii) $289 = 17^2$, so it is a perfect square.

(iii) and (iv) are not perfect square because a number having 2, 3, 7 or 8 at one's place is never a perfect square.

Example 2

Which of the following numbers are squares of even numbers ?

(i) 625 (ii) 324 (iii) 2401 (iv) 6651

Solution :

Only (b) is the square of an even number i.e., $18^2 = 324$.

Example 3

Determine the digit at the one's place in the square of the following numbers.

(i) 172 (ii) 399

Solution :

(i) The square of 172 will have 4 at one's place. [Using property 7(b)]

(ii) The square of 399 will have 1 at one's place. [Using property 7(a)]

4. SOME INTERESTING PATTERNS

(1) The square of any natural number n equals to the sum of first n odd numbers.

For example : $1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

(2) A number with n digits has either $(2n - 1)$ or $2n$ digits in its square.

For example : 1–digit number has 1 or 2 digits in its square i.e., $3^2 = 9$, $9^2 = 81$

A 2–digit number has either 3 or 4 digits in its square, i.e., $12^2 = 144$; $50^2 = 2500$

(3) The square of any odd number can be expressed as the sum of two consecutive positive numbers.

For example : $3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

(4) A perfect square number leaves a remainder 0 or 1 when divided by 3.

(5) The square of a natural number greater than 1 can be written as

(i) multiple of 3 or (multiple of 4) + 1

(ii) multiple of 4 or (multiple of 3) + 1.

For example : $3^2 = 9 = 3 \times 3$ or $4 \times 2 + 1$

$4^2 = 16 = 3 \times 5 + 1$ or 4×4

(6) Between the squares of any two consecutive numbers, n and (n + 1) there are 2n non-perfect square numbers. Between 1^2 and 2^2 i.e., 1 and 4 there are two non-perfect square numbers 2 and 3. It is same as 2×1 numbers. Between 2^2 and 3^2 i.e., 4 and 9, there are four non-perfect square numbers 5, 6, 7 and 8. It is same as 2×2 numbers.

(7) Product of two consecutive even or odd natural numbers is $(a + 1)(a - 1) = a^2 - 1$.

For example : $11 \times 13 = (12 - 1)(12 + 1) = 12^2 - 1$

$12 \times 14 = (13 - 1)(13 + 1) = 13^2 - 1$

(8) Squares of natural numbers having all digits 1 follow the following pattern :

$$1^2 = 1$$

$$11^2 = 1\ 2\ 1$$

$$111^2 = 1\ 2\ 3\ 2\ 1$$

$$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$$

$$11111^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$$

Another interesting pattern

$$7^2 = 49$$

$$67^2 = 4489$$

$$667^2 = 444889$$

$$6667^2 = 44448889$$

$$66667^2 = 4444488889$$

Example 4

Express 64 as the sum of 8 odd numbers.

Solution :

$64 = 8^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$

Example 5

Write $(27)^2$ as the sum of two consecutive positive numbers.

Solution :

$(27)^2 = 729 = 364 + 365$

Example 6

How many numbers lie between the square of the following numbers ?

(i) 4^2 and 5^2 (ii) 7^2 and 8^2

Solution :

(i) The number between 4^2 and 5^2 is $2 \times 4 = 8$

(ii) The number between 7^2 and 8^2 is $2 \times 7 = 14$

Example 7

Express the following as either in terms of multiple of 3 or multiple of 4.

(i) 6^2 (ii) 14^2

Solution :

(i) $6^2 = 36 = 3 \times 12$ or 4×9

(ii) $14^2 = 196 = 4 \times 49$ or $3 \times 65 + 1$

5. PYTHAGOREAN TRIPLETS

Three natural numbers m, n, p are said to form a Pythagorean triplet (m, n, p) if $(m^2 + n^2) = p^2$.

Note : For every natural number $m > 1$, we have $(2m, m^2 – 1, m^2 + 1)$ as a Pythagorean triplet.

Example 8

Find the pythagorean triplet whose smallest number is 12.

Solution :

For every natural number $m > 1$, $(2m, m^2 – 1, m^2 + 1)$ is a Pythagorean triplet.

Putting $2m = 12$, i.e., $m = 6$, we get the triplet (12, 35, 37).

6. FINDING THE SQUARE OF A NUMBER

We can also find the square of a number without actual multiplication.

For example : $43^2 = (40 + 3)^2 = (40 + 3)(40 + 3)$

$= 40(40 + 3) + 3(40 + 3)$

$= 1600 + 120 + 120 + 9$

$= 1600 + 240 + 9 = 1849$

7. OTHER PATTERNS IN SQUARE

Consider a number with unit digit 5 i.e., a5

$(a5)^2 = (10a + 5)^2 = (10a + 5)(10a + 5)$

$= 100a^2 + 50a + 50a + 25$

$= 100a(a + 1) + 25$

$= a(a + 1)$ hundred $+ 25$

For example : $95^2 = (90 + 5)^2 = (10 \times 9 + 5)^2$, where $a = 9$

$= 100 \times 9(9 + 1) + 25$

$= 100 \times 9(10) + 25$

$= 9(9 + 1)$ hundred $+ 25$

$= 9 \times 10$ hundred $+ 25$

$= 9000 + 25 = 9025$

8. SQUARE ROOTS

The square root of a number x is that number which when multiplied by itself gives x as its product. Thus, if b is the square root of a number x, then $b \times b = x \Rightarrow b^2 = x \Rightarrow b = \sqrt{x}$

The square root of a number x is denoted by $\sqrt{x}$.

Now, $\sqrt{9} = 3$

Also, the square of $(-3) = (-3) \times (-3) = 9$

So, 9 has two square roots; + 3 and – 3.

Therefore, every positive number has two square roots; one positive number and other is negative number.

9. METHODS OF FINDING SQUARE ROOTS

9.1 FINDING SQUARE ROOT THROUGH REPEATED SUBTRACTION

Step – 1 : Take the given number whose square root is to be found out.

Step – 2 : Subtract the odd numbers 1,3, 5, ... successively from the given number.

Step – 3 : If the given number is a perfect square, we will get zero at some stage. We stop at the point where we have got zero, and declare the number of times we have performed subtraction as the square root of the given number.

For example : Let us consider the number 49.

(i) 49 – 1 = 48
(ii) 48 – 3 = 45
(iii) 45 – 5 = 40
(iv) 40 – 7 = 33
(v) 33 – 9 = 24
(vi) 24 – 11 = 13
(vii) 13 – 13 = 0

So, we have performed subtraction 7 times. Therefore, $\sqrt{49} = 7$.

9.2 FINDING SQUARE ROOT THROUGH PRIME FACTORISATION

Step – 1 : Find prime factors of the given number.

Step – 2 : Make pairs of same factors obtained in step 1.

Step – 3 : Take one factor from each pair and multiply them together. The product thus obtained will be the required square root.

Example 9

Find the square root of 1444.

Solution :

2	1444
2	722
19	361
19	19
	1

$1444 = \underline{2 \times 2} \times \underline{19 \times 19}$

$\therefore \sqrt{1444} = 2 \times 19 = 38$

Hence, $\sqrt{1444} = 38$

Example 10

Is 3528 a perfect square ? If not, find the smallest number by which it must be multiplied so that product becomes a perfect square.

Solution :

$3528 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times \underline{7 \times 7}$ We find that the prime factor 2 does not occur in pair. Therefore, 3528 is not a perfect square. To make 3528 a perfect square,

2	3528
2	1764
2	882
3	441
3	147
7	49
7	7
	1

we multiply 3528 by 2 to get $3528 \times 2 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$.

Now, each prime factor is in pair. Therefore, $3528 \times 2 = 7056$ is a perfect square.

Therefore, the required smallest number is 2.

Example 11

Find the smallest number by which 2352 must be divided so that the quotient is a perfect square. Find the square root of the quotient.

Solution :

2	2352
2	1176
2	588
2	294
7	147
7	21
3	3
	1

We have, 2352 = $\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7} \times 3$

We find that 3 is not forming a pair. So, to make 2352 a perfect square, we have to eliminate 3.

So, we divide 2352 by 3 to get 2352 ÷ 3 = $\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

Now, each prime factor is in pair. Therefore 2352 ÷ 3 = 784 is a perfect square.

$\therefore \sqrt{784} = 2 \times 2 \times 7 = 28$

9.3 FINDING SQUARE ROOT BY LONG DIVISION METHOD

When the numbers are large and the method of finding square root by prime factorisation becomes inconvenient, then, we use long division method. To understand long division method effectively, it is important to know the number of digits in the square root of a perfect square.

We already know, $1^2 = 1$ and $9^2 = 81$. Also, $10^2 = 100$ and $99^2 = 9801$.

Further $100^2 = 10000$ and $999^2 = 998001$.

Now, we see that square root of a 1-digit or 2-digit number has 1-digit. Square root of a 3-digit or 4-digit has 2-digits and so on.

We can generalise it as : A perfect square number having n-digits, the corresponding square root will have

(a) $\frac{n}{2}$ digits, if n is even. (b) $\frac{(n+1)}{2}$ digits, if n is odd.

Let us consider a number 16641.

Since it is a 5-digit number, so the square root of the number will have 3 digits.

We have the following steps to find the square root by division method.

Step – 1 : Place a bar over every pair of digits starting from the one's digit (from right to left). If the number of digits in it is odd, then the left most single digit too will have a bar i.e., $\overline{1}\ \overline{66}\ \overline{41}$.

Step – 2 : Think of the largest number whose square is less than or equal to the number under the left most bar (in this case, 1).

Step – 3 : Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend (here 1). Divide and get the remainder (in this case, 0).

$$\begin{array}{r|l} & 1 \quad \leftarrow \text{Quotient} \\ \hline \text{Divisor} \rightarrow 1 & \overline{1}\,\overline{66}\,\overline{41} \\ & -1 \\ \hline & \;0 \end{array}$$

Step – 4 : Bring down the number under the bar to the right of the remainder and this becomes the new dividend (in this case 66).

```
        1
   ┌──────
 1 │ 1 66 41
   │−1
───┼──────
   │ 066
```

Step – 5 : Double the quotient and enter it with a blank on its right.

```
        1
   ┌──────
 1 │ 1 66 41
   │−1
───┼──────
 2 │ 066
```

Step – 6 : Choose the one's digit of the new divisor in such a way that the product of this digit and new divisor is equal to or less than the new dividend (in this case 22).

```
        12
    ┌──────
  1 │ 1 66 41
    │−1
────┼──────
 22 │  66
    │ − 44
────┼──────
    │  22
```

Step – 7 : Again bring down the number under the next bar to the right of the remainder in step 6. Repeat the steps 5 and 6.

```
         129
     ┌────────
   1 │ 1 66 41
     │−1
─────┼────────
  22 │  66
     │ − 44
─────┼────────
 249 │   2241
     │ − 2241
─────┼────────
     │      0
```

Step – 8 : Since, the remainder is 0 and no digits are left in the given number, therefore $\sqrt{16641} = 129$.

10. SQUARE ROOTS OF DECIMALS

Let us consider a number 54.76.

We have the following steps to find the square roots of decimals by long division method.

Step – 1 : To find the square root of a decimal number, we put bars on the integral part (i.e., 54) of the number in the usual manner. And place bars on the decimal part (i.e., 76) on every pair of digits beginning with the first decimal place. So, we get $\overline{54}.\overline{76}$

Step – 2 : Now proceed in a similar manner. The left most bar is on 54 and $7^2 < 54 < 8^2$. Take this number as the divisor and number under the left-most bar as the dividend i.e., 54. Divide and get the remainder.

$$\begin{array}{r|l} & \quad 7 \\ \hline 7 & \overline{54}.\overline{76} \\ & -49 \\ \hline & \quad 5 \end{array}$$

Step – 3 : The remainder is 5. Write the number under the next bar at the right of this remainder, to get 576

$$\begin{array}{r|l} & \quad 7 \\ \hline 7 & \overline{54}.\overline{76} \\ & -49 \\ \hline 14 & \quad 576 \end{array}$$

Step – 4 : Double the quotient and enter it with a blank on the right. Since, 76 is the decimal part so put a decimal point in the quotient.

$$\begin{array}{r|l} & \quad 7. \\ \hline 7 & \overline{54}.\overline{76} \\ & -49 \\ \hline 14 & \quad 576 \end{array}$$

Step – 5 : We know that $144 \times 4 = 576$, therefore the new digit is 4. Divide and get the remainder as 0.

$$\begin{array}{r|l} & \quad 7.4 \\ \hline 7 & \overline{54}.\overline{76} \\ & -49 \\ \hline 144 & \quad 576 \\ & \ -576 \\ \hline & \quad\ \ 0 \end{array}$$

Step – 6 : Since, the remainder is 0 and no bar left, therefore $\sqrt{54.76} = 7.4$.

11. SQUARE ROOT OF PRODUCT OF TWO NUMBERS AND FRACTIONAL NUMBERS

The square root of the product of two numbers is equal to the product of the square root of the one number and square root of the other number. For any two positive numbers a and b, we have $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$. The square root of a fractional number is equal to the quotient of the square root of its numerator and denominator. For any two positive numbers a and b, we have $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example 12

Find the square root of $\frac{625}{1296}$.

Sol.

We have, $\sqrt{\frac{625}{1296}} = \frac{\sqrt{625}}{\sqrt{1296}}$

Now, we find the square roots of 625 and 1296 separately as shown :

Thus, $\sqrt{625} = 25$ and $\sqrt{1296} = 36$

Hence, $\sqrt{\frac{625}{1296}} = \frac{\sqrt{625}}{\sqrt{1296}} = \frac{25}{36}$

	25
2	$\overline{6}\ \overline{25}$
	– 4
45	225
	– 225
	0

	36
3	$\overline{12}\ \overline{96}$
	– 9
66	396
	– 396
	0

12. ESTIMATING SQUARE ROOTS

We can estimate the square root by following ways. We know that $100 < 150 < 225$.

$\sqrt{100} = 10$ and $\sqrt{225} = 15$

But still we are not close to the square number.

We know that $12^2 = 144$ and $13^2 = 169$.

Therefore, $12 < \sqrt{150} < 13$ and 144 is much closer to 150 than 169.

So, $\sqrt{150}$ is approximately equal to 12.

SOLVED EXAMPLES

SE. 1

Is 144 a perfect square ? If yes, find the number whose square is 144.

Ans. The given number is 144. Writing 144 as the product of its prime factors, we have

2	144
2	72
2	36
2	18
3	9
3	3
	1

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= (2 \times 2) \times (2 \times 2) \times (3 \times 3)$

Clearly, the prime factors can be written in pairs as shown here.

Therefore, 144 is a perfect square.

So, $\sqrt{144} = 2 \times 2 \times 3 = 12$

SE. 2

Find the smallest number by which 25200 should be divided so that the result is a perfect square.

Ans. Resolving 25200 into prime factors, we get

2	25200
2	12600
2	6300
2	3150
3	1575
3	525
5	175
5	35
7	7
	1

$25200 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$

Now, grouping the factors into prime of equal factors, we get

$25200 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (5 \times 5) \times 7$

We observe that 7 cannot be paired. Thus, if we divide 25200 by 7, the quotient will be a perfect square. Hence, the smallest number by which 25200 must be divide to make the result a perfect square is 7.

SE. 3

Find whether 55 is a perfect square or not using repeated subtraction.

Ans. In order to check whether 55 is a perfect square or not, we successively subtract 1, 3, 5, 7, 9, ..., from it. We have,

(i) $55 - 1 = 54$ (ii) $54 - 3 = 51$

(iii) $51 - 5 = 46$ (iv) $46 - 7 = 39$

(v) $39 - 9 = 30$ (vi) $30 - 11 = 19$

(vii) $19 - 13 = 6$ (viii) $6 - 15 = -9$

This shows that 55 cannot be expressed as the sum of consecutive odd numbers starting from 1. Hence, it is not a perfect square.

SE. 4

Find the squares of the following numbers containing 5 in the unit's place :

(i) 15 (ii) 105

Ans. (i) $15^2 = 1 \times (1 + 1)$ hundred + 25

$= 1 \times 2$ hundred + 25

= 2 hundred + 25 $= 2 \times 100 + 25$

$= 200 + 25 = 225$

(ii) $105^2 = 10 \times (10 + 1)$ hundred + 25

$= (10 \times 11)$ hundred + 25

$= 110 \times 100 + 25 = 11000 + 25 = 11025$

SE. 5

Write a Pythagorean triplet whose smallest member is 20.

Ans. We can get Pythagorean triplet by using general form 2m, $m^2 - 1$, $m^2 + 1$.

Let $2m = 20 \Rightarrow m = 10$

So, $m^2 - 1 = (10)^2 - 1 = 99$, $m^2 + 1 = (10)^2 + 1 = 101$. The triplet is 20, 99, 101.

SE. 6

Determine whether squares of the following numbers are even or odd.

(i) 9777

(ii) 40028

Ans. (i) The square of 9777 is an odd number as its one's digit is odd.

(ii) The square of 40028 is an even number as its one's digit is even.

SE. 7

Which of the following numbers are not perfect squares ?

(i) 697

(ii) 900

(iii) 412

(iv) 1000

Ans. (i) A number having 7 at one's place is never a perfect square. So, 697 is not a perfect square.

(ii) $900 = (30)^2$ is a perfect square.

(iii) A number having 2 at one's place is never a perfect square. So, 412 is not a perfect square.

(iv) A number ending with odd number of zeroes is never a perfect square. So, 1000 is not a perfect square.

SE. 8

Determine the digit at one's place in the squares of the following numbers.

(i) 1146 (ii) 1955

(iii) 343

Ans. (i) The square of a number ending with 6 ends with 6.

(ii) The square of a number ending with 5 ends with 5.

(iii) The square of a number ending with 3 ends with 9.

SE. 9

Find the square of 491 using the identity $(a - b)^2 = a^2 - 2ab + b^2$.

Ans. We have $491^2 = (500 - 9)^2$

$= 500^2 - 2 \times 500 \times 9 + 9^2 = 250000 - 9000 + 81 = 241081$.

SE. 10

Find the square root of 8464 by prime factorization.

Ans. Resolving 8464 into prime factors, we get

$8464 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{23 \times 23}$

2	8464
2	4232
2	2116
2	1058
23	529
23	23
	1

Now, taking one factor from each pair, we obtain

$\sqrt{8464} = 2 \times 2 \times 23 = 92$

$\therefore \sqrt{8464} = 92$

SE. 11

Find the cost of erecting a fence around a square field whose area is 9 hectares if fencing costs Rs. 3.50 per metre.

Ans. Area of the square field = $(9 \times 10000)\text{m}^2$

$= 90000 \text{ m}^2$

Length of each side of the field = $\sqrt{90000}$ m

= 300 m. Perimeter of field = (4 × 300) m = 1200 m. Cost of fencing = Rs. $\left(1200 \times \frac{7}{2}\right)$ = Rs. 4200.

SE. 12

Find the greatest 4-digit number which is a perfect square.

Ans. Greatest number of 4-digits = 9999. We find $\sqrt{9999}$ by long division method.

$$\begin{array}{r|l} & 99 \\ \hline 9 & \overline{99}\ \overline{99} \\ & -81 \\ \hline 189 & 1899 \\ & -1701 \\ \hline & 198 \end{array}$$

The remainder is 198. This shows, 99^2 is less than 9999 by 198.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is 9999 – 198 = 9801.

And, $\sqrt{9801} = 99$

SE. 13

Find the square root of 11025 by prime factorization method.

Ans. Resolving 11025 into prime factors, we have

11025 = (3 × 3) × (5 × 5) × (7 × 7)

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

Taking one factor from each pair, we get

$\sqrt{11025} = 3 \times 5 \times 7 = 105$

SE. 14

Find the value of $\sqrt{37.0881}$.

Ans. We have, $\sqrt{37.0881} = \sqrt{\frac{370881}{10000}}$

$$\begin{array}{r|l} & 609 \\ \hline 6 & \overline{37}\ \overline{08}\ \overline{81} \\ & -36 \\ \hline 1209 & 10881 \\ & -10881 \\ \hline & 0 \end{array}$$

$\therefore \sqrt{37.0881} = \sqrt{\frac{370881}{10000}} = \frac{609}{100} = 6.09$

SE. 15

Find the value of $\sqrt{15625}$ and use it to find the value of $\sqrt{156.25}+\sqrt{1.5625}$.

Ans. We first find the value of $\sqrt{15625}$ by long division method as given below :

Divisor	125
1	$\overline{1}\ \overline{56}\ \overline{25}$
	– 1
22	56
	– 44
245	1225
	– 1225
	0

$\therefore \sqrt{15625}=125$

Now, $\sqrt{156.25}+\sqrt{1.5625}$

$=\sqrt{\dfrac{15625}{100}}+\sqrt{\dfrac{15625}{10000}}$

$=\dfrac{\sqrt{15625}}{\sqrt{100}}+\dfrac{\sqrt{15625}}{\sqrt{10000}}=\dfrac{125}{10}+\dfrac{125}{100}$

$= 12.5 + 1.25 = 13.75$

Space for Notes :

NCERT SOLUTIONS

EXERCISE – 2.1

NS. 1

What will be the unit digit of the squares of the following numbers ?

(i) 81 (ii) 272

(iii) 799 (iv) 3853

(v) 1234 (vi) 26387

(vii) 52698 (viii) 99880

(ix) 12796 (x) 55555

Ans. (i) The unit digit of $(81)^2$ is 1.

Because when we multiply the unit digit 1 by itself, we get 1.

(ii) The unit digit of $(272)^2$ is 4.

Because when we multiply the unit digit 2 by itself, we get 4.

(iii) The unit digit of $(799)^2$ is 1.

Because when we multiply the unit digit 9 by itself, we get 81.

(iv) The unit digit of $(3853)^2$ is 9.

Because when we multiply the unit digit 3 by itself, we get 9.

(v) The unit digit of $(1234)^2$ is 6.

Because when we multiply the unit digit 4 by itself, we get 16.

(vi) The unit digit of $(26387)^2$ is 9.

Because when we multiply the unit digit 7 by itself, we get 49.

(vii) The unit digit of $(52698)^2$ is 4.

Because when we multiply the unit digit 8 by itself, we get 64.

(viii) The unit digit of $(99880)^2$ is 0.

Because when we multiply the unit digit 0 by itself, we get 0.

(ix) The unit digit of $(12796)^2$ is 6.

Because when we multiply the unit digit 6 by itself, we get 36.

(x) The unit digit of $(55555)^2$ is 5.

Because when we multiply the unit digit 5 by itself, we get 25.

NS. 2

The following numbers are obviously not perfect squares. Give reason.

(i) 1057 (ii) 23453

(iii) 7928 (iv) 222222

(v) 64000 (vi) 89722

(vii) 222000 (viii) 505050

Ans. We know that number ending in 2, 3, 7 or 8 are not perfect squares.

(i) 1057 (ii) 23453

(iii) 7928 (iv) 222222

(vi) 89722

Clearly, all ends with the digits 7, 3, 8, 2.

$\therefore$ These are not perfect squares.

Since, for perfect squares, there should be even number of zeroes at the end.

(v) 64000 (vii) 222000

(viii) 505050

Clearly, all ends with the odd number of zeroes.

So, they are not perfect squares.

NS. 3

The square of which of the following would be odd numbers ?

(i) 431 (ii) 2826

(iii) 7779 (iv) 82004

Ans. (i) When we multiply the unit digit 1 by itself, we get 1 at the end, which shows that the square of 431 is an odd number.

(ii) When we multiply the unit digit 6 by itself we get 36, i.e., we get 6 at the end, which is an even number.

(iii) When we multiply the unit digit 9 by itself we get 81, i.e., the square of 7779 is an odd number.

(iv) When we multiply the unit digit 4 by itself we get 16, i.e., we get 6 at the end, which is an even number. So, the square of 82004 is an even number.

NS. 4

Observe the following pattern and find the missing digits.

11^2 = 121

101^2 = 10201

1001^2 = 1002001

100001^2 = 1............... 2............... 1

10000001^2 = ...

Ans. 100001^2 = 10000200001

100000012 = 100000020000001.

NS. 5

Observe the following pattern and find the missing numbers.

11^2 = 121

101^2 = 10201

1001^2 = 102030201

1010101^2 = ...

.............^{2} = 10203040504030201

Ans. 1010101^2 = 1020304030201

101010101^2 = 10203040504030201.

NS. 6

Using the given pattern, find the missing numbers.

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + \_^2 = 21^2$

$5^2 + \_^2 + 30^2 = 31^2$

$6^2 + 7^2 + \_^2 = \_^2$

Ans. $4^2 + 5^2 + \underline{20}^2 = 21^2$

$5^2 + \underline{6}^2 + 30^2 = 31^2$

$6^2 + 7^2 + \underline{42}^2 = \underline{43}^2$.

NS. 7

Without adding, find the sum.

(i) 1 + 3 + 5 + 7 + 9

(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19

(iii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23.

Ans. (i) We have to find the sum of first 5 odd numbers.

$1 + 3 + 5 + 7 + 9 = 5^2 = 25$.

(ii) We have to find the sum of the first 10 odd numbers.

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2 = 100$.

(iii) We have to find the sum of first 12 odd numbers.

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 12^2 = 144$.

NS. 8

(i) Express 49 as the sum of 7 odd numbers.

(ii) Express 121 as the sum of 11 odd numbers.

Ans. (i) 49 = 1 + 3 + 5 + 7 + 9 + 11 + 13.

(ii) 121 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21.

NS. 9

How many numbers lie between squares of the following numbers ?

(i) 12 and 13 (ii) 25 and 26

(iii) 99 and 100

Ans. (i) We have to find the number of terms between the squares of 12 and 13, by doubling the first term from 12 and 13. i.e., $2 \times 12 = 24$.

$\therefore$ Total number of terms = 24.

(ii) We can find the number of terms between the squares of 25 and 26, by doubling the first term from 25 and 26. i.e., $2 \times 25 = 50$.

$\therefore$ Total number of terms = 50.

(iii) We can find the number of terms between the squares of 99 and 100, by doubling the first term from 99 and 100. i.e., $2 \times 99 = 198$.

$\therefore$ Total number of terms = 198.

EXERCISE – 2.2

NS. 1

Find the square of the following numbers.

(i) 32 (ii) 35

(iii) 86 (iv) 93

(v) 71 (vi) 46

Ans. (i) $32^2 = (30 + 2)^2 = (30 + 2)\ (30 + 2)$

$= 30\ (30 + 2) + 2(30 + 2)$

$= 900 + 60 + 60 + 4 = 1024.$

(ii) $35^2 = (30 + 5)^2 = (30 + 5)\ (30 + 5)$

$= 30(30 + 5) + 5(30 + 5)$

$= 900 + 150 + 150 + 25 = 1225.$

(iii) $86^2 = (80 + 6)^2 = (80 + 6)\ (80 + 6)$

$80(80 + 6) + 6(80 + 6)$

$= 6400 + 480 + 480 + 36 = 7396.$

(iv) $93^2 = (90 + 3)^2 = (90 + 3)\ (90 + 3)$

$90(90 + 3) + 3(90 + 3)$

$= 8100 + 270 + 270 + 9 = 8649.$

(v) $71^2 = (70 + 1)^2 = (70 + 1)\ (70 + 1)$

$= 70(70 + 1) + 1(70 + 1)$

$= 4900 + 70 + 70 + 1 = 5041.$

(vi) $46^2 = (40 + 6)^2 = (40 + 6)\ (40 + 6)$

$= 40(40 + 6) + 6(40 + 6)$

$= 1600 + 240 + 240 + 36 = 2116.$

NS. 2

Write a Pythagorean triplet whose one member is

(i) 6 (ii) 14

(iii) 16 (iv) 18

Ans. (i) We can get Pythagorean triplet by using general form $2m, m^2 - 1, m^2 + 1$.

So, let us take $2m = 6 \Rightarrow m = 3$.

Thus, $m^2 - 1 = 3^2 - 1 = 9 - 1 = 8$

and $m^2 + 1 = 3^2 + 1 = 9 + 1 = 10$.

$\therefore$ The required triplet is 6, 8, 10.

(ii) We take $2m = 14 \Rightarrow m = 7$

Now, $m^2 - 1 = 7^2 - 1 = 49 - 1 = 48$

and $m^2 + 1 = 7^2 + 1 = 49 + 1 = 50$

$\therefore$ The required triplet is 14, 48, 50.

(iii) We take $2m = 16 \Rightarrow m = 8$

Now, $m^2 - 1 = 8^2 - 1 = 64 - 1 = 63$

and $m^2 + 1 = 8^2 + 1 = 64 + 1 = 65$.

Thus, the required triplet is 16, 63, 65.

(iv) We take $2m = 18 \Rightarrow m = 9$

Now, $m^2 - 1 = 9^2 - 1 = 81 - 1 = 80$

and $m^2 + 1 = 9^2 + 1 = 81 + 1 = 82$

Thus, the required triplet is 18, 80, 82.

EXERCISE – 2.3

NS. 1

What could be the possible 'ones' digits of the square root of each of the following numbers ?

(i) 9801 (ii) 99856

(iii) 998001 (iv) 657666025

Ans. (i) We know that the 'ones' place of the square of 1 and 9 is 1.

$\therefore$ The possible 'ones' digit of the square root of 9801 are 1 and 9.

(ii) We know that the 'ones' place of the square of 4 and 6 is 6.

$\therefore$ The possible 'ones' digit of the square root of 99856 are 4 and 6.

(iii) We know that the 'ones' place of the square of 1 and 9 is 1.

$\therefore$ The possible 'ones' digit of the square root of 998001 are 1 and 9.

(iv) We know that the 'ones' place of the square of 5 is 5.

$\therefore$ The possible 'ones' digit of the square root of 657666025 is 5.

NS. 2

Without doing any calculation, find the numbers which are surely not perfect squares.

(i) 153 (ii) 257

(iii) 408 (iv) 441

Ans. We know that the numbers ending with 2, 3, 7 or 8 are not a perfect squares. So, (i), (ii) and (iii) are surly not perfect squares.

(iv) Since, the number 441 ends with 1. Thus, 441 may or may not be a perfect square.

NS. 3

Find the square root of 100 and 169 by the method of repeated subtraction.

Ans. First consider 100.

(i) 100 – 1 = 99 (ii) 99 – 3 = 96

(iii) 96 – 5 = 91 (iv) 91 – 7 = 84

(v) 84 – 9 = 75 (vi) 75 – 11 = 64

(vii) 64 – 13 = 51 (viii) 51 – 15 = 36

(ix) 36 – 17 = 19 (x) 19 – 19 = 0

$\therefore \sqrt{100} = 10.$

Now, consider 169

(i) 169 – 1 = 168 (ii) 168 – 3 = 165

(iii) 165 – 5 = 160 (iv) 160 – 7 = 153

(v) 153 – 9 = 144 (vi) 144 – 11 = 133

(vii) 133 – 13 = 120 (viii) 120 – 15 = 105

(ix) 105 – 17 = 88 (x) 88 – 19 = 69

(xi) 69 – 21 = 48 (xii) 48 – 23 = 25

(xiii) 25 – 25 = 0

$\therefore \sqrt{169} = 13.$

NS. 4

Find the square roots of the following numbers by the Prime Factorisation Method.

(i) 729 (ii) 400

(iii) 1764 (iv) 4096

(v) 7744 (vi) 9604

(vii) 5929 (viii) 9216

(ix) 529 (x) 8100

Ans. (i) 729 = 3 × 3 × 3 × 3 × 3 × 3

By pairing the prime factors, we get

3	729
3	243
3	81
3	27
3	9
3	3
	1

$729 = \underline{3\times3}\times\underline{3\times3}\times\underline{3\times3}$

So, $\sqrt{729} = 3\times3\times3 = 27$.

(ii) 400 = 2 × 2 × 2 × 2 × 5 × 5

2	400
2	200
2	100
2	50
5	25
5	5
	1

By pairing the prime factors, we get

400 = $\underline{2\times2}\times\underline{2\times2}\times\underline{5\times5}$

So, $\sqrt{400}$ = 2 × 2 × 5 = 20.

(iii) 1764 = 2 × 2 × 3 × 3 × 7 × 7

2	1764
2	882
3	441
3	147
7	49
7	7
	1

By pairing the prime factors, we get

1764 = $\underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}$

So, $\sqrt{1764}$ = 2 × 3 × 7 = 42.

(iv) 4096 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

By pairing the prime factors, we get

4096 = $\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}$

So, $\sqrt{4096}$ = 2 × 2 × 2 × 2 × 2 × 2 = 64.

(v) 7744 = 2 × 2 × 2 × 2 × 2 × 2 × 11 × 11

2	7744
2	3872
2	1936
2	968
2	484
2	242
11	121
11	11
	1

By pairing the prime factors, we get

7744 = $\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{11\times11}$

So, $\sqrt{7744}$ = 2 × 2 × 2 × 11 = 88.

(vi) 9604 = 2 × 2 × 7 × 7 × 7 × 7

2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

By pairing the prime factors, we get

$9604 = \underline{2\times2} \times \underline{7\times7} \times \underline{7\times7}$

So, $\sqrt{9604} = 98$.

(vii) 5929 = 7 × 7 × 11 × 11

7	5929
7	847
11	121
11	11
	1

By pairing the prime factors, we get

$5929 = \underline{7\times7} \times \underline{11\times11}$

So, $\sqrt{5929} = 11 \times 7 = 77$.

(viii) 9216 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3

2	9216
2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

By pairing the prime factors, we get

$9216 = \underline{2\times2} \times \underline{2\times2} \times \underline{2\times2} \times \underline{2\times2} \times \underline{2\times2} \times \underline{3\times3}$

So, $\sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$.

(ix) 529 = 23 × 23

23	529
23	23
	1

By pairing the prime factors, we get

$529 = \underline{23\times23}$

So, $\sqrt{529} = 23$.

(x) 8100 = 3 × 3 × 3 × 3 × 2 × 2 × 5 × 5

3	8100
3	2700
3	900
3	300
2	100
2	50
5	25
5	5
	1

By pairing the prime factors, we get

$8100 = \underline{2\times2} \times \underline{3\times3} \times \underline{3\times3} \times \underline{5\times5}$

So, $\sqrt{8100} = 2 \times 3 \times 3 \times 5 = 90$.

NS. 5

For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.

(i) 252 (ii) 180

(iii) 1008 (iv) 2028

(v) 1458 (vi) 768

Ans. (i) We have, 252 = 2 × 2 × 3 × 3 × 7

The smallest whole number is 7 by which 252 should be multiplied so as to get a perfect square.

$252 \times 7 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

2	252
2	126
3	63
3	21
7	7
	1

Now each prime factor is in a pair.

Therefore, $252 \times 7 = 1764$ is a perfect square.

So, $\sqrt{1764} = 2 \times 3 \times 7 = 42$.

(ii) We have, $180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

The smallest whole number is 5, by which 180 should be multiplied so as to get a perfect square.

$180 \times 5 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5}$

2	180
2	90
3	45
3	15
5	5
	1

Now each prime factor is in pair.

Therefore, $180 \times 5 = 900$ is a perfect square.

So, $\sqrt{900} = 2 \times 3 \times 5 = 30$.

(iii) We have, $1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$

The smallest whole number is 7, by which 1008 should be multiplied so as to get a perfect square.

$1008 \times 7 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

Now each prime factor is in pair.

Therefore, $1008 \times 7 = 7056$ is a perfect square.

So, $\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$.

(iv) We have, $2028 = 2 \times 2 \times 3 \times 13 \times 13$

The smallest whole number is 3 by which 2028 should be multiplied to get a perfect square.

$2028 \times 3 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{13 \times 13}$

2	2028
2	1014
3	507
13	169
13	13
	1

Now each prime factor is in pair.

Therefore, $2028 \times 3 = 6084$ is a perfect square.

So, $\sqrt{6084} = 2 \times 3 \times 13 = 78$.

(v) We have, $1458 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

The smallest whole number is 2, by which 1458 should be multiplied so as to get a perfect square.

$1456 \times 2 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Now each prime factor is in pair.
Therefore, $1458 \times 2 = 2916$ is a perfect square.

So, $\sqrt{2916} = 2 \times 3 \times 3 \times 3 = 54$.

(vi) We have, $768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

The smallest whole number is 3, by which 768 should be multiplied so as to get a perfect square.

$768 \times 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Now each prime factor is in pair.

Therefore, $768 \times 3 = 2304$ is a perfect square.

So, $\sqrt{2304} = 2 \times 2 \times 2 \times 2 \times 3 = 48$.

NS. 6

For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.

(i) 252 (ii) 2925
(iii) 396 (iv) 2645
(v) 2800 (vi) 1620

Ans. (i) We have $252 = \underline{2 \times 2} \times \underline{3 \times 3} \times 7$

We find that 252 should be divided by 7, to get a perfect square.

2	252
2	126
3	63
3	21
7	7
	1

$252 \div 7 = 36 = \underline{2 \times 2} \times \underline{3 \times 3}$

Therefore, the required smallest number is 7.

Also, $\sqrt{36} = 6$.

(ii) We have, $2925 = \underline{5 \times 5} \times \underline{3 \times 3} \times 13$
We find that 2925 should be divided by 13, to get a perfect square.

5	2925
5	585
3	117
3	39
13	13
	1

$2925 \div 13 = 225 = \underline{5 \times 5} \times \underline{3 \times 3}$
Therefore, the required smallest number is 13.

Also, $\sqrt{225} = 15$.

(iii) We have, $396 = \underline{2\times2} \times \underline{3\times3} \times 11$

We find that 396 should be divided by 11, to get a perfect square.

3	396
3	132
2	44
2	22
11	11
	1

$396 \div 11 = 36 = \underline{2 \times 2} \times \underline{3 \times 3}$

Therefore, the required smallest number is 11.

Also, $\sqrt{36} = 6$.

(iv) We have $2645 = 5 \times \underline{23\times23}$

We find that 2645 should be divided by 5.

5	2645
23	529
23	23
	1

$2645 \div 5 = 529 = \underline{23 \times 23}$

Therefore, the required smallest number is 5.

Also, $\sqrt{529} = 23$.

(v) We have, $2800 = \underline{2\times2} \times \underline{2\times2} \times \underline{5\times5} \times 7$

We find that 2800 should be divided by 7, to get a perfect square.

2	2800
2	1400
2	700
2	350
5	175
5	35
7	7
	1

$2800 \div 7 = 400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

Therefore, the required smallest number is 7.

Also, $\sqrt{400} = 2 \times 2 \times 5 = 20$.

(vi) We have, $1620 = \underline{2\times2} \times 5 \times \underline{3\times3} \times \underline{3\times3}$

We find that 1620 should be divided by 5, to get a perfect square.

2	1620
2	810
5	405
3	81
3	27
3	9
3	3
	1

$1620 \div 5 = 324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

Therefore, the required smallest number is 5.

Also, $\sqrt{324} = 2 \times 3 \times 3 = 18$.

NS. 7

The students of Class VIII of a school donated Rs. 2401 in all, for the Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Ans. We have, $2401 = \underline{7\times7} \times \underline{7\times7}$

So, $\sqrt{2401} = 7 \times 7 = 49$

7	2401
7	343
7	49
7	7
	1

The number of students in the class are 49.

NS. 8

2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Ans. Total number of plants = 2025.

The plants are planted in a garden in such a way that each row contains as many plants as the number of rows.

3	2025
3	675
3	225
3	75
5	25
5	5
	1

$2025 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}$

$\therefore$ Number of plants in each row

$= 3 \times 3 \times 5 = 45$

So, number of rows = number of plants.

Thus, the number of rows = 45 and number of plants = 45.

NS. 9

Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.

Ans. The smallest number divisible by each 4, 9 and 10 is their L.C.M. The L.C.M. of 4, 9 and 10 is $2 \times 2 \times 3 \times 3 \times 5 = 180$.

2	4,	9,	10,
2	2,	9,	5,
3	1,	9,	5,
3	1,	3,	5,
5	1,	1,	5,
	1,	1,	1,

Now, prime factorisation of 180 is

$180 = \underline{2\times2} \times \underline{3\times3} \times 5$

In order to get a perfect square, each factor of 180 must be paired.

So, we need to make pair of 5.

$\therefore$ 180 should be multiplied by 5.

Hence, the required number is $180 \times 5 = 900$.

NS. 10

Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.

Ans. The smallest number divisible by each 8, 15 and 20, is their L.C.M. The L.C.M. of 8, 15 and 20 is $2 \times 2 \times 2 \times 3 \times 5 = 120$

2	8,	15,	20,
2	4,	15,	10,
2	2,	15,	5,
5	1,	15,	5,
3	1,	3,	1,
	1,	1,	1,

Now prime factorisation of 120 is

$120 = \underline{2\times2} \times 2\times3 \times 5$...........(i)

In order to get a perfect square, each factor of 120 must be paired. Thus we multiply (i) by $2 \times 3 \times 5 = 30$, we get $120 \times 30 = 3600$.

EXERCISE – 2.4

NS. 1

Find the square root of each of the following numbers by Division method.

(i) 2304 (ii) 4489
(iii) 3481 (iv) 529
(v) 3249 (vi) 1369
(vii) 5776 (viii) 7921
(ix) 576 (x) 1024
(xi) 3136 (xii) 900

Ans. (i) 2304

	48
4	$\overline{23}\ \overline{04}$
	– 16
88	704
	–704
	0

$\therefore \sqrt{2304} = 48.$

(ii) 4489

	67
6	$\overline{44}\ \overline{89}$
	– 36
127	889
	–889
	0

$\therefore \sqrt{4489} = 67.$

(iii) 3481

	59
5	$\overline{34}\ \overline{81}$
	– 25
109	981
	–981
	0

$\therefore \sqrt{3481} = 59.$

(iv) 529

	23
2	$\overline{5}\ \overline{29}$
	– 4
43	129
	–129
	0

$\therefore \sqrt{529} = 23.$

(v) 3249

	57
5	$\overline{32}\ \overline{49}$
	– 25
107	749
	–749
	0

$\therefore \sqrt{3249} = 57.$

(vi) 1369

	37
3	$\overline{13}\ \overline{69}$
	– 9
67	469
	– 469
	0

$\therefore \sqrt{1369} = 37.$

(vii) 5776

	76
7	$\overline{57}\ \overline{76}$
	– 49
146	876
	– 876
	0

$\therefore \sqrt{5776} = 76.$

(viii) 7921

	89
8	$\overline{79}\ \overline{21}$
	– 64
169	1521
	–1521
	0

$\therefore \sqrt{7921} = 89.$

(ix) 576

	24
2	$\overline{5}\ \overline{76}$
	– 4
44	176
	–176
	0

$\therefore \sqrt{576} = 24.$

(x) 1024

	32
3	$\overline{10}\,\overline{24}$
	−9
62	124
	−124
	0

$\therefore \sqrt{1024} = 32.$

(xi) 3136

	56
5	$\overline{31}\,\overline{36}$
	−25
106	636
	−636
	0

$\therefore \sqrt{3136} = 56.$

(xii) 900

	30
3	$\overline{9}\ \overline{00}$
	−9
60	00
	00
	0

$\therefore \sqrt{900} = 30.$

NS. 2

Find the number of digits in the square root of each of the following numbers (without any calculation).

(i) 64 (ii) 144

(iii) 4489 (iv) 27225

(v) 390625

Ans. (i) Since number of digits in 64 is 2 (= n), which is even.

Then its square root will have $\frac{n}{2}$ digits.

Number of digits $= \frac{2}{2} = 1$.

(ii) Since the number of digits in 144 is 3 (= n), which is odd.

Then its square root will have $\left(\frac{n+1}{2}\right)$ digits.

Number of digits $= \frac{3+1}{2} = \frac{4}{2} = 2$.

(iii) Since number of digits in 4489 is 4 (= n), which is even.

Then its square root will have $\frac{n}{2}$ digits.

Number of digits $= \frac{4}{2} = 2$ digits.

(iv) Since the number of digits in 27225 is 5 (= n), which is odd.

Then its square root will have $\left(\frac{n+1}{2}\right)$ digits.

Number of digits $= \frac{5+1}{2} = \frac{6}{2} = 3$ digits.

(v) Since number of digits in 390625 is 6 (= n), which is even.

Then its square root will have $\left(\frac{n}{2}\right)$ digits.

Number of digits $= \frac{6}{2} = 3$ digits.

NS. 3

Find the square root of the following decimal numbers.

(i) 2.56 (ii) 7.29

(iii) 51.84 (iv) 42.25

(v) 31.36

Ans. (i) 2.56

	1.6
1	$\overline{2}\cdot\overline{56}$
	−1
26	156
	−156
	0

$\therefore \sqrt{2.56} = 1.6$

(ii) 7.29

	2.7
2	$\overline{7}\cdot\overline{29}$
	-4
47	329
	-329
	0

$\therefore \sqrt{7.29} = 2.7$

(iii) 51.84

	7.2
7	$\overline{51}\cdot\overline{84}$
	-49
142	284
	-284
	0

$\therefore \sqrt{51.84} = 7.2$

(iv) 42.25

	6.5
6	$\overline{42}\cdot\overline{25}$
	-36
125	625
	-625
	0

$\therefore \sqrt{42.25} = 6.5$

(v) 31.36

	5.6
5	$\overline{31}\cdot\overline{36}$
	-25
106	636
	-636
	0

$\therefore \sqrt{31.36} = 5.6$

NS. 4

Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 402 (ii) 1989

(iii) 3250 (iv) 825

(v) 4000

Ans. (i) 402

	20
2	$\overline{4}\ \overline{02}$
	-4
40	02
	-00
	2

Thus, if we subtract 2 from 402, we get a perfect square number whose square root is 20.

(ii) 1989

	44
4	$\overline{19}\ \overline{89}$
	-16
84	389
	-336
	53

Thus, if we subtract 53 from 1989, we get a perfect square number whose square root is 44.

(iii) 3250

	57
5	$\overline{32}\ \overline{50}$
	-25
107	750
	-749
	1

Thus, if we subtract 1 from 3250, we get a perfect square number whose square root is 57.

(iv) 825

	28
2	$\overline{8}\ \overline{25}$
	-4
48	425
	-384
	41

Thus, if we subtract 41 from 825, we get a perfect square number whose square root is 28.

(v) 4000

```
        63
    ┌──────────
  6 │ 40 00
    │–36
────┼──────────
123 │  400
    │– 369
────┼──────────
    │   31
```

Thus, if we subtract 31 from 4000, we get a perfect square number whose square root is 63.

NS. 5

Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 525 (ii) 1750

(iii) 252 (iv) 1825

(v) 6412

Ans. (i) 525

```
       22
    ┌──────────
  2 │ 5 25
    │– 4
────┼──────────
 42 │ 125
    │– 84
────┼──────────
    │  41
```

Clearly, $22^2 = 484 < 525$, $23^2 = 529 > 525$.

Hence, the number to be added is 529 – 525 = 4 and square root of 529 is 23.

(ii) 1750

```
        41
    ┌──────────
  4 │ 17 50
    │–16
────┼──────────
 81 │  150
    │ – 81
────┼──────────
    │   69
```

Clearly, $41^2 = 1681 < 1750$

$42^2 = 1764 > 1750$.

Hence, the number to be added is

1764 – 1750 = 14

and square root of 1764 is 42.

(iii) 252

```
       15
    ┌──────────
  1 │ 2 52
    │– 1
────┼──────────
 25 │ 152
    │– 125
────┼──────────
    │  27
```

Clearly, $15^2 = 225 < 252$

$16^2 = 256 > 252$

∴ The number to be added is 256 – 252 = 4 and square root of 256 is 16.

(iv) 1825

```
        42
    ┌──────────
  4 │ 18 25
    │–16
────┼──────────
 82 │  225
    │ – 164
────┼──────────
    │   61
```

Clearly, $42^2 = 1764 < 1825$

$43^2 = 1849 > 1825$

∴ The number should be added is 1849 – 1825 = 24 and square root of 1849 is 43.

(v) 6412

```
        80
    ┌──────────
  8 │ 64 12
    │–64
────┼──────────
160 │    12
    │  – 00
────┼──────────
    │    12
```

Clearly, $80^2 = 6400 < 6412$

$81^2 = 6561 > 6412$

∴ The number should be added is

6561 – 6412 = 149 and square root of 6561 is 81.

NS. 6

Find the length of the side of a square whose area is 441 m^2.

Ans. Let the length of the side of a square is x m.

Area of the square = $x^2 \Rightarrow 441 = x^2$

$\Rightarrow 21 \times 21 = x^2 \Rightarrow 21 = x$

Thus, the required length of the square is 21 m.

NS. 7

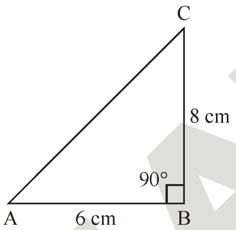
In a triangle ABC, $\angle B = 90°$.

(i) If AB = 6 cm, BC = 8 cm, find AC.

(ii) If AC = 13 cm, BC = 5 cm, find AB.

Ans. (i) AB = 6 cm, BC = 8 cm

By using Pythagoras theorem,



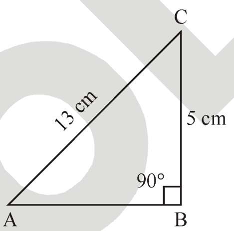
$AC^2 = AB^2 + BC^2 \quad \Rightarrow AC^2 = (6)^2 + (8)^2$

$\Rightarrow AC^2 = 36 + 64 \quad \Rightarrow AC^2 = 100 = 10 \times 10$

$\Rightarrow AC = 10$ cm.

(ii) AC = 13 cm, BC = 5 cm

By using Pythagoras theorem,



$AC^2 = AB^2 + BC^2 \quad \Rightarrow (13)^2 = AB^2 + (5)^2$

$\Rightarrow 169 = AB^2 + 25 \quad \Rightarrow 169 - 25 = AB^2$

$\Rightarrow 144 = AB^2 \quad \Rightarrow 12 \times 12 = AB^2$

$\Rightarrow AB = 12$ cm.

NS. 8

A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remains same. Find the minimum number of plants he needs more for this.

Ans. Total number of plants = 1000.

Since, the plants are planted in a garden in such a way that there are as many number of plants in a row as the number of rows.

	31
3	$\overline{10}\ \overline{00}$
	– 9
61	100
	– 61
	39

Clearly, $31^2 = 961 < 1000$

$32^2 = 1024 > 1000$

$\therefore 1024 - 1000 = 24$.

Thus, gardener needs 24 more plants.

NS. 9

There are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement ?

Ans. Total number of children = 500.

Since the number of rows is equal to the number of columns, in which students have to stand.

	22
2	$\overline{5}\ \overline{00}$
	– 4
42	100
	– 84
	16

$\therefore$ Clearly, 16 students would be left out in this arrangement.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. The value of $\sqrt{41+\sqrt{54+\sqrt{88+\sqrt{128+\sqrt{256}}}}}$ is

(A) 7 (B) 6

(C) 8 (D) 10

2. The value of $\left(\sqrt{\frac{225}{729}}-\sqrt{\frac{25}{144}}\right)\div\sqrt{\frac{16}{81}}$ is

(A) $\frac{1}{48}$ (B) $\frac{5}{48}$

(C) $\frac{5}{16}$ (D) $\frac{1}{16}$

3. If $\sqrt{0.04\times0.4\times a}=0.004\times0.4\times\sqrt{b}$, then $\frac{a}{b}$ is

(A) 16×10^{-3} (B) 16×10^{-4}

(C) 16×10^{-5} (D) 16×10^{-6}

4. If $\sqrt{1+\frac{x}{289}}=1\frac{1}{17}$, then x is equal to

(A) 1 (B) 13

(C) 35 (D) 15

5. If a = 0.1039, then the value of $3a-\sqrt{4a^2-4a+1}$ is

(A) 0.1039 (B) 0.2078

(C) 1.1039 (D) 2.1039

6. What is the least number which should be subtracted from 3.26 to make it a perfect square ?

(A) 2 (B) 0.2078

(C) 0.02 (D) 0.04

7. What is the digit at the one's place in the square of the number 2934 ?

(A) 4 (B) 6

(C) 1 (D) 5

8. The sum of 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 is

(A) 64 (B) 85

(C) 81 (D) 100

9. Evaluate : $\sqrt{6084}$

(A) 78 (B) 76

(C) 84 (D) 87

10. The square root of 1471369 is

(A) 1312 (B) 1211

(C) 1219 (D) 1213

11. The value of $\sqrt{1\frac{9}{16}}$ is

(A) $\frac{2}{3}$ (B) $\frac{2}{5}$

(C) $1\frac{1}{4}$ (D) $\frac{3}{4}$

12. What is the square root of 0.0009 ?

(A) 0.3 (B) 0.03

(C) 0.003 (D) 3

13. Evaluate : $\sqrt{175.2976}$

(A) 13.24 (B) 12.14

(C) 12.24 (D) 13.14

14. The value of $\sqrt{\frac{0.289}{0.00121}}$ is

(A) $\frac{17}{11}$ (B) $\frac{170}{11}$

(C) $\frac{70}{11}$ (D) $\frac{17}{110}$

15. The value of $\sqrt{0.121}$ upto three places of decimal is

(A) 0.011 (B) 0.11

(C) 0.347 (D) 1.1

16. The value of $\sqrt{\dfrac{0.16}{0.4}}$ is

(A) 0.02 (B) 0.2

(C) 0.63 (D) None of these

17. The least perfect square number divisible by 3, 4, 5, 6 and 8 is

(A) 900 (B) 1200

(C) 2500 (D) 3600

18. The least perfect square, which is divisible by each of 21, 36 and 66 is

(A) 213444 (B) 214344

(C) 214434 (D) 231444

19. Which of the following numbers is not a perfect square ?

(A) 612 (B) 529

(C) 900 (D) 100

20. Which of the following numbers is a perfect square ?

(A) 100 (B) 100000

(C) 25000 (D) 40

21. Which of the following numbers is a square of odd number ?

(A) 256 (B) 144

(C) 2601 (D) 400

22. The square of 33 is

(A) 1059 (B) 1069

(C) 1089 (D) 1079

23. A welfare association collected Rs. 52900 as donation from the students. If each paid as many rupees as there were students, find the number of students.

(A) 230 (B) 225

(C) 220 (D) 245

24. Which of the following are pythagorean triplets ?

(A) 2, 3, 5 (B) 5, 7, 9

(C) 6, 9, 11 (D) 8, 15, 17

25. Is 1600 a perfect square ?

(A) No (B) Yes

(C) Can't say (D) None of these

PARAGRAPH TYPE

PASSAGE # I

When a is the side of a square, its area is given by a^2. Area of a square plot is 2304 m^2.

26. The side of the square plot is :

(A) 42 m (B) 48 m

(C) 46 m (D) 50 m

27. If the area of a square is increased by 3172 units, then the new side of the square will be :

(A) 78 m (B) 76 m

(C) 72 m (D) 74 m

28. The area of a square with side 16 m will be :

(A) 96 m^2 (B) 56 m^2

(C) 256 m^2 (D) 196 m^2

PASSAGE # II

The area of two squares are respectively, 256 and 625 square metres.

29. The side of two squares respectively are :

(A) 16, 25 (B) 25, 16

(C) 17, 25 (D) 16, 5

30. The ratio of their sides are :

(A) 17 : 25 (B) 15 : 16

(C) 16 : 25 (D) 16 : 5

31. If both the squares are joined, then the perimeter of new figure will be :

(A) 140 m (B) 132 m

(C) 123 m (D) 164 m

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column – I and Column – II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the following :

Column – I	Column – II
(P) The least square number which is divisible by each of the numbers 6, 9, 15 and 20 is	(i) 18
(Q) The greatest number of six digits which is a perfect square is	(ii) 9
(R) The least square number which is subtracted from 2509 to make it a perfect square is	(iii) 900
(S) The least square number which must be added to 5607 to obtain a perfect square is	(iv) 998001

(A) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)
(B) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)
(C) (P) → (iii), (Q) → (ii), (R) → (iv), (S) → (i)
(D) (P) → (iv), (Q) → (iii), (R) → (i), (S) → (ii)

33. Match the numbers in List – I with their square roots in List – II.

Column– I	Column– II
(P) 1296	(i) 63
(Q) 5476	(ii) 231
(R) 3969	(iii) 36
(S) 53361	(iv) 74

(A) (P)→(ii), (Q)→(iii), (R)→(iv), (S)→(i)
(B) (P)→(i), (Q)→(iv), (R)→(iii), (S)→(ii)
(C) (P)→(iii), (Q)→(iv), (R)→(i), (S)→(ii)
(D) (P)→(iv), (Q)→(iii), (R)→(ii), (S)→(i)

Space for Notes :

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Find the value of $100^2 - 99^2$.
2. Find the squares of the following numbers using a pattern :
   (i) 205 (ii) 515
3. Find the square of the number 509 using the identity $(a + b)^2 = a^2 + 2ab + b^2$.
4. Find the square of 575 using the identity $(a - b)^2 = a^2 - 2ab + b^2$.
5. Find the square root of 11236 by prime factorisation.
6. Find the least square number which is exactly divisible by 10, 12, 15 and 18.
7. Find the value of $\sqrt{147} \times \sqrt{243}$.
8. Evaluate : $\sqrt{1.96}$
9. Find the value of $\sqrt{45} \times \sqrt{20}$.
10. What is the digit at the ones place in the square of 1819 ? Also, determine its square.

SHORT ANSWER TYPE

1. Find the least square number exactly divisible by each of the numbers 8, 12, 15 and 20.
2. A P.T. teacher wants to arrange maximum possible number of 6000 students in a field such that the number of rows is equal to the number of columns. Find the number of rows if 71 were left out after the arrangement.
3. What least number must be subtracted from 7250 to get a perfect square ? Also, find the square root of this perfect square.
4. Find the least number of six digits which is a perfect square. Find the square root of this number.
5. Find the square root of 298116 by prime factorisation.

LONG ANSWER TYPE

1. Find the value of $\sqrt{11025 \times 1024}$.
2. The area of a square field is $101\frac{1}{400}$ square metres. Find the length of one side of the field.
3. Find the square root of $52\frac{857}{2116}$.
4. A general wishes to draw up his 36581 soldiers in the form of a solid square. After arranging them , he found that some of them are left over. How many are left ?
5. A group of students decided to collect as many paise from each member of the group as is the number of members. If the total collection amounts to Rs. 59.29. Find the members in the group.

TRUE / FALSE TYPE

1. 3, 4, 5 are pythagorean triplet.
2. 9 is a perfect square.
3. 80 is a perfect square.
4. A perfect square is always an even number.
5. Square root of 21 is 4.5825.

FILL IN THE BLANKS

1. The digit at ones place of 37^2 is _______.
2. The square of even number is _________.
3. $\sqrt{4096}$ is _______.
4. There are ________ perfect squares between 1 and 100.
5. The square of proper fraction is ________ than to the fraction.

NUMERICAL PROBLEMS

1. Evaluate : $\sqrt{41-\sqrt{21+\sqrt{19-\sqrt{9}}}}$

2. The value of $7\left(\frac{\sqrt{625}}{11}\times\frac{14}{\sqrt{25}}\times\frac{11}{\sqrt{196}}\right)$ is

3. The square root of 15876 is

4. How many two-digit numbers satisfy this property : The last digit (unit digit) of the square of the two-digit number is 8 ?

5. The least number by which 294 must be multiplied to make it a perfect square is

ANALYTICAL PROBLEMS & BRAIN TEASER

1. What is the approximate value of $(15.01)^2\times\sqrt{728}$?
   (A) 6125
   (B) 6225
   (C) 6200
   (D) 6075

2. What must be subtracted from 16161 to get a perfect square ? Also, find the square root of this perfect square.
   (A) 32, 127
   (B) 35, 127
   (C) 32, 128
   (D) 35, 130

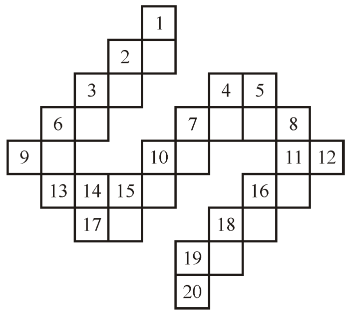
3. 4096 soldiers are arranged in an auditorium in such a manner that there are as many soldiers in a row as there are rows in the auditorium. How many rows are there in the auditorium ?
   (A) 94
   (B) 58
   (C) 44
   (D) 64

4. Find the least number which must be added to 304584 to make it a perfect square. Also, find the resulting number and the square root of the resulting number.
   (A) 120, 304704, 554
   (B) 120, 304704, 552
   (C) 115, 304704, 560
   (D) 115, 304704, 565

5. Find the value of $\sqrt{0.09}+\sqrt{0.81}+\sqrt{7.29}+\sqrt{98.01}$.
   (A) 13.7
   (B) 13.8
   (C) 13.6
   (D) 13.1

CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

1. The square of $\sqrt{5}$
2. 3^3
3. The square root of 256
4. $\sqrt{3249}$
6. $\left(\sqrt{8}\right)^4 \div 2$
7. $\sqrt{14}\times 1000$ (round off to nearest integer)
9. 6^2
10. $\sqrt{1156}$

11. $18^2 - 15^2$

13. $\sqrt{2040000}$ (round off to nearest integer)

16. $\sqrt{576} + \sqrt{4096}$

17. $\sqrt{6600 + 625}$

18. $2^6 - 6^2 + 1^2$

19. $\sqrt{85^2 - 77^2}$

20. $\sqrt{4}$

Down

1. $\sqrt{95^2 - 76^2}$

2. $5^2 + 1^2$

3. $6 \div \sqrt{\frac{1}{4}}$

4. $2^5 + 5^2$

5. $\sqrt{5476}$

6. 19^2

7. $16^2 - 15^2 + 3^1$

8. $\sqrt{88804}$

9. $\sqrt{9}$

10. $\sqrt{404 + 1040}$

12. $\sqrt{64} + \sqrt{0.81} + \sqrt{0.01}$

14. $\sqrt{2304}$

15. $\sqrt{24^2 + 7^2}$

16. $\sqrt{80^2 + 39^2}$

18. $\sqrt{441} + \sqrt{25}$

19. $8 \div \left(\frac{1}{2}\right)^2$

Space for Notes :

Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	C	C	C	C	B	C	A	D	C	B	A	B	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	D	A	A	A	C	C	A	D	B	B	D	C	A	C
31	32	33												
B	B	C												

EXERCISE II

VERY SHORT ANSWER TYPE

1. 199 **2.** (i) 42025, (ii) 265225 **3.** 259081 **4.** 330625 **5.** 106 **6.** 900

7. 189 **8.** 1.4 **9.** 30 **10.** 1

SHORT ANSWER TYPE

1. 3600 **2.** 77 **3.** 25, 85 **4.** 100489, 317 **5.** 546

LONG ANSWER TYPE

1. 3360 **2.** $10\frac{1}{20}$ m **3.** $7\frac{11}{46}$ **4.** 100 **5.** 77

TRUE / FALSE

1. T **2.** T **3.** F **4.** F **5.** T

FILL IN THE BLANKS

1. 9 **2.** even **3.** 64 **4.** θ **5.** Smaller

NUMERICAL PROBLEMS

1. 6 **2.** 35 **3.** 126 **4.** 0 **5.** 6

ANALYTICAL PROBLEMS & BRAIN TEASER

1. D **2.** A **3.** D **4.** B **5.** B

CROSSWORD PUZZLE

Across

1. 5 **2.** 27 **3.** 16 **4.** 57 **6.** 32 **7.** 3742 **9.** 36

10. 34 **11.** 99 **13.** 1428 **16.** 88 **17.** 85 **18.** 29 **19.** 36

20. 2

Down

1. 57 **2.** 26 **3.** 12 **4.** 57 **5.** 74 **6.** 361 **7.** 34

8. 298 **9.** 3 **10.** 38 **12.** 9 **14.** 48 **15.** 25 **16.** 89

18. 26 **19.** 32

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : SQUARES AND SQUARE ROOTS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Solutions			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put “completed” only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

CUBES AND CUBE ROOTS 6

Concepts

INTRODUCTION

This is a story about one of India's great mathematical geniuses, S. Ramanujan. Once another famous mathematician Prof.G.H. Hardy came to visit him in a taxi whose number was 1729. While talking to Ramanujan, Hardy described this number "a dull number". Ramanujan quickly pointed out that 1729 was indeed interesting. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways:

$1729 = 1728 + 1 = 12^3 + 1^3$

$1729 = 1000 + 729 = 10^3 + 9^3$

1729 has since been known as the Hardy – Ramanujan Number, even though this feature of 1729 was known more than 300 years before Ramanujan.

How did Ramanujan know this? Well, he loved numbers.All through his life, he experimented with numbers. He probably found numbers that were expressed as the sum of two squares and sum of two cubes also.

There are many other interesting patterns of cubes. Let us learn about cubes, cube roots and many other interesting facts related to them.

1. CUBE OF A NUMBER

When a number is multiplied three times by itself, we say that the number has been cubed and the product is called cube of that number or the number raised to the power of 3.

For example : 1 is a number and cube of $1 = 1 \times 1 \times 1 = 1^3$

2 is a number and cube of $2 = 2 \times 2 \times 2 = 2^3$

In general x is a number and cube of $x = x \times x \times x = x^3$

The following table gives the cubes'of numbers 1 to 10.

Number	Cube
1	$1^3 = 1 \times 1 \times 1 = 1$
2	$2^3 = 2 \times 2 \times 2 = 8$
3	$3^3 = 3 \times 3 \times 3 = 27$
4	$4^3 = 4 \times 4 \times 4 = 64$
5	$5^3 = 5 \times 5 \times 5 = 125$
6	$6^3 = 6 \times 6 \times 6 = 216$
7	$7^3 = 7 \times 7 \times 7 = 343$
8	$8^3 = 8 \times 8 \times 8 = 512$
9	$9^3 = 9 \times 9 \times 9 = 729$
10	$10^3 = 10 \times 10 \times 10 = 1000$

2. PERFECT CUBE NUMBER

A number x is said to be a perfect cube number if there is an integer y such that $x = y \times y \times y = y^3$.

For example : 216 is a perfect cube as there is an integer 6 such that $216 = 6 \times 6 \times 6 = 6^3$

3. CUBES AND THEIR PRIME FACTORS

Each prime factor appears three or multiple of three times in a perfect cube number.

Example : Observe the following table :

Number	Prime Factorisation	Prime Factorisation of its cube
2	2	$2^3 = 8 = 2\times2\times2 = 2^3$
3	3	$3^3 = 27 = 3\times3\times3 = 3^3$
4	2×2	$4^3 = 64 = 2\times2\times2\times2\times2\times2 = 2^3\times2^3$
6	2×3	$6^3 = 216 = 2\times2\times2\times3\times3\times3 = 2^3\times3^3$
9	3×3	$9^3 = 729 = 3\times3\times3\times3\times3\times3 = 3^3\times3^3$
15	3×5	$15^3 = 3375 = 3\times3\times3\times5\times5\times5 = 3^3\times5^3$

Example 1

Is 189 a perfect cube or not ?

Solution :

Resolving 189 into prime factors, we get $189 = \underline{3 \times 3 \times 3} \times 7$

3	189
3	63
3	21
7	7
	1

Making triplets, we find that one triplet is formed and we are left with one more factor. Thus, 189 cannot be expressed as a product of triplets.

Hence, 189 is not a perfect cube.

4. SMALLEST MULTIPLE THAT IS A PERFECT CUBE

To find the smallest number by which a number must be multiplied so that product is a perfect cube and the smallest number by which a number must be divided so that the quotient is a perfect cube.

Example 2

Is 1296 a perfect cube or not ? If not, find the smallest natural number by which 1296 must be divided so that the quotient is a perfect cube.

Solution :

$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 2 \times 3$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

Thus, 1296 is not a perfect cube. The prime factor 6 does not appear in the group of three, since 6 appears only one time, if we divide the number by 6, then we will get a perfect cube. So, $1296 \div 6 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

Hence the smallest number by which 1296 should be divided to make it perfect cube is 6.

Example 3

Examine if 1512 is a perfect cube. If not, find the smallest number by which it must be multiplied so that the product is a perfect cube. Also find the smallest number by which it must be divided so that the quotient is a perfect cube.

Solution :

$1512 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7$

Here the prime factor 7 does not occur in a group of three. Hence, 1512 is not a perfect cube. Further, 7 appears only once. If we multiply the number by 7×7, then in the product, 7 will also appear in a group of three and the product will be a perfect cube. Thus, the smallest number by which the given number should be multiplied is 7×7 i.e., 49. Finally, if we divide the given number 1512 by 7. The resulting number has prime factors in a group of three. In fact, $1512 \div 7 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$. Hence, the smallest number by which the given number should be divided so that quotient is a perfect cube is 7.

5. PROPERTIES OF CUBES OF NUMBERS

1. Cubes of all even numbers are even.
2. Cubes of all odd numbers are odd.
3. Cube of a negative number is always negative.

Cubes of the Digits 1 to 9.

x	1	2	3	4	5	6	7	8	9
x^3	1	8	27	64	125	216	343	512	729

From the table we observe that cubes of the digits 1, 4, 5, 6 and 9 are numbers ending in the same digits 1, 4, 5, 6 and 9 respectively. However 2 and 8 make a pair in the sense that the cube of 2 ends in 8 and the cube of 8 ends in 2. Numbers 3 and 7 also make a pair in the same way.

Focus Point

- The sum of the cubes of first n natural numbers is equal to the square of their sum.

  i.e., $1^3 + 2^3 + ... + n^3 = (1 + 2 + ... + n)^2 = \left(\frac{n(n+1)}{2}\right)^2$

Example 4

Find the cube of (i) 2 (ii) 4 (iii) 6 (iv) 12.

Solution :

(i) Cube of 2 : $(2)^3 = 2 \times 2 \times 2 = 8$ (Even)

(ii) Cube of 4: $(4)^3 = 4 \times 4 \times 4 = 64$ (Even)

(iii) Cube of 6 : $(6)^3 = 6 \times 6 \times 6 = 216$ (Even)

(iv) Cube of 12 : $(12)^3 = 12 \times 12 \times 12 = 1728$ (Even)

Example 5

Find the cube of (i) 3 (ii) 9 (iii) 13.

Solution :

(i) Cube of 3: $(3)^3 = 3 \times 3 \times 3 = 27$ (odd)

(ii) Cube of 9 : $(9)^3 = 9 \times 9 \times 9 = 729$ (odd)

(iii) Cube of 13 : $(13)^3 = 13 \times 13 \times 13 = 2197$ (odd)

Example 6

Find the value of $1^3 + 2^3 + ... + 7^3$.

Solution :

$1^3 + 2^3 + ... + 7^3 = (1 + 2 + ... + 7)^2$

$\left(\frac{7(7+1)}{2}\right)^2 = (7 \times 4)^2 = (28)^2 = 784$

Example 7

Write the digits in the unit place for the cube of each of the given numbers :

(i) 21 (ii) 35 (iii) 69 (iv) 22

(v) 98 (vi) 83 (vii) 27

Solution :

(i) 21; Unit digit of 2<u>1</u> is 1.

∴ The digit in the unit place for its cube is also 1.

$(21)^3 = 21 \times 21 \times 21 = 926\underline{1}$

(ii) 35; Unit digit of 3<u>5</u> is 5.

∴ The digit in the unit place for its cube is also 5.

$(35)^3 = 35 \times 35 \times 35 = 4287\underline{5}$.

(iii) 69; Unit digit of 6<u>9</u> is 9.

∴ The digit in the unit place for its cube is also 9.

$(69)^3 = 69 \times 69 \times 69 = 32850\underline{9}$

(iv) 22; Unit digit of 2<u>2</u> is 2.

∴ The digit in the unit place for its cube is 8.

$(22)^3 = 22 \times 22 \times 22 = 1064\underline{8}$

(v) 98; Unit digit of 9<u>8</u> is 8.

∴ The digit in the unit place for its cube is 2.

$(98)^3 = 98 \times 98 \times 98 = 94119\underline{2}$

(vi) 83; The digit in the unit place of 8<u>3</u> is 3.

∴ The digit in the unit place for its cube is 7.

$(83)^3 = 83 \times 83 \times 83 = 57178\underline{7}$

(vii) 27; The digit in the unit place of 2<u>7</u> is 7.

∴ The digit in the unit place for its cube is 3.

$(27)^3 = 27 \times 27 \times 27 = 1968\underline{3}$

6. CUBE ROOTS

Finding the cube root is the inverse operation of finding cube. The symbol $\sqrt[3]{\ }$ denotes 'cube root'. Consider the following :

Statement : Cube	Inference : Cube root
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$

7. METHODS FOR FINDING THE CUBE ROOTS

(i) Using unit digit method (For perfect cube numbers)

(ii) Prime factorisation method

7.1 USING UNIT DIGIT METHOD (FOR PERFECT CUBE NUMBERS)

Step I : Look at the digit at the unit's place and determine the digit at the unit's place in the cube root by using the table given in properties of cubes of numbers.

Step II : Strike out from the right, last three digits (unit's, ten's and hundred's) of the number. If no digit(s) is (are) left, then the digit obtained in Step I is the required cube root. Otherwise go to next step.

Step III : Consider the number left from step II. Find the number whose cube is less than or equal to this left over number. This number is the ten's digit of the cube root.

Step IV : Obtain the required cube root by forming a number whose unit digit is the number obtained in step I and ten's digit is the number obtained in step III.

Example 8

Find the cube roots of the following numbers :

(i) 4096 (ii) 857375

Solution :

(i) **Step 1 :** The unit's digit of 4096 is 6. Therefore, the digit at the unit's place in the cube root is 6.

Step 2 : Group 4096; two groups are 4 and 096.

Step 3 : Choose a number whose cube is less than 4.

$\because\ 1^3 = 1$ and $2^3 = 8$

$\therefore\ 1 < 4 < 8 \Rightarrow 1^3 < 4 < 2^3$

Hence 1 is ten's place digit $\therefore \sqrt[3]{4096} = 16$

(ii) The given number is 857375.

Step 1 : The unit's digit of 857375 is 5. Therefore, unit's digit of its cube root is also 5.

Step 2 : Group 857375; two groups are 857 and 375.

Step 3 : Choose a number whose cube is less than 857.

$\because\ 9^3 = 729$ and $10^3 = 1000$ and $729 < 857 < 1000$

$\Rightarrow 9^3 < 857 < 10^3 \Rightarrow 9$ is ten's place digit.

Hence, $\sqrt[3]{857375} = 95$

7.2 PRIME FACTORISATION METHOD

In order to find the cube root of a perfect cube by factor method we follow the following procedure.

Step I : Obtain the given number.

Step II : Resolve it into prime factors.

Step III : Group the factors into triplets such that all the three factors in each triplet are same.

Step IV : If no factor is left ungrouped, choose one factor from each group and take the product. The product is cube root of number. If some prime factors are left ungrouped, the number is not a perfect cube and the process stops.

Example 9

Find the cube root of the following :

(a) 4913 (b) 19683

Solution :

(a) Resolving the given number into prime factors, we get

$4913 = 17 \times 17 \times 17$

17	4913
17	289
17	17
	1

So, $\sqrt[3]{4913} = 17$

(Taking one factor from triplet)

(b) Resolving 19683 into prime factors, we get

$19683 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$\sqrt[3]{19683} = 3 \times 3 \times 3$

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

(Taking one factor from each triplet)

$\therefore \sqrt[3]{19683} = 27$

8. CUBE ROOT OF PRODUCT OF INTEGERS

We have, $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.

Example 10

Evaluate : $\sqrt[3]{125 \times 64}$

Solution :

We have,

$\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$

$\sqrt[3]{5 \times 5 \times 5} \times \sqrt[3]{4 \times 4 \times 4}$

= (5 × 4) = 20.

Example 11

Evaluate : $\sqrt[3]{216\times(-343)}$

Solution :

We have,

$$\sqrt[3]{216\times(-343)}=\sqrt[3]{216}\times\sqrt[3]{-343}=\sqrt[3]{216}\times\left(-\sqrt[3]{343}\right).$$

$$=-\sqrt[3]{6\times6\times6}\times\sqrt[3]{7\times7\times7}$$

= – [6 × 7] = – 42.

9. CUBE ROOT OF A RATIONAL NUMBER

The cube root of a rational number $\frac{a}{b},(b\neq0)$, is given by $\sqrt[3]{\frac{a}{b}}=\frac{\sqrt[3]{a}}{\sqrt[3]{b}}$, where a, b are integers.

Example 12

Evaluate : (i) $\sqrt[3]{\frac{216}{2197}}$ (ii) $\sqrt[3]{\frac{-125}{512}}$

Solution :

(i) $\sqrt[3]{\frac{216}{2197}}=\frac{\sqrt[3]{216}}{\sqrt[3]{2197}}=\frac{\sqrt[3]{6\times6\times6}}{\sqrt[3]{13\times13\times13}}=\frac{6}{13}$

(ii) $\sqrt[3]{\frac{-125}{512}}=\frac{\sqrt[3]{-125}}{\sqrt[3]{512}}=\frac{-\sqrt[3]{125}}{\sqrt[3]{512}}=\frac{-\sqrt[3]{5\times5\times5}}{\sqrt[3]{8\times8\times8}}=\frac{-5}{8}$

Example 13

Evaluate : $\frac{\sqrt[3]{15625\times216}}{\sqrt[3]{3375}}$

Solution :

We have,

$$\frac{\sqrt[3]{15625}\times\sqrt[3]{216}}{\sqrt[3]{3375}}$$

$$=\frac{\sqrt[3]{25\times25\times25}\times\sqrt[3]{6\times6\times6}}{\sqrt[3]{15\times15\times15}}$$

$$=\frac{25\times6}{15}=10$$

SOLVED EXAMPLES

SE. 1

Is 53240 a perfect cube ? If not, then by which smallest natural number should 53240 be divided so that the quotient is a perfect cube ?

Ans. 53240 = 2 × 2 × 2 × 11 × 11 × 11 × 5

The prime factor 5 does not appear in a group of three. So 53240 is not a perfect cube. To get rid of this extra five we divide the number by 5. So factorisation of the quotient will not contain 5.

So, 53240 ÷ 5 = $\underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

2	53240
2	26620
2	13310
11	6655
11	605
11	55
5	5
	1

Hence the smallest number by which 53240 should be divided to make it a perfect cube is 5. The perfect cube in that case is = 10648.

SE. 2

Is 246 a perfect cube ?

Ans. Resolving 246 into prime factors, we have

246 = 2 × 3 × 41

2	246
3	123
3	41
	1

Clearly, we cannot group the factors in the triplets. Therefore, 246 is not a perfect cube.

SE. 3

Is 1331 a perfect cube ?

Ans. Resolving 1331 into prime factors, we get

1331 = $\underline{11 \times 11 \times 11}$

We find that prime factors of 1331 can be grouped into triplets of equal factors (as shown above) and no factor is left.

∴ 1331 is a perfect cube.

SE. 4

Is 68600 a perfect cube ? If not, find the smallest number by which 68600 must be multiplied to get a perfect cube.

Ans. We have, 68600 = $\underline{2 \times 2 \times 2} \times 5 \times 5 \times \underline{7 \times 7 \times 7}$.

2	68600
7	34300
7	4900
7	700
2	100
2	50
5	25
5	5
	1

In this factorisation we find that there is no triplet of 5. So, 68600 is not a perfect cube. To make it a perfect cube we multiply it by 5 to complete the triplet of 5. Thus,

68600 × 5 = $\underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}$

= 343000, which is a perfect cube.

SE. 5

Prove that if a number is doubled, then its cube is eight times the cube of the given number.

Ans. Let the given number be a. Let b denote the double of a i.e., b = 2a. Then,

$b^3 = b \times b \times b$

$= 2a \times 2a \times 2a$

$= 2 \times 2 \times 2 \times a \times a \times a$

$= 8 \times a \times a \times a = 8 \times a^3$

$\Rightarrow b^3 = 8 \times$ (Cube of a).

SE. 6

Evaluate : $\{(24^2 + 7^2)^{1/2}\}^3$

Ans. We have,

$$\{(24^2 + 7^2)^{1/2}\}^3 = \{(576 + 49)^{1/2}\}^3 = \left\{\sqrt{625}\right\}^3$$

$$= \left\{\sqrt{25 \times 25}\right\}^3 = 25^3 = 25 \times 25 \times 25 = 15625]$$

SE. 7

Find the cube root of $\sqrt[3]{392} \times \sqrt[3]{448}$.

Ans. Resolving 392 and 448 into prime factors Now,

$$\sqrt[3]{392} \times \sqrt[3]{448} = \sqrt[3]{392 \times 448}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}$$

$= 2 \times 2 \times 2 \times 7 = 56$

2	392
2	196
2	98
7	49
7	7
	1

2	448
2	224
2	112
2	56
2	28
2	14
7	7
	1

SE. 8

Is $\frac{27}{125}$ a cube of a rational number ?

Ans. We have,

$$\frac{27}{125} = \frac{3 \times 3 \times 3}{5 \times 5 \times 5}$$

$$\Rightarrow \frac{27}{125} = \frac{3^3}{5^3} = \left(\frac{3}{5}\right)^3$$

$\therefore \frac{27}{125}$ is cube of $\frac{3}{5}$ i.e., a rational number.

SE. 9

Is 1728 a perfect cube ? If yes, find its cube root.

Ans. Resolving 1728 into prime factors, we have

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Grouping the factors in triplets of equal factors, we get $1728 = \{2 \times 2 \times 2\} \times \{2 \times 2 \times 2\}$ $\{3 \times 3 \times 3\}$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

We find that the prime factors of 1728 can be grouped into triplets of equal factors and no factor is left over.

$\therefore$ 1728 is a perfect cube.

Taking one factors from each triplet, we obtain

$\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

Hence, 1728 is the cube of 12.

SE. 10

Find the cube root of 1.331.

Ans. We have, $1.331 = \frac{1331}{1000}$

$\therefore \sqrt[3]{1.331} = \sqrt[3]{\frac{1331}{1000}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{1000}} = \frac{11}{10} = 1.1$

SE. 11

Observe the following pattern.

$1^3 = 1$

$1^3 + 2^3 = (1 + 2)^2$

$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2$

Write the next three rows and calculate the value of $1^3 + 2^3 + 3^3 + ... + 9^3 + 10^3$ by above pattern.

Ans. If we observe the above pattern, the next three rows can be written as follows –

$1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$

$= (1 + 2 + 3 + 4 + 5 + 6)^2$

The value of

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$

$= (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)^2$

$= \left[\frac{10(10+1)^2}{2}\right]^2 = [5 \times 11]^2 = (55)^2 = 3025$

SE. 12

Find the volume of a cube whose surface area is 150 m^2.

Ans. Let the length of each edge of the given cube be x metres. Then, Surface area = 150 m^2

$\Rightarrow 6x^2 = 150 \Rightarrow x^2 = \frac{150}{6} = 25 \Rightarrow x = \sqrt{25} = 5\,m$

$\therefore$ Volume of the cube = $x^3 = 5^3$ cubic metres

= $(5 \times 5 \times 5)$ cubic metres = 125 cubic metres.

SE. 13

For a big icecream of volume 2744 cm^3, Mukti wants to make a box. What should be the edge of box so that the block can be put into it ?

Ans. Volume of box = 2744 cm^3

Let edge of the box = x cm

$\therefore x^3 = 2744 \Rightarrow x = \sqrt[3]{2744}$

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$\Rightarrow x = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7} = 2 \times 7 = 14$

$\therefore$ Edge of the box = 14 cm.

SE. 14

Consider the following pattern :

$2^3 - 1^3 = 1 + 2 \times 1 \times 3$

$3^3 - 2^3 = 1 + 3 \times 2 \times 3$

$4^3 - 3^3 = 1 + 4 \times 3 \times 3$.

Find the value of $12^3 - 11^3$, using the above pattern.

Ans. $12^3 - 11^3 = 1 + 12 \times 11 \times 3 = 1 + 396 = 397$.

SE. 15

Find the value of $(27 \times 2744)^{1/3}$.

Ans. $(27 \times 2744)^{1/3}$

$= \sqrt[3]{27} \times \sqrt[3]{2744}$

$= 3 \times 14 = 42$

Space for Notes :

NCERT SOLUTIONS

EXERCISE – 7.1

NS. 1

Which of the following numbers are not perfect cubes ?

(i) 216 (ii) 128

(iii) 1000 (iv) 100

(v) 46656

Ans. (i) $216 = \underline{2\times2\times2} \times \underline{3\times3\times3}$

Since the prime factors of 216 appear in a group of three.

$\therefore$ 216 is a perfect cube.

(ii) $128 = \underline{2\times2\times2} \times \underline{2\times2\times2} \times 2$

Since the prime factors of 128 don't appear in a group of three.

$\therefore$ 128 is not a perfect cube.

(iii) $1000 = \underline{2\times2\times2} \times \underline{5\times5\times5}$

Since the prime factors of 1000 appear in a group of three.

$\therefore$ 1000 is a perfect cube.

(iv) $100 = 2 \times 2 \times 5 \times 5$.

Since the prime factors of 100 don't appear in a group of three.

$\therefore$ 100 is not a perfect cube.

(v) $46656 = \underline{2\times2\times2} \times \underline{2\times2\times2} \times \underline{3\times3\times3} \times \underline{3\times3\times3}$

Since the prime factors of 46656 appear in a group of three.

$\therefore$ 46656 is a perfect cube.

NS. 2

Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(i) 243 (ii) 256

(iii) 72 (iv) 675

(v) 100

Ans. (i) $243 = \underline{3\times3\times3} \times3\times3$

Clearly the prime factor 3 doesn't appear in a group of three.

$\therefore$ 243 is not a perfect cube.

3	243
3	81
3	27
3	9
3	3
	1

So to make 243 a perfect cube we multiply it by 3.

In that case $243 \times 3 = \underline{3\times3\times3} \times \underline{3\times3\times3} = 729$, which is a perfect cube.

(ii) $256 = \underline{2\times2\times2} \times \underline{2\times2\times2} \times2\times2$

Clearly the prime factor 2 doesn't appear in a group of three.

$\therefore$ 256 is not a perfect cube.

So to make 256 a perfect cube we multiply it by 2.

In that case

$256 \times 2 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

= 512, which is a perfect cube.

(iii) $72 = \underline{2 \times 2 \times 2} \times 3 \times 3$

Clearly the prime factor 3 doesn't appear in a group of three.

$\therefore$ 72 is not a perfect cube.

So to make 72 a perfect cube, we must multiply it by 3.

In that case $72 \times 3 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

= 216, which is a perfect cube.

(iv) 675 = $\underline{3 \times 3 \times 3} \times 5 \times 5$

Clearly the prime factor 5 doesn't appear in a group of three.

∴ 675 is not a perfect cube.

3	675
3	225
3	75
5	25
5	5
	1

So we multiply it by 5, to make a perfect cube.

In that case 675 × 5 = $\underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

= 3375, which is a perfect cube.

(v) 100 = 2 × 2 × 5 × 5.

Clearly both the prime factors 2 and 5 don't appear in a group of three.

∴ 100 is not a perfect cube.

So we multiply it by 2 and 5, to make it a perfect cube.

In that case 100 × 2 × 5 = $\underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

= 1000, which is a perfect cube.

NS. 3

Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

(i) 81 (ii) 128

(iii) 135 (iv) 192

(v) 704

Ans. (i) 81 = $\underline{3 \times 3 \times 3} \times 3$

Clearly the prime factor 3 doesn't appear in a group of three.

∴ 81 is not a perfect cube.

So to make it a perfect cube, we must divide it by 3.

3	81
3	27
3	9
3	3
	1

$\frac{81}{3} = \frac{3\times3\times3\times3}{3} = 27$, which is a perfect cube.

(ii) 128 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$

Clearly the prime factor 2 doesn't appear in a group of three.

∴ 128 is not a perfect cube.

So to make it a perfect cube, we must divide it by 2.

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\frac{128}{2} = \frac{2\times2\times2\times2\times2\times2\times2}{2} = 64$, which is a perfect cube.

(iii) 135 = $\underline{3 \times 3 \times 3} \times 5$

Clearly the prime factor 5 doesn't appear in a group of three.

∴ 135 is not a perfect cube.

So, to make it a perfect cube, we must divide it by 5.

5	135
3	27
3	9
3	3
	1

i.e., $\frac{135}{5} = \frac{3 \times 3 \times 3 \times 5}{5} = 27$

Thus the smallest number is 5 by which 135 must be divided.

(iv) $192 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$

Clearly the prime factor 3 doesn't appear in a group of three.

$\therefore$ 192 is not a perfect cube.

So to make it a perfect cube, we must divide it by 3.

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

$\frac{192}{3} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3}{3} = 64$, which is a perfect cube.

(v) $704 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11$

Clearly the prime factor 11 doesn't appear in a group of three.

$\therefore$ 704 is not a perfect cube.

So to make it a perfect cube, we must divide it by 11.

2	704
2	352
2	176
2	88
2	44
2	22
11	11
	1

$\therefore \frac{704}{11} = \frac{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11}{11} = 64$, which is a perfect cube.

NS. 4

Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube ?

Ans. We have $2 \times 5 \times 5$

Clearly in above the prime factors 2 and 5 both don't appear in a group of three.

$\therefore$ To make it a perfect cube we need to multiply it by $2 \times 2 \times 5$, we get

$2 \times 5 \times 5 \times 2 \times 2 \times 5 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

= 1000, which is a perfect cube.

Thus, he needs 20 cuboids more to make it a perfect cube.

EXERCISE – 7.2

NS. 1

Find the cube root of each of the following numbers by prime factorisation method.

(i) 64 (ii) 512

(iii) 10648 (iv) 27000

(v) 15625 (vi) 13824

(vii) 110592 (viii) 46656

(ix) 175616 (x) 91125

Ans. (i) Prime factors of 64

2	64
2	32
2	16
2	8
2	4
2	2
	1

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\therefore \sqrt[3]{64} = \sqrt[3]{\underline{2\times2\times2}\times\underline{2\times2\times2}} = 2\times2 = 4.$

(ii) Prime factors of 512

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$512 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$\therefore \sqrt[3]{512} = 2\times2\times2 = 8.$

(iii) Prime factors of 10648

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

$10648 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

$\therefore \sqrt[3]{10648} = 2\times11 = 22.$

(iv) Prime factors of 27000

3	27000
3	9000
3	3000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$27000 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$\therefore \sqrt[3]{27000} = 2\times3\times5 = 30.$

(v) Prime factors of 15625

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$15625 = \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$

$\therefore \sqrt[3]{15625} = 5\times5 = 25.$

(vi) Prime factors of 13824

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$13824 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$\therefore \sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24.$

(vii) Prime factors of 110592

2	110592
2	55296
2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$110592 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$\therefore \sqrt[3]{110592} = 2 \times 2 \times 2 \times 2 \times 3 = 48.$

(viii) Prime factors of 46656

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$46656 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$\therefore \sqrt[3]{46656} = 2 \times 2 \times 3 \times 3 = 36.$

(ix) Prime factors of 175616

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

175616 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

$\therefore \sqrt[3]{175616} = 2 \times 2 \times 2 \times 7 = 56.$

(x) Prime factors of 91125

3	91125
3	30375
3	10125
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

91125 = $\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$\therefore \sqrt[3]{91125} = 3 \times 3 \times 5 = 45.$

NS. 2

State true or false.

(i) Cube of any odd number is even.

(ii) A perfect cube does not end with two zeros.

(iii) If square of a number ends with 5, then its cube ends with 25.

(iv) There is no perfect cube which ends with 8.

(v) The cube of a two digit number may be a three digit number.

(vi) The cube of a two digit number may have seven or more digits.

(vii) The cube of a single digit number may be a single digit number.

Ans. (i) It is false.

Because the cube of any odd number is also odd.

(ii) It is true.

Because to make a perfect cube we need each and every factor to occur three times in a group. So we need (3, 6, 9, ...) zero's to make a perfect cube.

(iii) It is false.

(iv) It is false.

$\because$ $12^3 = 1728$, which ends with 8.

(v) It is false.

$\because$ $10^3 = 1000$, where 10 is the smallest two digit number, and its cube contains 4 digits.

(vi) It is false.

$\because$ $99^3 = 970299$, where 99 is the largest two digit number and doesn't contain seven or more digits.

(vii) It is true.

$\because$ $1^3 = 1$, which is a single digit number.

NS. 3

You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root ? Similarly, guess the cube roots of 4913, 12167, 32768.

Ans. The given number is 1331.

Step – 1 : Start making groups of digits starting from the right most digit of the number we get 1 and 331 as two groups of one and three digits.

Step – 2 : First group i.e., 331 will give the one's digit (unit's) of the required cube root. The number 331 ends with 1. We know that 1 comes at the unit's place of a number only when its cube root ends in 1. So, we get 1 at the unit's place of the cube root.

Step – 3 : Now, take another group i.e., 1. We know that $1^3 = 1$ and $2^3 = 8$. Also $1 \leq 1 \leq 8$. We take the smaller number 1 as the ten's place of the required cube root. So we get $\sqrt[3]{1331} = 11$.

Similarly cube root of 4913.

Step – 1 : 4913; Groups are 4 and 913

Step – 2 : One's place digit of first group is 3.

$\therefore$ We take one's place digit of required cube root is 7 ($\because$ $7^3 = 343$)

Step – 3 : Now, the second group is 4.

$1^3 = 1$, $2^3 = 8$

$1 < 4 < 8$, hence 1 is the ten's place digit of required cube root.

So, we get $\sqrt[3]{4913} = 17$.

Cube root of 12167.

Step – 1 : 12167; two groups 12 and 167

Step – 2 : First group is 167.

$\therefore$ Unit's place digit of required cube root is 3.

Step – 3 : Second group is 12

$2^3 = 8$ and $3^3 = 27$, $8 < 12 < 27$

$\Rightarrow 2^3 < 12 < 3^3$

$\therefore$ 2 is the ten's place digit of required cube root.

So, $\sqrt[3]{12167} = 23$.

Cube root of 32768

Step – 1 : 32768; two groups 32 and 768.

Step – 2 : First group is 768.

$\therefore$ Unit's place digit of required cube root is 2.

Step – 3 : Second group is 32

$3^3 = 27$ and $4^3 = 64$, $27 < 32 < 64$

$\Rightarrow 3^3 < 32 < 4^3$

$\therefore$ 3 is the ten's place digit of required cube root.

So, $\sqrt[3]{32768} = 32$.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. Which of the following numbers is a perfect cube ?
   (A) 1525 (B) 1728
   (C) 1458 (D) 3993
2. Which of the following numbers is not a perfect cube ?
   (A) 2197 (B) 512
   (C) 2916 (D) 343
3. What least number must be multiplied to 3456 so that the product becomes a perfect cube ?
   (A) 2 (B) 3
   (C) 4 (D) 6
4. $\sqrt[3]{5832} =$
   (A) 22 (B) 18
   (C) 16 (D) 14
5. $\sqrt[3]{4\frac{12}{125}}$ equals
   (A) $1\frac{3}{5}$ (B) $1\frac{2}{5}$
   (C) $7\frac{1}{5}$ (D) $7\frac{2}{3}$
6. Possible unit digit of cube root of a number ending with 5 is
   (A) 0 (B) 5
   (C) 7 (D) 9
7. If $(125)^x = 3125$, then x equals
   (A) $\frac{3}{5}$ (B) $\frac{5}{3}$
   (C) $\frac{1}{4}$ (D) $\frac{1}{5}$
8. By what least number should 9720 be multiplied to get a perfect cube ?
   (A) 15 (B) 25
   (C) 5 (D) 75
9. If $\sqrt[3]{(156+x)} = 12$, then the value of x is
   (A) 1570 (B) 1572
   (C) 1560 (D) 1512
10. The value of $(0 - 0.4)^3$ is
    (A) 0.640 (B) 0.064
    (C) – 0.064 (D) – 0.640
11. $3^3 - (-0.6)^3 =$
    (A) 27.216 (B) 26.784
    (C) – 26.784 (D) – 27.216
12. $\frac{16}{9} \times \left(-1\frac{1}{2}\right)^3 =$
    (A) – 12 (B) – 6
    (C) $-\frac{8}{3}$ (D) $\frac{8}{9}$
13. $\sqrt[3]{0.125} + 3 =$
    (A) 8 (B) 3.5
    (C) 2 (D) 0.35
14. Calculate the value of $\sqrt[3]{\frac{192}{81}}$.
    (A) $\frac{5}{3}$ (B) $\frac{4}{3}$
    (C) $\frac{3}{2}$ (D) $\frac{13}{9}$
15. Which of the following is the cube of a negative number ?
    (A) – 396 (B) 4096
    (C) – 81 (D) – 2744
16. Which of the following numbers is the cube of an odd number ?
    (A) 79507 (B) 2744
    (C) 32768 (D) 1728

17. Which of the following numbers is the cube of an even number ?
(A) 6859 (B) 648
(C) 13824 (D) 42875

18. The symbol $\sqrt[3]{\ }$ denotes
(A) square (B) cube
(C) square root (D) cube root

19. The cube root of 97336 is
(A) 17 (B) 18
(C) 46 (D) 23

20. What is the least number by which 13720 must be divided so that the quotient is a perfect cube ?
(A) 2 (B) 3
(C) 5 (D) 6

21. The value of $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is
(A) 2 (B) 2.273
(C) 2.327 (D) None of these

22. The value of $\frac{\sqrt[3]{8} + \sqrt[3]{27} - \sqrt[3]{343}}{(2)^2 - 3}$ is
(A) 7 (B) – 2
(C) 8 (D) – 5

23. Observe the pattern given below
$1^3 = 1$
$2^3 = 3 + 5$
$3^3 = 7 + 9 + 11$
$4^3 = 13 + 15 + 17 + 19$
$5^3 = 21 + 23 + 25 + 27 + 29$
According to this pattern, the number of consecutive odd numbers whose sum equals 9^3 is
(A) 3 (B) 9
(C) 12 (D) 15

24. Evaluate : $\sqrt[3]{27} + \sqrt[3]{0.008}$
(A) 3.4 (B) 3.1
(C) 3.3 (D) 3.2

25. The unit digit in the $(3723)^3$ is :
(A) 5 (B) 6
(C) 9 (D) 7

PARAGRAPH TYPE

PASSAGE # I :

Three numbers are in the ratio 2 : 3 : 4.

26. The sum of their cubes is 33957. Find the numbers.
(A) 2, 4, 8 (B) 14, 21, 28
(C) 6, 9, 18 (D) 5, 10, 15

27. The cubes of these numbers are
(A) 8, 64, 512
(B) 2744, 7261, 8849
(C) 2744, 9261, 21952
(D) 7261, 125, 1000

28. The sum of squares of these numbers is :
(A) 1421 (B) 1423
(C) 1424 (D) 1429

PASSAGE # II :

The sum of the cubes of first n natural number is equal to the square of their sum.

i.e. $(1^3 + 2^3 + 3^3 + \ldots\ldots + n^3) =$

$$(1 + 2 + 3 + \ldots + n)^2 = \left[\frac{n(n+1)}{2}\right]^2$$

pattern

$1^3 = 1^2$

$1^3 + 2^3 = (1 + 2)^2 = 9$

$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2 = 36$ and 500n

29. The sum of cubes of first 20 natural number is :

(A) 44100 (B) 44000

(C) 40000 (D) None

30. The sum of $3^3 + 4^3 + ... + 13^3$ is :

(A) 8722 (B) 8272

(C) 8227 (D) 8281

31. If $1^3 + 2^3 + 3^3 + ... + n^3 = 225$ then the value of n is :

(A) 3 (B) 4

(C) 5 (D) 6

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column – I and Column – II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the numbers given in **Column – I** with their cube roots given in **Column – II**.

Column – I	**Column – II**
(P) 148877	(i) 24
(Q) 13824	(ii) 53
(R) 35937	(iii) 33
(S) 17576	(iv) 26

(A) (P)→(ii), (Q)→(iii), (R)→(iv), (S)→(i)

(B) (P)→(ii), (Q)→(i), (R)→(iii), (S)→(iv)

(C) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)

(D) (P)→(i), (Q)→(iii), (R)→(ii), (S)→(iv)

33. Match the following :

Column – I	**Column – II**
(P) The smallest number by which 392 must be multiplied so that the product is a perfect cube is	(i) 5
(Q) The smallest number by which 8640 must be divided so that the quotient is a perfect cube is	(ii) 3
(R) The smallest number by which 3087 must be multiplied so that product is a perfect cube is	(iii) 25
(S) The smallest number by which 33275 must be divided so that the quotient becomes a perfect cube is	(iv) 7

(A) (P)→(iv), (Q)→(i), (R)→(ii), (S)→(iii)

(B) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)

(C) (P)→(ii), (Q)→(iii), (R)→(iv), (S)→(i)

(D) (P)→(iv), (Q)→(ii), (R)→(i), (S)→(iii)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Is 1024 a perfect cube ?
2. Find the unit digit of cube of 8888.
3. Is 196 a perfect cube ?
4. Find the smallest number by which 96 must be multiplied so that the product is a perfect cube.
5. Find the smallest number by which 3087 must be divided so that the quotient is a perfect cube.
6. Show that 1331 is a pefect cube.
7. Consider the following pattern.

   $2^3 - 1^3 = 1 + 2 \times 1 \times 3$

   $3^3 - 2^3 = 1 + 3 \times 2 \times 3$

   $4^3 - 3^3 = 1 + 4 \times 3 \times 3$

   Using the above pattern find the value of $7^3 - 6^3$.
8. Find the cube root of 17576.
9. Find the cube root of 0.003375.
10. Evaluate the following : $\left\{\sqrt{15^2 + 8^2}\right\}^3$.

SHORT ANSWER TYPE

1. Is 8000 a pefect cube ? What is the number whose cube is 8000 ?
2. Find the cube root of 389017.
3. Find the smallest number which when multiplied with 3600 make the product a perfect cube. Further, find the cube root of the product.
4. Find the smallest number by which 8192 must be divided so that quotient is a perfect cube. Also, find the cube root of the quotient so obtained.
5. Three numbers are in the ratio 1 : 2 : 3. The sum of their cubes is 98784. Find the numbers.

LONG ANSWER TYPE

1. Find the cube root of 85184 by prime factorisation method.
2. Find the cube root of 2300×5290.
3. Evaluate : $\sqrt[3]{1372} \times \sqrt[3]{1458}$.
4. Find the cube root of $\frac{3375}{2744}$.
5. Find the cube root of 110592.

TRUE / FALSE TYPE

1. A perfect cube can end with even number of zeroes.
2. The cube of a 2 digit number may be a 3 digit number.
3. The cube of a single digit number may be a single digit number.
4. If n ends in 3, then n^3 ends in 7.
5. The cube root of a negative perfect cube is negative.

NUMERICAL PROBLEMS

1. If $\sqrt[3]{\frac{a^6 \times b^3 \times c^{21}}{c^9 \times a^{12}}} = \frac{bc^k}{a^{k/2}}$, then k = ________.
2. The cube of the number p is 16 times the number. Then find p where $p \neq 0$ and $p \neq -4$.
3. A number is multiplied 3 times by itself and then 61 is subtracted from the product obtained. If the final result is 9200, then the number is.
4. $\sqrt[3]{0.125} + \sqrt[3]{0.729} = \frac{n}{10}$. Find n.
5. If a = 2b and b = 4c, then $\sqrt[3]{\frac{a^2}{16bc}}$ = _______.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Simplify : $\left(\sqrt[6]{27}-\sqrt{6\frac{3}{4}}\right)^2$

(A) $\frac{3}{4}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{3\sqrt{3}}{4}$ (D) $\frac{3}{2}$

2. By what least number 3600 must be divided to make it a perfect cube ?

(A) 9 (B) 50

(C) 300 (D) 450

3. The value of $\sqrt{\frac{0.00001225}{0.00005329}}-\sqrt[3]{\sqrt{0.000064}}$ is

(A) 0.2 (B) 0.279

(C) 0.479 (D) 0

4. Evaluate : $\sqrt[3]{0.008}-\sqrt[3]{-512}+\sqrt[3]{2.197}$

(A) 9.3 (B) – 6.5

(C) 9.5 (D) 6.5

5. $\frac{3\sqrt[3]{13824}}{2\sqrt[3]{-15625}}\div\frac{2\sqrt[3]{-13824}}{\sqrt[3]{5832}}=$

(A) $\frac{50}{27}$ (B) $\frac{-50}{27}$

(C) $\frac{27}{50}$ (D) $\frac{-27}{50}$

6. Cube root of a number when divided by the smallest prime number gives square of the smallest prime number. Find the number.

(A) 512 (B) 8

(C) 64 (D) 125

7. If a number has digit 2 at unit place, then its cube has digit ______ at its unit place.

(A) 1 (B) 2

(C) 8 (D) 4

8. Which of the following is incorrect ?

(A) The cube of an even natural number is always even.

(B) The cube root of a rational number $\frac{x}{y}$ is $\frac{\sqrt[3]{x}}{\sqrt[3]{y}}$.

(C) The cube of a negative number is always positive.

(D) 2197 is a perfect cube.

9. $\left(\sqrt[3]{3}+\sqrt[3]{2}\right)\left(\sqrt[3]{9}+\sqrt[3]{4}-\sqrt[3]{6}\right)=$

(A) 5 (B) $\sqrt[9]{5}$

(C) $\sqrt[6]{5}$ (D) $\sqrt[3]{5}$

10. $\frac{\sqrt[3]{1.728}-\sqrt[3]{0.216}}{\sqrt[3]{2.197}-\sqrt[3]{0.343}}=$

(A) 1 (B) – 1

(C) 2 (D) – 2

11. If $3^9 + 3^{12} + 3^{15} + 3^n$ is a perfect cube, $n \in N$, then the value of n is

(A) 18 (B) 17

(C) 14 (D) 16

12. If $\sqrt{\sqrt[3]{x\times 0.000009}}=0.3$, then the value of $\sqrt{x}$ is :

(A) 27 (B) 81

(C) 9 (D) 18

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	C	C	B	A	B	B	D	B	C	A	B	B	B	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	C	D	C	C	A	B	B	D	D	B	C	A	A	B
31	32	33												
C	B	A												

EXERCISE II

VERY SHORT ANSWER TYPE

1. No **2.** 2 **3.** No **4.** 18 **5.** 9 **6.** 11

7. $1 + 7 \times 6 \times 3 = 127$ **8.** 26 **9.** 0.15 **10.** 4913

SHORT ANSWER TYPE

1. Yes, 20 **2.** 73 **3.** 60,60 **4.** 2 **5.** 14, 28, 42

LONG ANSWER TYPE

1. 44 **2.** 230 **3.** 126 **4.** $\frac{15}{14}$ **5.** 48

TRUE / FALSE

1. T **2.** **F** **3.** T **4.** T **5.** T

NUMERICAL PROBLEMS

1. 4 **2.** 4 **3.** 21 **4.** 14 **5.** 1

ANALYTICAL PROBLEMS & BRAIN TEASER

1. A **2.** D **3.** B **4.** C **5.** C **6.** A **7.** C

8. C **9.** A **10.** A **11.** C **12.** B

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : CUBES AND CUBE ROOTS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Solutions			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

COMPARING QUANTITIES 7

Concepts

INTRODUCTION

When we compare the quantities using concepts of ratio, proportions, percentage, etc. that is called comparing quantities.

1. RATIO

The quotient of two numbers or quantities indicating their relative sizes.

The ratio of a to b is written as a : b or $\frac{a}{b}$. The first term is antecedent and the second term is consequent.

Example 1

If $\frac{x}{5} = \frac{y}{8}$, then find (x + 5) : (y + 8).

Solution :

Let $\frac{x}{5} = \frac{y}{8} = k$. Then x = 5k and y = 8k

$\therefore \frac{x+5}{y+8} = \frac{5k+5}{8k+8} = \frac{5(k+1)}{8(k+1)} = \frac{5}{8}$

$\Rightarrow x + 5 : y + 8 = 5 : 8$

Example 2

If 2A = 3B and 4B = 5C, then find A : C.

Solution :

2A = 3B and 4B = 5C

$\Rightarrow \frac{A}{B} = \frac{3}{2}$ and $\frac{B}{C} = \frac{5}{4}$

$\Rightarrow \frac{A}{C} = \left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{3}{2} \times \frac{5}{4}\right) = \frac{15}{8}$

$\Rightarrow A : C = 15 : 8$

2. PERCENTAGE

Percent : Indicating hundredths. A fraction can be expressed as a percentage by multiplying it by 100. A change in a quantity from a to b is a change of $\frac{100(b-a)}{a}$ percent.

Finding a percentage of a number : To find a percent of a given number, we proceed as follows :

Step – 1 : Obtain the number, say x.

Step – 2 : Obtain the required percent, say P %.

Step – 3 : Multiply x by P and divide by 100 to obtain the required P % of x i.e., P % of x = $\frac{P}{100} \times x$.

Example 3

Find 30 % of Rs. 150.

Solution :

$$30\ \%\ \text{of Rs. } 150 = \frac{30}{100} \times 150 = \text{Rs. } 45$$

Example 4

Find x, if x % of 250 + 25 % of 68 = 67.

Solution :

$$\frac{x}{100} \times 250 + \frac{25}{100} \times 68 = 67$$

$$\Rightarrow \frac{25x}{10} = 67 - 17 \Rightarrow x = \frac{50 \times 10}{25} \Rightarrow x = 20$$

Finding how much percent one quantity is of another quantity :

To find : What percent is x of y, where x and y are any two numbers ?

Since, 'is' is placed before 'of', then required percent = $\frac{x}{y} \times 100\ \%$

To find : What percent of a is b, where a and b are any two numbers ?

Since, 'of' is placed before 'is', then required percent = $\frac{b}{a} \times 100\ \%$

Example 5

What percent of 35 kg is 2.5 kg ?

Solution :

Here 'of' is before 'is', so

$$\text{Required percent} = \frac{2.5}{35} \times 100\%$$

$$= \frac{25}{35 \times 10} \times 100\% = \frac{50}{7}\% = 7\frac{1}{7}\%$$

Thus, 2.5 kg is $7\frac{1}{7}\%$ of 35 kg.

Example 6

What percent is 18 hours of 3 days ?

Solution :

3 days = 3 × 24 hours = 72 hours.

[To find percentage both quantities should be in same unit]

Here 'is' is before 'of', so

Required percent = $\frac{18}{72}\times 100\% = 25\ \%$

Thus, 18 hours is 25 % of 3 days.

3. INCREASE OR DECREASE PERCENT

We calculate increase or decrease percent as follows :

Increase % = $\frac{\text{Increase}}{\text{Original Value}}\times 100\%$; Decrease % = $\frac{\text{Decrease}}{\text{Original Value}}\times 100\%$

Example 7

The price of a scooter which was Rs. 34000 last year increased by 20 % this year. What is the price now ?

Solution :

Let original price be Rs. 100.

Increase in price = Rs. 20

$\therefore$ Increased price = 100 + 20 = Rs. 120

If original price is Rs. 100, increased price = Rs. 120.

If original price is Rs. 34000, increased price = $\frac{120}{100}\times 34000 = \text{Rs.}\,40800$.

4. PROFIT AND LOSS

Cost Price : The amount paid to purchase an article or the price at which an article is made is known as its cost price. The cost price is abbrevited as C.P.

The overhead expenses like taxes, labour charges, etc. are included in the cost price. If overhead expenses are not included in the cost price, then

Effective Cost Price = Pyment made while purchasing the goods + Overhead expenses

Selling Price : The price at which an article is sold is known as its selling price.

The selling price is abbreviated as S.P.

Profit : If the selling price (S.P.) of an article is greater than the cost price (C.P.), then the diffrence between the selling price and cost price is called profit.

Profit = S.P. – C.P.

Profit Percentage : The profit percent is the profit that would be obtained for a C.P. of Rs. 100 i.e.,

Profit percent $= \left(\frac{\text{Profit}}{\text{C.P.}} \times 100\right)\%$.

Formulas for calculating S.P. and C.P., when profit % is given.

$$\text{S.P.} = \left(\frac{100 + \text{Profit}\,\%}{100}\right) \times \text{C.P.};\ \text{C.P.} = \frac{100 \times \text{S.P.}}{(100 + \text{Profit}\,\%)}$$

Loss : If the selling price (S.P.) of an article is less than the cost price (C.P.) then the difference between the cost price (C.P.) and the selling price (S.P.) is called loss.

Loss = C.P. – S.P.

Loss Percentage : The loss percent is the loss that would be made for a C.P. of Rs. 100.

That is, loss percent $= \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\%$.

Formulas for calculating S.P. and C.P. when loss % is given :

$$\text{S.P.} = \left(\frac{100 - \text{Loss}\,\%}{100}\right) \times \text{C.P.};\ \text{C.P.} = \frac{100 \times \text{S.P.}}{(100 - \text{Loss}\,\%)}$$

Example 8

Rishi bought a wrist watch for Rs. 2200 and sold it for Rs. 1980. Find loss and loss percent.

Solution :

We have, C.P. of watch = Rs. 2200

S.P. of watch = Rs. 1980

Since S.P. < C.P. So, there is loss given by

Loss = C.P. – S.P. = 2200 – 1980 = Rs. 220

Now, loss % $= \left(\frac{\text{Loss}}{\text{C.P.}} \times 100\right)\% = \left(\frac{220}{2200} \times 100\right)\% = 10\%$

Hence, loss = Rs. 220 and loss % = 10 %.

Example 9

Raghu bought an almirah for Rs. 6250 and spent Rs. 375 on its repairs. Then, he sold it for Rs. 6890. Find his gain or loss percent.

Solution :

C.P. of the almirah = Rs. 6250, overheads = Rs. 375.

Total cost price = (6250 + 375) = Rs. 6625.

Selling price = Rs. 6890.

Since (S.P.) > (C.P.), Raghu gains.

Gain = (6890 – 6625) = Rs. 265.

$$\text{Gain \%} = \left(\frac{\text{Gain}}{\text{Total C.P.}} \times 100\right)\% = \frac{265}{6625} \times 100\ \% = 4\%$$

Example 10

On selling a fan for Rs. 810, Samrat gains 8 %. For how much did he purchase it ?

Solution :

S.P. of the fan = Rs. 810, Gain % = 8 %.

$$\therefore \text{ C.P. of the fan} = \left\{\frac{100}{(100 + \text{gain }\%)} \times \text{S.P.}\right\}$$

$$= \left\{\frac{100}{(100+8)} \times 810\right\} = \left(\frac{100}{108} \times 810\right) = \text{Rs.}750.$$

Hence, Samrat purchased the fan for Rs. 750.

Example 11

Mayank lost 20% by selling a bicycle for Rs. 1536. What percent shall he gain or loss by selling it for Rs. 2000 ?

Solution :

S.P. = Rs. 1536, Loss % = 20 %

$$\therefore \text{ C.P.} = \left(\frac{100}{80} \times 1536\right) = \text{Rs.}1920$$

New S.P. = Rs. 2000.

Gain = (2000 – 1920) = Rs. 80.

$$\therefore \text{ Gain \%} = \frac{80}{1920} \times 100 = \frac{25}{6}\ \%$$

5. DISCOUNT

Market price or list price : The price written on the article or tagged with the article is called the marked price (M.P.) or list price (L.P.).

Discount : The deduction made on the marked price is called discount. Discount is generally given as a certain percent of the marked price. It is always calculated on the marked price or list price.

Selling Price : The difference between the marked price (M.P.) and discount is called the selling price (S.P.) of the article.

- Discount = M.P. – S.P.

- Rate of discount = Discount % = $\left(\frac{\text{Discount}}{\text{M.P.}}\times 100\right)\%$

- S.P. = M.P. – Discount

$\Rightarrow$ S.P. = M.P. – $\frac{\text{Discount}\,\%\times\text{M.P.}}{100}$ $\Rightarrow$ S.P. = M.P. $\times\left(1-\frac{\text{Discount}\,\%}{100}\right)$

$\Rightarrow$ S.P. = M.P. $\times\left(\frac{100-\text{Discount}\,\%}{100}\right)$

- M.P. = $\frac{100\times\text{S.P.}}{100-\text{Discount}\,\%}$

Example 12

The marked price of a pen set is Rs. 450. The shopkeeper gives a discount of 20%. What is his selling price ?

Solution :

Given that M.P. = Rs. 450 and discount % = 20 %

Discount = 20 % of Rs. 450 = $\frac{20}{100}$ × Rs. 450 = Rs. 90

$\therefore$ S.P. = M.P. – Discount = Rs. 450 – Rs. 90 = Rs. 360

Thus, the pen set was sold for Rs. 360.

Example 13

Which is better offer : two successive discounts of 10% and 8% or a single discount of 18% ?

Solution :

Let the marked price = Rs. 100

First discount = Rs. 10

First reduced price = (100 – 10) = Rs. 90. Second discount = $\left(\frac{8}{100}\times 90\right)$ = Rs. 7.20

Net selling price = Rs. (90 – 7.20) = Rs. 82.80

Total discount allowed = Rs. (100 – 82.80) = Rs. 17.20

$\therefore$ Single discount equivalent to two successive discounts of 10% and 8% is 17.2%.

$\therefore$ Single discount of 18% is better.

6. SALES TAX/VALUE ADDED TAX

Tax is the money collected by the government from the citizens of the country in order to provide them services. There are various types of taxes – Income Tax, Sales Tax, Service Tax and Value Added Tax etc.

Sales tax or Value added tax : It is collected by the government on the sale of different commodities. It is charged by the shopkeeper from the customer and given to the government. Therefore, this is always on the selling price of an article and is added to the value of the bill (known as VAT).

- The rate of this tax depends upon the item sold.
- When C.P. is given exclusive of tax. Tax to be paid = $\frac{\text{Tax}\,\%}{100} \times \text{C.P.}$ So, bill amount = C.P. + Sales Tax
- When C.P. is given inclusive of tax or VAT. Original Price = $\frac{100}{100 + \text{Tax}\,\%} \times \text{C.P.}$

Focus Point

Goods and Services Tax : Goods and Services Tax (G.S.T.) is an Indirect Tax, which has replaced many Indirect taxes like VAT, Central Excise duty, Sales Tax, Service Tax, etc. It is described as one tax for one nation. G.S.T. is classified under three categories :

♦ **C.G.S.T. :** The revenue collected under "Central Goods and Services Tax". (C.G.S.T.) is for centre government.

♦ **S.G.S.T. :** The revenue collected under "State Goods and Services Tax". (S.G.S.T.) is for state government.

♦ **I.G.S.T. :** "Integrated Goods and Services Tax" (I.G.S.T.) is charged when movement of goods and services from one state to another. The revenue collected out of I.G.S.T. is shared by state government and central government as per the rates fixed by the authorities.

Example 14

The cost of a pair of shoes at a shop was Rs. 450. The sales tax charged was 5%. Find the bill amount.

Solution :

On Rs. 100, the tax paid was Rs. 5.

On Rs. 450, the tax paid would be = $\frac{5}{100} \times 450$ = Rs. 22.50

Bill amount = Cost of item + Sales tax = Rs. 450 + Rs. 22.50 = Rs. 472.50

Example 15

Sanjana bought a sofa for Rs. 13500. If 8% VAT is included in the price, find the original price of sofa.

Solution :

The price includes the VAT, i.e., the value added tax. Thus, 8% VAT means if the price without VAT is Rs. 100, then price including VAT is Rs. 108.

Now, when price including VAT is Rs. 108, the original price is Rs. 100.

Hence, when price including tax is Rs. 13500,

The original price $= \frac{100}{108} \times 13500 = \text{Rs. } 12500$

Example 16

Reena goes to a shop to buy a washing machine, costing Rs. 11800. The rate of G.S.T. is 18%. She tells the shopkeeper to reduce the price of the washing machine to such an extent that she has to pay Rs. 11800, inclusive of G.S.T. Find the reduction needed in the price of the washing machine.

Solution :

Let the reduced price, excluding the G.S.T., of the washing machine be x. Then,

G.S.T. = 18% of x = Rs. $\frac{18x}{100}$

$\therefore$ Selling price of the washing machine $= \text{Rs.} \left(x + \frac{18x}{100}\right) = \text{Rs. } \frac{118x}{100}$

But, the selling price of the washing machine is Rs. 11800.

$\therefore \frac{118x}{100} = 11800 \Rightarrow x = \frac{11800 \times 100}{118} \Rightarrow x = \text{Rs.}10,000$

Hence, the reduction needed in the price of the washing machine = Rs. (11800 – 10000) = Rs. 1800

7. INTEREST

(i) Simple Interest : If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called simple interest.

(ii) Compound Interest : Sometimes it happens that the borrower and the lender agree to fix up a certain unit of time, say yearly or half-yearly or quarterly to settle the account. In such case, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on. Difference between the amount and the money borrowed is called the compound interest (abbreviated as C.I. for that period). [When interest is calculated on the amount of previous year, it is compound interest].

Example 17

Find the amount of Rs. 8000 for 3 years, compounded annually at 5% per annum. Also, find the compound interest.

Solution :

Here, P = Rs. 8000, R = 5% per annum and n = 3 years. Using the formula $A = P\left(1+\frac{R}{100}\right)^n$, we get

Amount after 3 years $= 8000 \times \left(1+\frac{5}{100}\right)^3 = \left(8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}\right) = \text{Rs.}9261.$

Thus, amount after 3 years = Rs. 9261. And, compound interest = Rs. (9261 – 8000) = Rs. 1261.

When the interest is compounded annually but rates are different for different years :

Let P = principal, time = 2 years, and let the rates of interest be p% p.a. during the first year and q% p.a. during the second year.

Then, amount after 2 years $= \left\{P \times \left(1+\frac{p}{100}\right) \times \left(1+\frac{q}{100}\right)\right\}$

This formula may similarly be extended for any number of years.

Example 18

Find the amount of Rs. 12000 after 2 years, compounded annually; the rate of interest being 5% p.a. during the first year and 6% p.a. during the second year. Also, find the compound interest.

Solution :

Here, P = Rs. 12000, p = 5% p.a. and q = 6% p.a.

Using the formula, $A = P \times \left(1+\frac{p}{100}\right) \times \left(1+\frac{q}{100}\right)$,

Amount after 2 years $= \left\{12000 \times \left(1+\frac{5}{100}\right) \times \left(1+\frac{6}{100}\right)\right\} = \left(12000 \times \frac{21}{20} \times \frac{53}{50}\right) = \text{Rs.}13356$

And, compound interest = (13356 – 12000) = Rs. 1356.

When interest is compounded annually but time is a fraction :

For example, P = principal, R = rate of interest and suppose time is $2\frac{3}{5}$ years. Then,

$$\text{Amount} = P \times \left(1+\frac{R}{100}\right)^2 \times \left(1+\frac{3/5}{100} \times R\right)$$

Example 19

Find the compound interest on Rs. 31250 at 8% per annum for $2\frac{3}{4}$ years.

Solution :

Here P = Rs. 31250, R = 8%, n = $2\frac{3}{4}$ years. $A = \left[31250\times\left(1+\frac{8}{100}\right)^2\times\left\{1+\frac{3/4}{100}\right\}\times 8\right]$

$$=\left\{31250\times\left(\frac{27}{25}\right)^2\times\left(\frac{53}{50}\right)\right\}=\left(31250\times\frac{27}{25}\times\frac{27}{25}\times\frac{53}{50}\right)=\text{Rs.}38637$$

Hence, compound interest = (38637 – 31250) = Rs. 7387.

When interest is componded half yearly :

Let Principal = P, Rate = R% per annum, Time = n years.

Then, rate = $\left(\frac{R}{2}\right)\%$ per half-year, time = (2n) half-years, and amount = $P\times\left(1+\frac{R}{2\times100}\right)^{2n}$

Compound interest = (amount) – (principal).

Example 20

Find the compound interest on Rs. 15625 for $1\frac{1}{2}$ years, at 8% per annum when compounded half-yearly.

Solution :

Here, principal = Rs. 15625, Rate = 8% per annum = 4% per half-year,

Time = $1\frac{1}{2}$ years = 3 half-years.

$$\therefore \text{Amount}=\left[15625\times\left(1+\frac{4}{100}\right)^3\right]=\left(15625\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}\right)=\text{Rs.}17576$$

∴ Compound interest = (17576 – 15625) = Rs. 1951.

When interest is compounded quarterly :

Let P = principal, Rate = R% per annum, Time = n years. Suppose that the interest is componded quarterly.

Then, rate = $\left(\frac{R}{4}\right)\%$ per quarter, Time = (4n) quarters, ∴ Amount = $P\times\left(1+\frac{R}{4\times100}\right)^{4n}$

Compound interest = (amount) – (principal).

Example 21

Find compound interest on Rs. 125000 for 9 months 8% per annum, compounded quarterly.

Solution :

Here, principal = Rs. 125000, Rate = 8% p.a. = $\left(\frac{8}{4}\right)\%$ = 2% per quarter, Time = 9 months = 3 quarters

$$\therefore \text{Amount} = \left\{125000 \times \left(1+\frac{2}{100}\right)^3\right\} = \left(125000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50}\right) = \text{Rs.}132651$$

$\therefore$ Compound interest = (132651 – 125000) = Rs. 7651.

SOLVED EXAMPLES

SE. 1

Find the ratio of :

(i) 75 paise to Rs. 3 (ii) 3 weeks to 3 days

Ans. (i) Rs. 3 = 3 × 100 paise = 300 paise

Ratio of 75 paise to Rs. 3

= Ratio of 75 paise to 300 paise

= 75 : 300 = 1 : 4

(ii) 1 week = 7 days

So, 3 weeks = 3 × 7 days = 21 days

Ratio of 3 weeks to 3 days

= Ratio of 21 days to 3 days

= 21 : 3 = 7 : 1

SE. 2

The ratio of the sum of money in three bags A, B and C is 4 : 3 : 2. If Rs. 50 is added to each of the bags, the ratio becomes 14 : 13 : 12. Find the sum of money in each of the bags.

Ans. Let the sum of money in the bags be Rs. x, Rs. y, Rs. z respectively.

As these are in the ratio 4 : 3 : 2, we get

x = 4k, y = 3k, z = 2k.

We are given that Rs. 50 is added to each bag.

So, the sums are now 4k + 50, 3k + 50, 2k + 50 respectively. But these are in the ratio 14 : 13 : 12

$\therefore$ 4k + 50 = 14p .....(i)

3k + 50 = 13p .....(ii)

2k + 50 = 12p .....(iii)

Subtracting equation (ii) from equation (i)

[or equations (iii) from (ii) or (iii) from (i)]

We get, k = p

$\therefore$ 4k + 50 = 14p $\Rightarrow$ 4k + 50 = 14k

$\Rightarrow$ 10k = 50 $\Rightarrow$ k = 5

$\therefore$ The sum of money in the bags are,

Rs. 4 × 5, Rs. 3 × 5, Rs. 2 × 5

i.e., Rs. 20, Rs. 15, Rs. 10

SE. 3

If 23% of a is 46, then find a.

Ans. We have, 23% of a = $\frac{23}{100} \times a$.

But, 23% of a is given as 46.

$\therefore \frac{23}{100} \times a = 46 \Rightarrow a = 46 \times \frac{100}{23} \Rightarrow a = 200$

SE. 4

A football team loss 10 games from the total they played. This was 20% of the total. How many games were played in all ?

Ans. Let total games played be x.

So, 20% of x = 10 or, $x \times \frac{20}{100} = 10$

or, $x = \frac{10 \times 100}{20} = 50$

Thus, total games were 50.

SE. 5

Convert the following into percent.

(i) $12\frac{1}{2}$ (ii) 4 : 5

Ans. (i) $12\frac{1}{2} = \frac{25}{2} = \frac{25}{2} \times 100\% = 1250\%$

(ii) $4:5 = \frac{4}{5} = \frac{4}{5} \times 100\% = (4 \times 20)\% = 80\%$

SE. 6

Find :

(i) 75% of 500 (ii) $4\frac{1}{2}\%$ of 1800 m

Ans. (i) 75% of 500 = $\frac{75}{100} \times 500 = 375$

(ii) $4\frac{1}{2}\%$ of 1800 m = $\frac{9}{2}\%$ of 1800 m

$= \frac{9}{2} \times \frac{1}{100} \times 1800\,m = 81\,m$

SE. 7

If Kamini had Rs. 500 left after spending 50% of her money, how much did she have in the beginning?

Ans. Let the money in the beginning be Rs. 100.

Now, money spent = Rs. 50

$\therefore$ Money left = 100 – 50 = Rs. 50.

If money left is Rs. 50, then amount in the beginning = Rs. 100

If money left is Rs. 500, then amount in the beginning $= \frac{100}{50} \times 500 = Rs.1000$

SE. 8

The population of a town increases by 6% every year. If the present population is 16000, find its population after a year.

Ans. Original (present) population = 16000

$\therefore$ Increase in population = 6% of original

$= \frac{6}{100} \times 16000 = 960$

Thus, population after a year

= Present population + Increase in population

= 16000 + 960 = 16960

SE. 9

A number is increased by 10% and then it is decreased by 10%. Find the net increase or decrease percent.

Ans. Let the number be = 100

Increase in the number = 10% of 100 = 10

$\therefore$ Increased number = 100 + 10 = 110

This number is decreased by 10%.

$\therefore$ Decrease in the number = 10% of 110

$= \left(\frac{10}{100} \times 110\right) = 11$

$\therefore$ New number = 110 – 11 = 99

Thus, net decrease = 100 – 99 = 1

Hence, net percentage decrease

$= \left(\frac{1}{100} \times 100\right)\% = 1\%$

SE. 10

x is 5% of y, y is 24% of z. If x = 480, find the values of y and z.

Ans. $x = 5\%$ of $y \Rightarrow x = \frac{5}{100}y$ ...(i)

and $y = 24\%$ of $z \Rightarrow y = \frac{24}{100}z$ ...(ii)

Now, x = 480 (Given)

Substituting the value of x in (i), we get

$480 = \frac{5y}{100} \Rightarrow 480 \times 100 = 5y$

$\Rightarrow y = \frac{480 \times 100}{5} = 9600 \Rightarrow y = 9600$ ...(iii)

Substituting the value of y from (iii) in (ii),

we get $9600 = \frac{24}{100}z \Rightarrow 9600 \times 100 = 24z$

$\Rightarrow z = \frac{9600 \times 100}{24} \Rightarrow z = 40000$

SE. 11

A bookseller bought a book for Rs. 20 and sold it for Rs. 24. Find his gain percent.

Ans. Cost price of the book = Rs. 20

Selling price of the book = Rs. 24

Gain = S.P. – C.P. = (24 – 20) = Rs. 4

$$\text{Gain\%} = \frac{\text{Total gain}}{\text{C.P.}} \times 100$$

$$= \left(\frac{4}{20} \times 100\right) = 20\%$$

SE. 12

An electronic goods dealer purchased a second hand colour T.V. for Rs. 7250. He spent Rs. 250 on its repairs and sold it for Rs. 7800. Find his gain or loss percent.

Ans. C.P. of the television = Rs. 7250

Overhead (repair) charges = Rs. 250

Total cost of the T.V. = Rs. 7500

S.P. of the T.V. = Rs. 7800

$\therefore$ Gain = 7800 – 7500 = Rs. 300

$$\therefore \text{Gain\%} = \frac{\text{Total gain}}{\text{C.P.}} \times 100 = \frac{300 \times 100}{7500} = 4\%$$

SE. 13

The cost price of 16 articles is equal to the selling price of 12 article. Find the gain or loss percent.

Ans. L.C.M. of 16 and 12 = 48

Let C.P. of 16 articles = Rs. 48

$\therefore$ C.P. of 1 article = 48 ÷ 16 = Rs. 3

S.P. of 12 articles = Rs. 48

$\therefore$ S.P. of 1 article = 48 ÷ 12 = Rs. 4

$\therefore$ Gain on one article = 4 – 3 = Rs. 1

$$\therefore \text{Gain percent} = \frac{\text{Total gain}}{\text{C.P.}} \times 100$$

$$= \frac{1}{3} \times 100 = 33\frac{1}{3}\%$$

SE. 14

A man purchases two fans for Rs. 2160. By selling one fan at a profit of 15% and the other at a loss of 9% he neither gains nor loss in the whole transaction. Find the cost price of each fan.

Ans. Let the cost price of first fan be Rs. x. Then,

Cost price of second fan = Rs. (2160 – x)

It is given that in the whole transaction, the man neither gains nor losses.

$\therefore$ Gain on the sale of first fan = Loss in the sale of second fan

$\Rightarrow$ 15% of Rs. x = 9% of Rs. (2160 – x)

$$\Rightarrow \frac{15}{100} \times x = \frac{9}{100} \times (2160 - x)$$

$\Rightarrow 15x = 9(2160 - x)$

$\Rightarrow 5x = 3(2160 - x) \Rightarrow 5x = 6480 - 3x$

$\Rightarrow 5x + 3x = 6480 \Rightarrow 8x = 6480$

$$\Rightarrow x = \frac{6480}{8} \Rightarrow x = 810$$

$\therefore$ C.P. of first fan = Rs. 810

C.P. of second fan = (2160 – x) = (2160 – 810) = Rs. 1350.

SE. 15

By selling a towel for Rs. 126.90, a draper loses 6%. For how much should he sell the towel to gain 4% ?

Ans. We have, C.P. of the towel

$$= \left(\frac{100}{100 - \text{Loss\%}} \times \text{S.P.}\right) = \left(\frac{100}{100 - 6} \times 126.90\right)$$

$$= \left(\frac{100}{94} \times 126.90\right) = \text{Rs.}135$$

Now, C.P. of the towel = Rs. 135 and

Required gain% = 4%

$$\therefore \text{S.P.} = \left(\frac{100 + \text{Gain\%}}{100}\right) \times \text{C.P.}$$

$$\Rightarrow \text{S.P.} = \left(\frac{100 + 4}{100} \times 135\right) = \left(\frac{104}{100} \times 135\right)$$

= Rs. 140.40

Hence, the draper should sell the towel for Rs. 140.40

SE. 16

A dealer buys 50 chairs for Rs. 50,000 but 20 of them are damaged. He decides to sell each damaged one at three fourths the price of the normal one. What should this price be in order that he may make a profit of 35% on the whole transaction ?

Ans. We have, C.P. of 50 chairs = Rs. 50000

Required profit percent = 35%

$\therefore$ Overall profit = 35% of Rs. 50000

$= \left(\frac{35}{100} \times 50000\right) = \text{Rs.}17500.$

$\therefore$ Desired S.P. of 50 chairs

= (50000 + 17500) = Rs. 67500

$\Rightarrow$ S.P. of one chair $= \left(\frac{67500}{50}\right) = \text{Rs.}1350$

$\Rightarrow$ S.P. of one damaged chair $= \frac{3}{4}$ of 1350

$= \left(\frac{3}{4} \times 1350\right) = \text{Rs.}1012.50$

SE. 17

The marked price of a ceiling fan is Rs. 1250 and the shopkeeper allows a discount of 6% on it. Find the selling price of the fan.

Ans. Marked price = Rs. 1250 and discount = 6%

Discount = 6% of M.P. = (6% of Rs. 1250)

$= \left(1250 \times \frac{6}{100}\right) = \text{Rs.}75.$

Selling price = (M.P.) – (discount)

= (1250 – 75) = Rs. 1175.

Hence, the selling price of the fan is Rs. 1175.

SE. 18

How much percent above the cost price should a shopkeeper mark his goods so that after allowing a discount of 25% on the marked price, he gains 20% ?

Ans. Let the cost price be Rs. 100.

Gain% = 20%.

$\therefore$ Selling price = Rs. 120.

Let the marked price be Rs. x.

Then, discount = 25% of Rs. x

$= \left(x \times \frac{25}{100}\right) = \text{Rs.}\frac{x}{4}.$

$\therefore$ Selling price = (M.P.) – (discount)

$= \left(x - \frac{x}{4}\right) = \text{Rs.}\frac{3x}{4}.$

$\therefore \frac{3x}{4} = 120 \Rightarrow x = \left(120 \times \frac{4}{3}\right) = 160.$

$\therefore$ Marked price = Rs. 160.

Hence, the marked price is 60% above cost price

SE. 19

If 8% VAT is included in the prices, find the original price of

(i) a TV bought for Rs. 14,500.

(ii) a shampoo bottle bought for Rs. 180.

Ans. (i) Let original price be Rs. 100.

Then VAT = 8% of Rs. 100 = Rs. 8

$\therefore$ C.P. = Rs. 108

Here, C.P. = Rs. 108, original price = Rs. 100

then for C.P. = Rs. 1, original price = Rs. $\frac{100}{108}$

So, for C.P. = Rs. 14,500, original price

$= \frac{100}{108} \times 14,500 = \text{Rs.}13425.90 \text{ (approx)}$

(ii) Let original price be Rs. 100.

Then, VAT = 8% of Rs. 100 = Rs. 8

So, C.P. = Rs. 108

For C.P. = Rs. 108, original price = Rs. 100

Then for C.P. = Rs. 180,

original price $= \frac{100}{108} \times 180$

$= \frac{18000}{108} = \text{Rs.}166.70$ (approx).

Space for Notes :

EXERCISE – 8.1

NS. 1

Find the ratio of the following.

(a) Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.

(b) 5 m to 10 km.

(c) 50 paise to Rs. 5.

Ans. (a) The ratio is 15 km/hr : 30 km/hr or 1 : 2.

(b) The ratio is 5 m : 10 km

or 5 m : 10×10^3 m {$\because$ 1 km = 10^3 m}

or 1 : 2000.

(c) The ratio is 50 paise : Rs. 5

or 50 paise : 500 paise {$\because$ 1 Rs. = 100 paise}

or 1 : 10.

NS. 2

Convert the following ratios to percentages.

(a) 3 : 4 (b) 2 : 3

Ans. To convert ratio to its percentage counterpart, we multiply it by 100. Thus,

(a) Ratio = 3 : 4

and percentage = $\frac{3}{4} \times 100 = 75\%$

(b) Ratio = 2 : 3

and percentage = $\frac{2}{3} \times 100 = 66\frac{2}{3}\%$.

NS. 3

72% of 25 students are good in Mathematics. How many are not good in Mathematics ?

Ans. We have, 72% of students are good in Mathematics.

$\therefore$ Percentage of students not good in Mathematics = 100% – 72% = 28%

Also, total number of students = 25

Hence, number of students not good in Mathematics = $\frac{28}{100} \times 25 = 7$ students.

NS. 4

A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all ?

Ans. We have, matches won = 10

Win % = 40%

Let total number of matches = t

$\therefore$ 40 % of t = 10 or $\frac{40}{100} \times t = 10$

or t = 25 matches.

NS. 5

If Chameli had Rs. 600 left after spending 75% of her money, how much did she have in the beginning ?

Ans. We know, money left with Chameli = Rs. 600%

% of money she spent = 75 %

$\therefore$ % of money left = 100% – 75% = 25%

Let total money in the beginning be x.

$\therefore$ 25 % of x = Rs. 600

or, $\frac{25}{100} \times x$ = Rs. 600 or, x = Rs. 2400

NS. 6

If 60% people in a city like cricket, 30% like football and the remaining like other games, then what percent of the people like other games ? If the total number of people are 50 lakh, find the exact number who like each type of game.

Ans. In the city, People who like cricket = 60 %

people who like football = 30 %

People who like other games

= 100 % – (60 + 30) % = 10 %

Total population = 50 lakhs

Hence, people who like cricket

$$= \frac{60}{100} \times 50\,\text{lakh} = 30\,\text{lakh}$$

People who like football

$$= \frac{30}{100} \times 50\,\text{lakh} = 15\,\text{lakh}$$

People who like other games

$$= \frac{10}{100} \times 50\,\text{lakh} = 5\,\text{lakh}$$

EXERCISE – 8.2

NS. 1

A man got a 10% increase in his salary. If his new salary is Rs. 1,54,000, find his original salary.

Ans. We have, Increase in salary = 10 %

New salary = 154000

Let original salary be Rs. s.

$$\therefore s + \frac{10}{100} \times s = 154000$$

$$\Rightarrow \frac{110}{100} \times s = 154000 \Rightarrow s = \frac{154000 \times 100}{110}$$

$\Rightarrow$ s = Rs. 140000.

NS. 2

On Sunday 845 people went to the Zoo. On Monday only 169 people went. what is the percent decrease in the people visiting the Zoo on Monday ?

Ans. People went to zoo on Sunday = 845

People went to zoo on Monday = 169

Decrease in people = 845 – 169 = 676

$$\therefore \% \text{ decrease} = \frac{676}{845} \times 100 = 80\%$$

NS. 3

A shopkeeper buys 80 articles for Rs. 2,400 and sells them for a profit of 16%. Find the selling price of one article.

Ans. We have, Cost price = Rs. 2400

Profit = 16%

Number of articles = 80

$$\text{S.P.} = 2400\left(\frac{100+16}{100}\right)$$

$\therefore$ Selling price of 80 articles

$$= 2400\left(1 + \frac{16}{100}\right) = \text{Rs.}\,2784.$$

Thus, the selling price of one article

$$= \frac{2784}{80} = \text{Rs.}\,34.80$$

NS. 4

The cost of an article was Rs. 15,500. Rs. 450 were spent on its repairs. If it is sold for a profit of 15%, find the selling price of the article.

Ans. Cost of article = Rs. 15500; Repairs = Rs. 450

$\therefore$ Total cost = Cost + Repairs = Rs. 15950

Profit = 15 %

$$\therefore \text{Selling price} = \frac{15950(100+15)}{100}$$

$$= 15950\left(1 + \frac{15}{100}\right) = \text{Rs.}\,18342.50.$$

NS. 5

A VCR and TV were bought for Rs. 8000 each. The shopkeeper made a loss of 4% on the VCR and profit of 8% on the TV. Find the gain or loss percent on the whole transaction.

Ans. Cost of VCR = Rs. 8000

Cost of TV = Rs. 8000

Total cost = 8000 + 8000 = Rs. 16000

Loss on VCR = 4 %

$\therefore$ Selling price of VCR

$$= 8000\left(1-\frac{4}{100}\right) = \text{Rs.}7680$$

Profit on TV = 8 %

$\therefore$ Selling price of TV

$$= 8000\left(1+\frac{8}{100}\right) = \text{Rs.}8640$$

$\therefore$ Total selling price = 7680 + 8640 = Rs. 16320

Thus, profit = 16320 – 16000 = Rs. 320

Hence, profit % = $\frac{320}{16000} \times 100 = 2\%$.

NS. 6

During a sale, a shop offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at Rs. 1450 and two shirts marked at Rs. 850 each ?

Ans. We have, cost of pair of jeans = Rs. 1450

Cost of two shirts = 2 × 850 = Rs. 1700

$\therefore$ Total cost = 1450 + 1700 = Rs. 3150

Discount = 10 %

Hence, selling price

$$= 3150\left(1-\frac{10}{100}\right) = \text{Rs.}2835.$$

NS. 7

A milkman sold two of his buffaloes for Rs. 20,000 each. On one he made a gain of 5 % and on the other a loss of 10%. Find his overall gain or loss. (Hint : Find C.P. of each).

Ans. We have, selling price of each buffalo = Rs. 20000

On buffalo 1, profit = 5 %

$\therefore$ Cost price of buffalo 1

$$= \frac{20000}{\left(1+\frac{5}{100}\right)} = \text{Rs. } 19047.62$$

On buffalo 2, loss = 10 %

$$\therefore \text{Cost price} = \frac{20000}{\left(1-\frac{10}{100}\right)} = \text{Rs. } 22222.22$$

$\therefore$ Total cost price = 19047.62 + 22222.22

= Rs. 41269.84

Hence, loss = 41269.84 – 40000 = Rs. 1269.84.

NS. 8

The price of a TV is Rs. 13000. The sales tax charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it.

Ans. Cost price of TV = Rs. 13000

Sales tax = 12 %

$$\therefore \text{Amount} = \text{C.P.}\left(1+\frac{12}{100}\right)$$

$$= 13000\left(1+\frac{12}{100}\right) = \text{Rs. } 14560$$

NS. 9

Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is Rs. 1600, find the marked price.

Ans. Selling price = Rs. 1600

Discount = 20 %

$$\text{We know, M.P.} = \frac{\text{S.P.}}{\left(1-\frac{\text{Discount\%}}{100}\right)} = \frac{1600}{\left(1-\frac{20}{100}\right)}$$

= Rs. 2000.

NS. 10

I purchased a hair-dryer for Rs. 5400 including 8% VAT. Find the price before VAT was added.

Ans. Cost price = Rs. 5400; VAT = 8%

Original price $= \frac{100}{100 + \text{Tax\%}} \times \text{C.P.}$

$= \frac{100}{100+8} \times 5400 = \frac{100}{108} \times 5400 = \text{Rs. } 5000$

EXERCISE – 8.3

NS. 1

Calculate the amount and compound interest on

(a) Rs. 10,800 for 3 years at $12\frac{1}{2}\%$ per annum compounded annually.

(b) Rs. 18,000 for $2\frac{1}{2}$ years at 10% per annum compounded annually.

(c) Rs. 62,500 for $1\frac{1}{2}$ years at 8% per annum compounded half yearly.

(d) Rs. 8,000 for 1 year at 9% per annum compounded half yearly.

(e) Rs. 10,000 for 1 year at 8% per annum compounded half yearly.

Ans. (a) We have, P = Rs. 10,800

R = 12.5 % per annum; n = 3 years

Total amount, $A = P\left(1 + \frac{R}{100}\right)^n$

$= 10800\left(1 + \frac{12.5}{100}\right)^3 = \text{Rs. } 15377.34$

$\therefore$ Interest = A – P = 15377.34 – 10800

= Rs. 4577.34

(b) We have, P = Rs. 18000

R = 10 % per annum; n = 2.5 years or $2\frac{1}{2}$ years

$\therefore$ Amount $A = P\left(1 + \frac{R}{100}\right)^n$

At the end of 2 years,

$A = 18000\left(1 + \frac{10}{100}\right)^2 = \text{Rs. } 21780.$

Now, we calculate SI on this amount for $\frac{1}{2}$ year at 10 % per annum.

$\therefore$ Amount after 2.5 years

$= 21780\left(1 + \frac{10 \times \frac{1}{2}}{100}\right) = \text{Rs. } 22869$

$\therefore$ Interest = A – P = 22869 – 18000

= Rs. 4869.

(c) We have, P = Rs. 62500

R = 8% per annum = 4% per half year

$n = 1\frac{1}{2}$ years = 3 half years

Amount $A = P\left(1 + \frac{R}{100}\right)^n$

$= 62500\left(1 + \frac{4}{100}\right)^3 = \text{Rs. } 70304.$

$\therefore$ Interest = A – P = Rs. 7804

(d) We have, P = Rs. 8000

R = 9% per annum = 4.5 % per half year

n = 1 year = 2 half years

So, amount $A = P\left(1 + \frac{R}{100}\right)^n$

$$= 8000\left(1+\frac{4.5}{100}\right)^2 = \text{Rs.}8736.20$$

Hence, interest = A – P = Rs. 736.20

(e) We have, P = Rs. 10000

R = 8% per annum = 4% per half year

n = 1 year = 2 half years.

Thus, amount $A = P\left(1+\frac{R}{100}\right)^n$

$$= 10000\left(1+\frac{4}{100}\right)^2 = \text{Rs. } 10816$$

∴ Interest = A – P = Rs. 816.

NS. 2

Kamla borrowed Rs. 26,400 from a bank to buy a scooter at a rate of 15% per annum compounded yearly. What amount will she pay at the end of 2 years and 4 months to clear the loan? (Hint : Find A for 2 years with interest is compounded yearly and then find SI on the 2nd year amount for $\frac{4}{12}$ years).

Ans. We have, P = Rs. 26400

R = 15% per annum; n = 2 years 4 months

At the end of 2 years, $A = P\left(1+\frac{R}{100}\right)^n$

$$A = 26400\left(1+\frac{15}{100}\right)^2 = \text{Rs.}34914$$

Now, P = Rs. 34914

R = 15% per annum; n = 4 months = $\frac{1}{3}$ years

At the end of 2 years, 4 months

$$A = \left(1+\frac{15\times 1}{100\times 3}\right)34914 = \text{Rs.}36659.70$$

NS. 3

Fabina borrows Rs. 12500 at 12% per annum for 3 years at simple interest and Radha borrows the same amount for the same time period at 10% per annum, compounded annually. Who pays more interest and by how much ?

Ans. For Fabina, P = Rs. 12500

R = 12% per annum; n = 3 years

For simple interest, amount $A = P\left(1+\frac{Rn}{100}\right)$

$$= 12500\left(1+\frac{12\times 3}{100}\right) = \text{Rs.}17000$$

Interest = 17000 – 12500 = Rs. 4500

For Radha, P = Rs. 12500

R = 10% per annum; n = 3 years

As this is compound interest

Amount $A = P\left(1+\frac{R}{100}\right)^n$

$$= 12500\left(1+\frac{10}{100}\right)^3 = \text{Rs.}16637.5$$

Interest = 16637.5 – 12500 = Rs. 4137.50

Hence, Fabina pays more interest by

4500 – 4137.50 = Rs. 362.50

NS. 4

I borrowed Rs. 12000 from Jamshed at 6% per annum simple interest for 2 years. Had I borrowed this sum at 6% per annum compound interest, what extra amount would I have to pay ?

Ans. We have, P = Rs. 12000

R = 6% per annum; n = 2 years

For simple interest,

$$I_1 = \frac{P \times R \times T}{100} = \frac{12000 \times 6 \times 2}{100} = Rs.1440$$

For compound interest, $I_2 = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$

$$= 12000\left[\left(1+\frac{6}{100}\right)^2 - 1\right] = Rs.1483.20$$

So, he would have to pay 1483.20 – 1440

= Rs. 43.20

NS. 5

Vausudevan invested Rs. 60000 at an interest rate of 12% per annum compounded half yearly. What amount would he get

(i) after 6 months (ii) after 1 year

Ans. We have, P = Rs. 60000

R = 12% per annum = 6% per half year

(i) n = 6 months = 1 half year

$$\therefore A = P\left(1+\frac{R}{100}\right)^n = 60000\left(1+\frac{6}{100}\right)$$

= Rs. 63600.

(ii) n = 1 year = 2 half years

$$\therefore A = P\left(1+\frac{R}{100}\right)^n = 60000\left(1+\frac{6}{100}\right)^2$$

= Rs. 67416.

NS. 6

Arif took a loan of Rs. 80000 from a bank. If the rate of interest is 10% per annum, find the difference in amounts he would be paying after $1\frac{1}{2}$ years if the interest is :

(i) compounded annually.

(ii) compounded half yearly.

Ans. We have, P = Rs. 80000

R = 10% per annum = 5% per half year

(i) If interest is compounded annually

$n = 1\frac{1}{2}$ years = 1 years $+\frac{1}{2}$ year

Amount after 1 year

$$A = 80000\left(1+\frac{10}{100}\right) = Rs.\ 88000$$

Amount after $1\frac{1}{2}$ year

$$A = 88000\left(1+\frac{10 \times 1}{100 \times 2}\right) = Rs.\ 92400$$

(ii) If interest is compounded half-yearly

$n = 1\frac{1}{2}$ years = 3 half years

$$\therefore \text{Amount } A = P\left(1+\frac{R}{100}\right)^n$$

$$= 80000\left(1+\frac{5}{100}\right)^3 = Rs.92610$$

∴ Difference in amounts = 92610 – 92400

= Rs. 210.

NS. 7

Maria invested Rs. 8000 in a business. She would be paid interest at 5% per annum compounded annually. Find :

(i) The amount credited against her name at the end of the second year.

(ii) The interest for the 3rd year.

Ans. We have, P = Rs. 8000

R = 5% per annum

(i) n = 2 years

$$\therefore A = P\left(1+\frac{R}{100}\right)^n = 8000\left(1+\frac{5}{100}\right)^2$$

= Rs. 8820.

(ii) For n = 3 years

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 8000\left(1+\frac{5}{100}\right)^3 = \text{Rs.}9261$$

Hence, interest for 3rd year

= 9261 – 8820 = Rs. 441

NS. 8

Find the amount of the compound interest on Rs. 10000 for $1\frac{1}{2}$ years at 10% per annum, compounded half yearly. Would this interest be more than the interest he would get if it was compounded annually ?

Ans. We have, P = Rs. 10000

R = 10% per annum = 5% per half year

$n = 1\frac{1}{2}$ year = 3 half years

(i) If interest is compounded half yearly

$$A = 10000\left(1+\frac{5}{100}\right)^3 = \text{Rs. } 11576.25$$

Interest = 11576.25 – 10000 = Rs. 1576.25

(ii) If interest is compounded annually.

Amount after 1 year,

$$A = 10000\left(1+\frac{10}{100}\right) = \text{Rs. } 11000$$

Amount after $1\frac{1}{2}$ year

$$A = 11000\left(1+\frac{10\times 1}{100\times 2}\right) = \text{Rs. } 11550$$

Interest = 11550 – 10000 = Rs. 1550

Thus, more interest would be generated if interest is calculated half yearly.

NS. 9

Find the amount which Ram will get on Rs. 4096, if he gave it for 18 months at $12\frac{1}{2}\%$ per annum, interest being compounded half yearly.

Ans. We have, P = Rs. 4096

R = 12.5% per annum = 6.25% per half year

n = 18 months = 3 half years

$$\therefore A = P\left(1+\frac{R}{100}\right)^n = 4096$$

$$\left(1+\frac{6.25}{100}\right)^3 = \text{Rs.}4913$$

NS. 10

The population of a place icreased to 54000 in 2003 at a rate of 5% per annum :

(i) find the population in 2001.

(ii) what would be its population in 2005 ?

Ans. We have, population in 2003 = 54000

Rate = 5% per annum.

(i) Let population in 2001 be x.

The population in 2003 will be

$$= x\left(1+\frac{5}{100}\right)^2 = 54000 \qquad \text{(as given)}$$

$$\therefore x = \frac{54000}{(1+0.05)^2}$$

Population = 48980 in 2001.

(ii) For population in 2005 n = 2 years

$\therefore$ Population $= 54000\left(1+\frac{5}{100}\right)^2 = 59535$

NS. 11

In a laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5,06,000.

Ans. Initial count of bacteria = 506000

Rate = 2.5 % per hour

n = 2 hours

$\therefore$ Count after 2 hours = 506000

$\left(1+\frac{2.5}{100}\right)^2 = 531616.25$

NS. 12

A scooter was bought at Rs. 42000. Its value depreciated at the rate of 8% per annum. Find its value after one year.

Ans. Initial price = Rs. 42000

rate of depreciation = 8% per annum

n = 1 year

Price after 1 year = 42000

$\left(1-\frac{8}{100}\right)$ = Rs. 38640.

Space for Notes :

EXERCISE – I

ONLY ONE CORRECT TYPE

1. If x is 90% of y, then what percent of x is y ?
(A) 90 (B) 190
(C) 101.1 (D) 111.1

2. A number exceeds 20% of itself by 40. The number is :
(A) 50 (B) 60
(C) 80 (D) 320

3. 12.5% of 192 = 50% of x, then x =
(A) 48 (B) 96
(C) 24 (D) None of these

4. One-third of 1206 is what percent of 134 ?
(A) 3 (B) 30
(C) 300 (D) None of these

5. 8/40 is equivalent to :
(A) 20% (B) 40%
(C) 25% (D) 8%

6. $\sqrt{(3.6\% \text{of } 40)}$ is equal to :
(A) 2.8 (B) 1.8
(C) 1.2 (D) None of these

7. 75% of a number when added to 75 is equal to the number. The number is :
(A) 150 (B) 200
(C) 225 (D) 300

8. If 4.6% of x is 23, find x :
(A) 400 (B) 200
(C) 100 (D) 500

9. If A's salary is 30% more than B's, then how much percent is B's salary less than A's ?
(A) 30% (B) 25%
(C) $23\frac{1}{13}\%$ (D) $33\frac{1}{3}\%$

10. Bananas are bought at 15 for a rupee and sold at the rate of 9 for a rupee. The gain percent is :
(A) 30% (B) 60%
(C) $66\frac{2}{3}\%$ (D) $33\frac{1}{3}\%$

11. A trader allows a trade discount of 20% and a cash discount of $6\frac{1}{4}\%$ on the marked price of the goods and gets a net gain of 20% on the cost. By how much above the cost should the goods be marked for sale ?
(A) 40% (B) 50%
(C) 60% (D) 70%

12. A shopkeeper marks his goods 20% higher than the cost price and allows a discount of 5%. The percentage of his profit is :
(A) 10% (B) 14%
(C) 15% (D) 20%

13. By selling a vehicle for Rs. 455000, Sumant suffers 25% loss, what was his loss ?
(A) Rs. 115370 (B) Rs. 113570
(C) Rs. 113750 (D) None of these

14. If the difference between selling a shirt at a profit of 10% and 15% is Rs. 10, then the cost price is :
(A) Rs. 110 (B) Rs. 115
(C) Rs. 150 (D) Rs. 200

15. By selling an article for Rs. 100, one gains Rs. 10. Then, the gain percent is :
(A) 9%
(B) 10%
(C) $11\frac{1}{9}\%$
(D) None of these

16. A man sold a radio for Rs. 1980 and gained 10%. The radio was bought for :
(A) Rs. 1782 (B) Rs. 1800
(C) Rs. 2178 (D) None of these

17. By selling an article for Rs. 247.50, we get a profit of $12\frac{1}{2}$%. The cost of the article is :
(A) Rs. 210 (B) Rs. 220
(C) Rs. 224 (D) Rs. 225

18. What is the cost price of an article which is sold at a loss of 25% for Rs. 150 ?
(A) Rs. 125 (B) Rs. 175
(C) Rs. 200 (D) Rs. 225

19. What single discount is equivalent to two successive discounts of 25% and 5% ?
(A) 28% (B) 27%
(C) 28.75% (D) 28.2%

20. By selling a watch for Rs. 1140, a man loses 5%. In order to gain 5%, the watch must be sold for :
(A) Rs. 1311 (B) Rs. 1197
(C) Rs. 1254 (D) Rs. 1260

21. There would be 10% loss if a toy is sold at Rs. 10.80 per piece. At what price should it be sold to earn a profit of 20% ?
(A) Rs. 12 (B) Rs. 12.96
(C) Rs. 14.40 (D) None of these

22. A fruit seller had some apples. He sells 40% and still has 420 apples. Originally, he had :
(A) 588 apples
(B) 600 apples
(C) 672 apples
(D) 700 apples

23. At what rate of interest will a sum of money amounts to 7/4 of itself in 8 years and 4 months ?
(A) 9% (B) 8%
(C) 7% (D) 10%

24. If every side of a right angled triangle is doubled, then percentage increase in the area of the triangle is
(A) 100% (B) 200%
(C) 300% (D) 400%

25. Joseph bought a scooter for Rs. 8400 and sold it at a gain of 6%. At what price did he sell the scooter ?
(A) Rs. 8204 (B) Rs. 8601
(C) Rs. 8904 (D) Rs. 8900

PARAGRAPH TYPE

PASSAGE # I

$$P\% \text{ of } x = \frac{P}{100} \times x$$

26. Find 135% of 80 cm.
(A) 10800 m (B) 10.8 m
(C) 1.08 m (D) 108 m

27. What percent of 24m is 6m ?
(A) 60% (B) 25%
(C) 5% (D) 6.25%

28. Ajit lost 20% of his money after which he was left with Rs. 6400. The amount he had in the beginning with him is
(A) Rs. 8000 (B) Rs. 9000
(C) Rs. 7000 (D) Rs. 1000

PASSAGE # II

If the selling price of an article is greater than the cost price, the difference between selling price and cost price is called profit :

29. C.P. = Rs. 620 and S.P. = Rs. 713, then profit is :

(A) Rs. 73 (B) Rs. 93

(C) Rs. 1333 (D) Rs. 13

30. C.P. = Rs. 345, Profit = Rs. 98, then S.P. =

(A) Rs. 443 (B) Rs. 247

(C) Rs. 434 (D) Rs. 742

31. If S.P. = Rs. 840, Profit = Rs. 62 then C.P. =

(A) Rs. 247 (B) Rs. 877

(C) Rs. 902 (D) Rs. 778

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from List – I and List – II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the quantities given in List I to the respective ratios given in List II

List – I	List – II
(P) 36 to 64	(i) 2 : 5
(Q) 40 paise to Rs. 3	(ii) 9 : 16
(R) 24 minutes to an hour	(iii) 1 : 16
(S) 125 ml to 2 litres	(iv) 2 : 15

(A) (P) → (ii), (Q) → (iv), (R) → (i), (S) → (iii)

(B) (P) → (ii), (Q) → (i), (R) → (iv), (S) → (iii)

(C) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(D) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)

33. Match the statements given in List-I with the corresponding amount and compound interest given in List II

List – I	List – II
(P) Rs. 12600 for 2 years at 10% p.a. compounded annually	(i) Rs. 22781.25 Rs. 6781.25
(Q) Rs. 18000 for 2 years at 10% p.a. compounded annually	(ii) Rs. 21780 Rs. 3780
(R) Rs. 16000 for 3 years at $12\frac{1}{2}$% p.a. compounded annually	(iii) Rs. 15246 Rs. 2646
(S) Rs. 1500 for $2\frac{1}{2}$ years at 10% p.a.compounded annually	(iv) Rs. 1905.75, Rs. 405.75

(A) (P)→(i), (Q)→(ii), (R)→(iii), (S)→(iv)

(B) (P)→(iii), (Q)→(i), (R)→(ii), (S)→(iv)

(C) (P)→(iii), (Q)→(ii), (R)→(i), (S)→(iv)

(D) (P)→(ii), (Q)→(iii), (R)→(i), (S)→(iv)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Find the ratio compounded of the three ratios 2a : 3b, 6ab : $5c^2$, c : a.
2. Find the ratio of :
   (i) 35 minutes of 1 hour
   (ii) 8 kg to 400 gms.
3. Find the ratio of the following :
   (i) 5m to 10 km
   (ii) 50 paise to Rs. 5
   (iii) Speed of cycle which is 15 km per hour to the speed of the scooter which is 30 km per hour.
4. A certain company has 80 employees who are engineers. In this company engineers constitute 40% of its work force. How many people are employed in the company ?
5. During one year, the population of a town increased by 5% and during the next year, the population decreased by 5%. If the total population is 9975 at the end of the second year, then what was the population size in the beginning of the first year ?
6. Express 75 paise as a percent of Rs. 8
7. The ratio of bus and train fares from Calcutta to a certain place is 3 : 4. If the train fare increases by 20% and bus fare by 10% then what will be new ratio of bus and train fares ?
8. Rakesh purchased a car for Rs. 135000 and soon after he sold it for Rs. 142500. Find his gain percent.
9. A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 960 grams for 1 kg. Find gain percent
10. Find the buying price of a towel when 5% sales tax is added on the purchase of it at Rs. 50.

SHORT ANSWER TYPE

1. Malvika gets 98 marks in her exams. This amounts to 56% of the total marks. What are the maximum marks ?
2. A nursery has 5000 plants. 5% of the plants are roses and 1% are mango plants. What is the total number of other plants ?
3. The price of a Maruti car rises by 30%, while the sales of the car goes down by 20%. What is the percentage change in the total revenue ?
4. By selling 144 pens, Jyoti lost the S.P. of 6 pens. Find her loss percent.
5. Mohit bought a CD for Rs. 750 and sold it for Rs. 875, find his gain percent.
6. Ishaan purchased an old scooter for Rs. 12000 and spent Rs. 2850 on its overhauling. Then, he sold it to his friend Sohan for Rs. 13860. How much percent did he gain or lose ?
7. 200 kg of sugar was purchased at the rate of Rs. 15 per kg and sold at a profit of 5%. Compute the profit and the selling price per kg.
8. Calculate amount and compound interest on Rs. 9600 for 3 years at $12\frac{1}{2}$% p.a. compounded annually.
9. Simple interest on a sum of money for 2 years at $6\frac{1}{2}$% per annum is Rs. 5200. What will be the compound interest on that sum at the same rate for the same time period ?

10. Find the compound interest on Rs. 160000 for 2 years at 10% per annum when compounded semi-annually.

LONG ANSWER TYPE

1. Ruchi's weight is 25% that of Sneha and 40% that of Tameena. What percentage of Tameena's weight is Sneha's weight ?
2. A man buys a plot of agricultural land for Rs. 300000. He sells one-third at a loss of 20% and two-fifths at a gain of 25%. At what price must he sell the remaining land so as to make an over-all profit of 10% ?
3. A man sold two articles at Rs. 25920 each. These were sold at 8% gain and 4% loss respectively. Find the gain or loss percent in the whole transaction.
4. A man bought two T.V. sets for Rs. 42500. He sold one at a loss of 10% and other at a profit of 10%. If the selling price of each T.V. set is same, determine the C.P. of each set.
5. At what rate, will a sum of Rs. 1000 amount to Rs. 1102.50 in 6 months at compound interest, if the interest is compounded quarterly ?

TRUE / FALSE TYPE

1. To convert a fraction into a percent multiply by 100.
2. The additional money paid by the borrower to the lender after a specified period of time is called amount.
3. 40 % of 60 is greater than 25 % of 90.
4. If C.P. = Rs. 400 and loss = 15 %, then SP = Rs. 400 – 15 % of 400.
5. Profit and loss are always calculated on SP.

NUMERICAL PROBLEMS

1. 75% of 20 students are not good at English. How many of them are good in English ?
2. If 50% of (x – y) = 30% of (x + y), then x = ky, find the value of 10k.
3. A shopkeeper buys 50 articles for 2000 and sells them for a profit of 10%. Find the selling price of one article.
4. The value of a T.V. depreciates every year by 25%. The value after a year if its present value is Rs. 45,000 will be 3375×10^{n}. Find n.
5. A person sells an article for Rs. 550, gaining 1/10 of its C.P. If gain % is 10^{n}. Find n.
6. Shobha's Mathematics test had 75 problems i.e., 10 arithmetic, 30 algebra and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of the algebra and 60 % of the geometry problems correctly, she did not pass the test because in algebra she got less than 60% of the problems right. How many more questions she would have needed to answer correctly to score a 60% passing grade ?
7. If $n\frac{n}{3}$ kg of pure salt must be added to 30 kg of 2% solution of salt and water to increase it to a 10% solution, then find n.
8. Tanuj sold a sofa set for Rs. 8800, thereby losing 12%. Had he sold it for Rs. 9600, what percent profit or loss would he have made ?
9. In what time will Rs. 1000 amount to Rs. 1331 at 10% per annum compounded annually ?

10. The present population of a town is 16000. If it increases at rate of 5% p.a., what will be the population after 2 years ?

ANALYTICAL PROBLEMS & BRAIN TEASER

1. A sum of money fetches a certain interest when it is invested for 4 years at the rate of 8 % per annum. If it is invested for 6 years, then the interest increases by Rs. 640. Find the sum of money.
(A) Rs. 4050 (B) Rs. 2000
(C) Rs. 2050 (D) Rs. 4000

2. The compound interest earned by Suresh on a certain amount at the end of two years at the rate of 8 % per annum was Rs. 1414.40. What was the total amount that Suresh got back at the end of two years ?
(A) Rs. 9414.40
(B) Rs. 9914.40
(C) Rs. 9014.40
(D) Rs. 8914.40

3. Amar wrote exams in four subject – Maths, Science, Hindi and Social Studies. The ratio of marks he got in these exams was 2 : 3 : 4 : 5. He got an aggregate of 70 % in these exams. Each exam had the same maximum marks. In how many of these subjects did he get more than 50 % ?
(A) 1 (B) 2
(C) 3 (D) 4

4. If 540 is 10 % of y and z % of y is 16200, then find the values of y and z respectively.
(A) 5400, 20 (B) 5400, 300
(C) 300, 5400 (D) 20, 5800

5. Two successive discounts of x % and y % on a western gown is same as the single discount of :
(A) $\left(x+y+\frac{xy}{100}\right)\%$ (B) $\left(x-y-\frac{xy}{100}\right)\%$
(C) $\left(x+y-\frac{xy}{100}\right)\%$ (D) $\left(y-x-\frac{xy}{100}\right)\%$

6. If A : B = 5 : 6 and B : C = 7 : 8, then by approximately what percent is C more than A ?
(A) 36 % (B) 40 %
(C) 37.14 % (D) 48.23 %

7. Sonika spent Rs. 45760 on the interior decoration for her home, Rs. 27896 on buying air conditioner and the remaining 28 % of the total amount she had as cash with her. What was the total amount?
(A) Rs. 98540 (B) Rs. 102300
(C) Rs. 134560 (D) Rs. 97500

8. Simple interest on a sum of money for 1 year at 12 % per annum is Rs. 1200. What will be the compound interest when compounded half-yearly on that sum at the same rate for the same period?
(A) Rs. 1236 (B) Rs. 1326
(C) Rs. 11236 (D) Rs. 13260

9. A man borrowed Rs. 4000 at 10 % per annum at compound interest. At the end of each year he has repaid Rs. 1000. The amount of money he still incurs after the third year is :
(A) Rs. 2740 (B) Rs. 2104
(C) Rs. 2014 (D) Rs. 3400

10. A shopkeeper fixes the marked price of a pair of shoes 45 % above its cost price. What is the percentage of discount allowed to gain 16 % ?
(A) 30 % (B) 25 %
(C) 15 % (D) 20 %

11. A man sells two articles each at Rs. 198. He makes a profit of 10% on one article and a loss of 10% on the other. Net profit or loss of the person is :

(A) 2% profit (B) 2% loss

(C) 1% profit (D) 1% loss

12. The present population of a city is 8000. If it increases by 10 % during the first year and by 20% during the second year, then population after two years will be :

(A) 12400 (B) 14400

(C) 10560 (D) None of these

13. A businessman fixed the selling price of an article after increasing the cost price by 40 %. Then he allowed his customers a discount of 20 % and gained Rs. 48. The cost price of the article is :

(A) Rs. 200 (B) Rs. 248

(C) Rs. 400 (D) Rs. 448

14. The compound interest on Rs. 1000 in 2 years at 4 % per annum, the interest being compounded half yearly, is :

(A) Rs. 6360.80 (B) Rs. 82.43

(C) Rs. 912.86 (D) Rs. 828.82

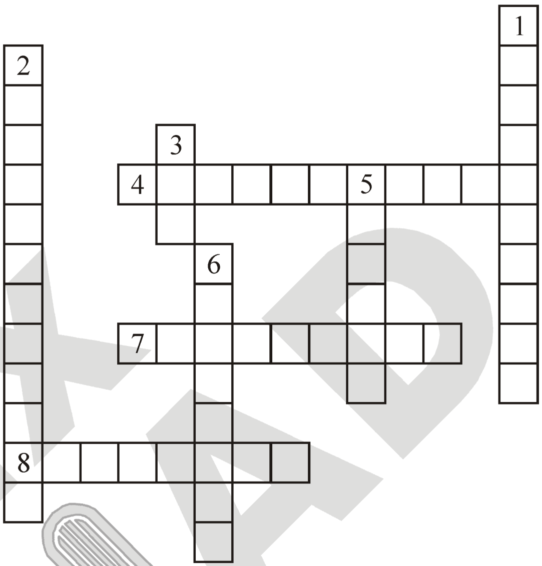
15. The difference between the compound interest (compounded annually) and simple interest on a certain sum at 10 % per annum for 2 years is Rs. 631. Find the sum.

(A) Rs. 80100 (B) Rs. 63100

(C) Rs. 60000 (D) Rs. 73100

CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

4. Shopkeeper fixes ________ over the cost price which is greater than the cost price and then, he allows us discount at that price. [6, 5]

7. Profit and loss are always calculated on the _________. [4, 5]

8. When the interest due at the end of a certain period is added to the principal of the previous period to get the principle of the next period, then such an interest is called _________ interest. [8]

Down

1. If two or more discounts are allowed one after the other, then such discounts are known as _______ discounts. [10]

2. Sales tax is always calculated on the _____. [7, 5]

3. Money is collected by the government from the citizens in the form of _________. [3]

5. When selling price is greater than the cost price, then we will have a _________. [6]

6. Reduction in the cost price of an article by the shopkeeper is called a _________. [8]

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	A	A	C	A	C	D	D	C	C	C	B	D	D	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	B	C	C	D	C	D	A	C	C	C	A	B	B	A
31	32	33												
D	A	C												

EXERCISE–I

VERY SHORT ANSWER TYPE

1. $\frac{4a}{5c}$ **2.** (i) 7 : 12, (ii) 20 : 1 **3.** (i) 1 : 2000, (ii) 1 : 10, (iii) 1 : 2 **4.** 200

5. 10000 **6.** $\frac{75}{8}\%$ **7.** 11 : 16 **8.** $\frac{50}{9}\%$ **9.** $4\frac{1}{6}\%$ **10.** Rs. 52.5

SHORT ANSWER TYPE

1. 175 **2.** 4700 **3.** 4% **4.** 4% **5.** $16\frac{2}{3}\%$ **6.** Loss = $6\frac{2}{3}\%$

7. Rs. 15.75 **8.** Rs. 4068.75 **9.** Rs. 5369 **10.** Rs. 34481

LONG ANSWER TYPE

1. 160% **2.** Rs. 100000 **3.** Gain% = $1\frac{11}{17}\%$ **4.** Rs. 19125 **5.** 20% per annum

TRUE / FALSE

1. True **2.** False **3.** True **4.** True **5.** False

CROSSWORD PUZZLE

Across

4. Marked price **7.** Cost price **8.** Compound

Down

1. Successive **2.** Selling price **3.** Tax **5.** Profit **6.** Discount

NUMERICAL PROBLEMS

1. 5 **2.** 40 **3.** 44 **4.** 1 **5.** 1 **6.** 5 **7.** 2

8. 4 **9.** 3 **10.** 17640

ANALYTICAL PROBLEMS & BRAIN TEASER

1. D **2.** B **3.** C **4.** B **5.** C **6.** C **7.** B

8. A **9.** C **10.** D **11.** D **12.** C **13.** C **14.** B

15. B

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : COMPARING QUANTITIES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Exercises			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

ALGEBRAIC EXPRESSIONS AND IDENTITIES

8

Concepts

INTRODUCTION

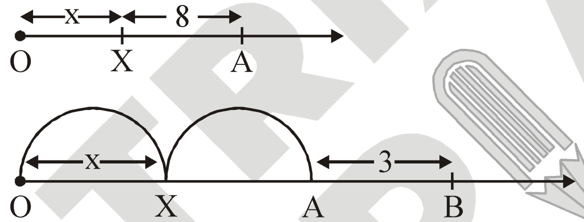
Algebraic expressions : A combination of constants and variables connected by the signs of fundamental operations of addition, subtraction, multiplication and division is called an algebraic expression.

For example : $x + 8$, $4z^2$, $9xy + 15$, etc are algebraic expressions.

Here, 8, 4, 9, 15 are constants as these are symbols having a fixed numerical value and *x, y, z* are called variables or literal numbers as they can take various numerical values.

1. NUMBER LINE AND AN EXPRESSION

Let us have the expression x + 8, where the variable x has a position X on the number line. X may be anywhere on the number line, but it is definite that the value of x + 8 is given by a point A, 8 units to the right of X. Similarly, the value of x – 5 will be 5 units to the left of X. The position of 2x + 3 will be as follows. The position of 2x will be point A and position B of 2x + 3 will be 3 units to the right of A.



2. TERMS, FACTORS AND COEFFICIENTS

Terms : Various parts of an algebraic expression which are separated by the signs + or – are called the 'terms' of the expression.

For example : 5b has one term, i.e., 5b, 2x + 9 has two terms, i.e., 2x and 9.

$3x^2 + 2y - 7$ has three terms i.e., $3x^2$, 2y and – 7.

Factors : Each term in an algebraic expression is a product of one or more number(s) and literal number(s). These number(s) and literal number(s) are known as the factors of that term.

For example : 3x is the product of 3 and x.

Coefficient : The numerical factor of the term is called coefficient. In expression 9xy – 7x, the coefficient of the term 9xy is 9 and the coefficient of the term – 7x is – 7.

3. TYPES OF ALGEBRAIC EXPRESSIONS

Depending upon the number of terms, an algebraic expression can be categorised in the following manner :

Types	No.of terms	Examples
Monomial	1	2x
Binomial	2	3x + 7y
Trinomial	3	3x + 7y + 2
Polynomial	1 or more than 1	a + b + c + d

1. Monomial : An expression which contains only one term.

2. Binomial : An expression which contains only two terms.

3. Trinomial : An expression which contains only three terms.

4. Polynomial : An expression containing one or more terms with non-zero coefficient (with variables having non-negative exponents).

4. LIKE AND UNLIKE TERMS

Like Terms : When the terms have same literal factors or variables, they are called like terms.

For example : 3x, – 2x, – 18x are like terms.

Unlike Terms : When the terms have different literal factors or variables, they are called unlike terms.

For example : 7x, 5y, 3z are unlike terms.

5. ADDITION OF ALGEBRAIC EXPRESSIONS

To add two algebraic expressions, we collect different groups of like terms and find the sum of like terms in each group.

Example 1

Add : $5x^2 + 3x - 8$ and $4x^2 - 5x + 6$.

Solution :

$$\begin{array}{r} 5x^2 + 3x - 8 \\ +4x^2 - 5x + 6 \\ \hline 9x^2 - 2x - 2 \\ \hline \end{array}$$

Example 2

Add the given algebraic expressions :

$$2 + \frac{2y}{3} - \frac{5y^2}{3} + \frac{5y^3}{3} \text{ and } -\frac{4}{3} - \frac{2y^2}{3} - \frac{y}{2} + \frac{5y^3}{3}$$

Solution :

$$\begin{array}{r} 2 + \frac{2y}{3} - \frac{5y^2}{3} + \frac{5y^3}{3} \\ + \; -\frac{4}{3} - \frac{y}{2} - \frac{2y^2}{3} + \frac{5y^3}{3} \\ \hline \frac{2}{3} + \frac{y}{6} - \frac{7y^2}{3} + \frac{10y^3}{3} \\ \hline \end{array}$$

6. SUBTRACTION OF ALGEBRAIC EXPRESSIONS

In order to subtract an algebraic expression from another, we change the signs (from + to – or from – to +) of all the terms of the expression which is to be subtracted and then the two expressions are added.

Example 3

Subtract $3x^2 - 6x - 4$ from $5 + x - 2x^2$.

Solution :

Arranging the terms of the expression in descending powers of x and subtracting column wise, we get :

$$\begin{array}{l} -2x^2 + x + 5 \\ +3x^2 - 6x - 4 \\ \underline{(-) \quad (+) \quad (+)} \\ -5x^2 + 7x + 9 \end{array}$$

Note : Once the signs are changed the old signs will not be used.

Example 4

Subtract $\left(-2y^2 + \frac{1}{2}y - 3\right)$ from $7y^2 - 2y + 10$.

Solution :

$$\begin{array}{l} 7y^2 - 2y + 10 \\ -2y^2 + \frac{1}{2}y - 3 \\ \underline{(+) \quad (-) \quad (+)} \\ 9y^2 - \frac{5}{2}y + 13 \end{array}$$

7. MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

Let us have a look at the rules of sign and laws of exponents which will be useful for multiplication of two algebraic expressions.

(a) The product of two factors with like signs is positive and product of two factors with unlike signs is negative.

(i) (+) × (+) = + (ii) (+) × (–) = – (iii) (–) × (+) = – (iv) (–) × (–) = +

(b) If x is any variable and m, n are positive integers, then $x^m \times x^n = x^{m+n}$.

8. MULTIPLICATION OF TWO, THREE OR MORE MONOMIALS

We multiply constants as we multiply rational numbers and multiply the same variables by using laws of exponents.

Example 5

Multiply : $(3xy) \times (4y)$

Solution :

$(3xy) \times (4y) = (3 \times 4) \times (x) \times (y \times y) = 12xy^2$

Example 6

Find the area of rectangle with $(2x^2, 5y^2)$ as a pair of monomials as their length and breadth respectively.

Solution :

We know that the area of a rectangle is the product of its length and breadth.

So, Area = length × breadth = $2x^2 \times 5y^2 = (2 \times 5) \times (x^2 \times y^2) = 10x^2y^2$

Example 7

Find the product of

(i) $-3x, 4y, -3z$ (ii) $3q, 4q^2, 8q^3$.

Solution :

(i) $-3x \times 4y \times (-3z) = (-3x \times 4y) \times (-3z) = -12xy \times (-3z) = 36xyz$

(ii) $3q \times 4q^2 \times 8q^3 = (3q \times 4q^2) \times 8q^3 = 12q^3 \times 8q^3 = 96q^6$

9. MULTIPLICATION OF A MONOMIAL BY A POLYNOMIAL

Multiplication of a monomial by a binomial of the type a(b + c)

Step 1 : Multiply a by b to get 1st term.

Step 2 : Multiply a by c to get 2nd term.

Step 3 : Add the resulting terms obtained in step 1 and step 2.

Example 8

Multiply : $(7xy + 5y) \times 3xy$

Solution :

$7xy \times 3xy + 5y \times 3xy = 21x^{1+1}y^{1+1} + 15xy^{1+1} = 21x^2y^2 + 15xy^2$

Example 9

Find the following products :

(i) $100x \times (0.01x^4 - 0.01x^2)$ (ii) $0.1a \times (0.01a + 0.001b)$

Solution :

(i) We have, $100x \times (0.01x^4 - 0.01x^2)$

$= 100x \times 0.01x^4 - 100x \times 0.01x^2 = (100 \times 0.01)x^5 - (100 \times 0.01)x^3$

$= \left(100 \times \frac{1}{100}\right)x^5 - \left(100 \times \frac{1}{100}\right)x^3 = x^5 - x^3$

(ii) We have, $0.1a \times (0.01a + 0.001b) = 0.1a \times 0.01a + 0.1a \times 0.001b = 0.001a^2 + 0.0001ab$

Multiplication of a monomial by a trinomial :

To multiply a monomial by a trinomial, we use the following property :

$A \times (B + C + D) = A \times B + A \times C + A \times D$

Example 10

Multiply : $2x(z - x - y)$

Solution :

$2x(z - x - y) = (2x) \times (z) - (2x) \times (x) - (2x) \times (y)$

$= 2xz - 2x^2 - 2xy$

Example 11

Multiply : $5x^2(a^2x - 4y + 3z^2b)$

Solution :

$5x^2(a^2x - 4y + 3z^2b)$

$= (5x^2) \times (a^2x) - (5x^2) \times (4y) + (5x^2) \times (3z^2b)$

$= 5a^2(x^2 \times x) - (5 \times 4) \times (x^2 \times y) + (5 \times 3) \times (x^2 \times z^2 \times b)$

$= 5a^2x^3 - 20x^2y + 15x^2z^2b$

10. MULTIPLICATION OF A POLYNOMIAL BY A POLYNOMIAL

Multiplication of a Binomial by a Binomial

Working Rule : For multiplication of binomial by binomial of the type $(a + b)(c + d)$.

Step 1 : Apply distributive property and obtain $(a + b)(c + d) = a(c + d) + b(c + d)$.

Step 2 : Multiply further to get $ac + ad + bc + bd$.

Example 12

Multiply $(3x + 5y)$ and $(5x - 7y)$.

Solution :

We have,

$(3x + 5y) \times (5x - 7y) = 3x \times (5x - 7y) + 5y \times (5x - 7y)$

$= (3x \times 5x - 3x \times 7y) + (5y \times 5x - 5y \times 7y)$

$= (15x^2 - 21xy) + (25xy - 35y^2)$

$= 15x^2 - 21xy + 25xy - 35y^2 = 15x^2 + 4xy - 35y^2$

Example 13

Multiply $(3x^2 + y^2)$ by $(2x^2 + 3y^2)$.

Solution :

We have,

$(3x^2 + y^2) \times (2x^2 + 3y^2)$

$= 3x^2(2x^2 + 3y^2) + y^2(2x^2 + 3y^2)$

$= (6x^4 + 9x^2y^2) + (2x^2y^2 + 3y^4)$

$= 6x^4 + 9x^2y^2 + 2x^2y^2 + 3y^4 = 6x^4 + 11x^2y^2 + 3y^4$

Multiplication of a binomial by a trinomial :

In this type of multiplication, we multiply each of the three terms in the trinomial by each of the two terms in the binomial and we get $3 \times 2 = 6$ terms, which may be reduced to 5 or less

Example 14

Multiply :

(i) $(a + b)$ and $(2a - 3b + c)$

(ii) $(x + y)$ and $(x^2 - xy + y^2)$

Solution :

(i) $(a + b)$ and $(2a - 3b + c)$

$= a \times (2a - 3b + c) + b \times (2a - 3b + c)$

$= 2a^2 - 3ab + ac + 2ab - 3b^2 + bc$

$= 2a^2 - ab + ac + bc - 3b^2$

(ii) $(x + y)(x^2 - xy + y^2)$

$= x \times (x^2 - xy + y^2) + y \times (x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3$

11. AN IDENTITY

Consider the equality $(x + 2)(x + 3) = x^2 + 5x + 6$

Let us evaluate its both sides for $x = 2$

For $x = 2$, L.H.S. $= (x + 2)(x + 3) = (2 + 2)(2 + 3) = 4 \times 5 = 20$

R.H.S. $= x^2 + 5x + 6 = (2)^2 + 5 \times 2 + 6 = 4 + 10 + 6 = 20$

Thus, L.H.S. = R.H.S. for $x = 2$

Let us now take $x = -5$

L.H.S. $= (x + 2)(x + 3) = (-5 + 2)(-5 + 3) = (-3) \times (-2) = 6$

R.H.S. $= x^2 + 5x + 6 = (-5)^2 + 5 \times (-5) + 6 = 25 - 25 + 6 = 6$

Note : Equations and identities are different. Equations are only true for particular value of variable but identities are true for all values of variable.

Thus, L.H.S.= R.H.S. also for $x = -5$. We find that for any value of x, L.H.S. = R.H.S.. Such an equality which is true for every value of the variable in it, is called an Identity.

So, we can define an identity as follows : "An unconditional equation, which is true for all values of its variable is known as identity."

An equation is true for only certain values of the variable in it. It is not true for all values of the variable.

For example, $b^2 + 3b + 2 = 132$.

It is true for $b = 10$, but it is not true for $b = -5$ or for $b = 0$.

12. STANDARD IDENTITIES

Identity I : $(a + b)^2 = a^2 + 2ab + b^2$

Proof : We have, $(a + b)^2 = (a + b)(a + b) = a(a + b) + b(a + b) = a^2 + ab + ba + b^2$

$\Rightarrow (a + b)^2 = a^2 + 2ab + b^2$

Identity II : $(a - b)^2 = a^2 - 2ab + b^2$

Proof : We have, $(a - b)^2 = (a - b)(a - b) = a(a - b) - b(a - b) = a^2 - ab - ba + b^2$

$\Rightarrow (a - b)^2 = a^2 - ab - ab + b^2 \Rightarrow (a - b)^2 = a^2 - 2ab + b^2$

Identity III : $(a + b)(a - b) = a^2 - b^2$

Proof : We have, $(a + b)(a - b) = a(a - b) + b(a - b) = a^2 - ab + ba - b^2$

$\Rightarrow (a + b)(a - b) = a^2 - b^2$

Example 15

Simplify the following :

(i) $(2a + 3b)^2$ (ii) $(4y - 7)^2$

Solution :

(i) $(2a + 3b)^2 = (2a)^2 + (3b)^2 + 2(2a) \times (3b)$

$= 4a^2 + 9b^2 + 12ab$

(ii) $(4y - 7)^2 = (4y)^2 - 2(4y) \times 7 + (7)^2$

$= 16y^2 - 56y + 49.$

Example 16

Evaluate the following, using identities :

(i) $(105)^2$ (ii) $(47)^2$ (iii) (8.3×7.7)

Solution :

We have,

(i) $(105)^2 = (100 + 5)^2 = (100)^2 + (5)^2 + 2 \times 100 \times 5$

[using $(a + b)^2 = a^2 + b^2 + 2ab$]

$= 10000 + 25 + 1000 = 11025.$

(ii) $(47)^2 = (50 - 3)^2 = (50)^2 + (3)^2 - 2 \times 50 \times 3$

[Using $(a - b)^2 = a^2 + b^2 - 2ab$]

$= 2500 + 9 - 300 = 2209.$

(iii) $(8.3 \times 7.7) = (8 + 0.3)(8 - 0.3) = (8)^2 - (0.3)^2$ [Using $(a + b)(a - b) = (a^2 - b^2)$] $= 64 - 0.09 = 63.91.$

Example 17

Find the value of expression :

$(81x^2 + 16y^2 - 72xy)$, when $x = \frac{2}{3}$ and $y = \frac{3}{4}$.

Solution :

We have,

$(81x^2 + 16y^2 - 72xy) = (9x)^2 + (4y)^2 - 2 \times 9x \times 4y = (9x - 4y)^2$

[Using $a^2 + b^2 - 2ab = (a - b)^2$]

$= \left(9\times\frac{2}{3} - 4\times\frac{3}{4}\right)^2$, when $x = \frac{2}{3}$ and $y = \frac{3}{4}$ $= (6 - 3)^2 = (3)^2 = 9.$

13. A USEFUL IDENTITY

We have, $(x + a)(x + b) = x(x + b) + a(x + b) = x^2 + xb + ax + ab = x^2 + bx + ax + ab$

$= x^2 + ax + bx + ab = x^2 + (a + b)x + ab$

Thus, we have the following identity :

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Using this identity, we can derive the results :

(i) $(x + a)(x - b) = x^2 + (a - b)x - ab$

(ii) $(x - a)(x + b) = x^2 + (b - a)x - ab$

(iii) $(x - a)(x - b) = x^2 - (a + b)x + ab$

Example 18

Find the following products :

(i) $(x + 4)(x + 7)$ (ii) $(x - 11)(x + 4)$ (iii) $(x - 3)(x - 2)$ (iv) $\left(y^2 + \frac{5}{7}\right)\left(y^2 - \frac{14}{5}\right)$

Solution :

Using the identity :

$(x + a)(x + b) = x^2 + (a + b)x + ab,$

We have,

(i) $(x + 4)(x + 7) = x^2 + (4 + 7)x + 4 \times 7$

$= x^2 + 11x + 28$

(ii) $(x - 11)(x + 4) = \{x + (-11)\}(x + 4)$

$= x^2 + \{(-11) + 4\}x + (-11) \times 4 = x^2 - 7x - 44$

(iii) $(x - 3)(x - 2) = \{x + (-3)\}\{x + (-2)\}$

$= x^2 + \{(-3) + (-2)\}x + (-3) \times (-2) = x^2 - 5x + 6$

(iv) $\left(y^2 + \frac{5}{7}\right)\left(y^2 - \frac{14}{5}\right) = \left(y^2 + \frac{5}{7}\right)\left\{y^2 + \left(\frac{-14}{5}\right)\right\}$

$= y^4 + \left\{\frac{5}{7} + \left(\frac{-14}{5}\right)\right\}y^2 + \frac{5}{7}\left(\frac{-14}{5}\right)$

$= y^4 + \left(\frac{-73}{35}\right)y^2 - 2 = y^4 - \frac{73}{35}y^2 - 2$.

SE. 1

Identify the terms, their coefficients for each of the following expressions :

(i) $7x^2yz - 5xy$ (ii) $x^2 - x - 1$

(iii) $9 - ab + bc - ca$.

Ans.

No.	Terms	Coefficients
(i)	$7x^2yz$	7
	$-5xy$	-5
(ii)	x^2	1
	$-x$	-1
	-1	-1
(iii)	$-ab$	-1
	bc	1
	$-ca$	-1
	9	9

SE. 2

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any category ?

(i) 50 (ii) $y - y^2 - y^3 + y^4$

(iii) $7 + a + 5b$ (iv) $2b - 3b^2$

Ans. Monomials : 50

Binomials : $2b - 3b^2$

Trinomials : $7 + a + 5b$

None of these : $y - y^2 - y^3 + y^4$

SE. 3

Add $5x^2 + 3x - 8$, $4x + 7 - 2x^2$ and $6 - 5x + 4x^2$.

Ans. Writing the given expressions in descending powers of x in the form of rows with like terms below each other and adding columnwise, we get

$$\begin{array}{r} 5x^2 + 3x - 8 \\ -2x^2 + 4x + 7 \\ 4x^2 - 5x + 6 \\ \hline 7x^2 + 2x + 5 \\ \hline \end{array}$$

SE. 4

Subtract. $\frac{3}{2}x^2y + \frac{4}{5}y - \frac{1}{2}x^2yz$ from

$\frac{12}{5}x^2yz - \frac{3}{5}xyz + \frac{2}{3}x^2y$.

Ans. We have,

$$\begin{array}{r} \frac{12}{5}x^2yz - \frac{3}{5}xyz + \frac{2}{3}x^2y \\ -\frac{1}{2}x^2yz \qquad\quad + \frac{3}{2}x^2y + \frac{4}{5}y \\ (+) \qquad\quad (-) \qquad\quad (-) \\ \hline \frac{29}{10}x^2yz - \frac{3}{5}xyz - \frac{5}{6}x^2y - \frac{4}{5}y \\ \hline \end{array}$$

SE. 5

Find each of the following products :

(i) $(-2x^2) \times (7a^2x^7) \times (6a^5x^5)$

(ii) $(4s^2t) \times (3s^3t^3) \times (2st^4) \times (-2)$

(iii) $(5x^6) \times (-10xy^4) \times (-2x^6y^6) \times (10xy)$.

Ans. (i) We have, $(-2x^2) \times (7a^2x^7) \times (6a^5x^5)$

$= (-2 \times 7 \times 6) \times (x^2 \times x^7 \times x^5 \times a^2 \times a^5)$

$= -84x^{14}a^7$

(ii) We have, $(4s^2t) \times (3s^3t^3) \times (2st^4) \times (-2)$

$= (4 \times 3 \times 2 \times (-2)) \times (s^2 \times s^3 \times s \times t \times t^3 \times t^4)$

$= -48s^6t^8$

(iii) We have, $(5x^6) \times (-10xy^4) \times (-2x^6y^6) \times (10xy)$

$= (5 \times (-10) \times (-2) \times 10) \times (x^6 \times x \times x^6 \times x \times y^4 \times y^6 \times y) = 1000x^{14}y^{11}$

SE. 6

Find the value of $(5a^6) \times (-10ab^2) \times (-2.1a^2b^3)$ for $a = 1$ and $b = \frac{1}{2}$.

Ans. We have, $(5a^6) \times (-10ab^2) \times (-2.1a^2b^3)$

$= (5 \times (-10) \times (-2.1)) \times (a^6 \times a \times a^2 \times b^2 \times b^3)$

$$= \left(5 \times (-10) \times -\frac{21}{10}\right) \times (a^6 \times a \times a^2 \times b^2 \times b^3) = 105a^9b^5$$

Putting $a = 1$ and $b = \frac{1}{2}$, we have

$$105a^9b^5 = 105 \times (1)^9 \times \left(\frac{1}{2}\right)^5$$

$$= 105 \times 1 \times \frac{1}{32} = \frac{105}{32}.$$

SE. 7

Evaluate each of the following when $x = 2$, $y = -1$

(i) $(2xy) \times \left(\frac{x^2y}{4}\right) \times (x^2) \times (y^2)$

(ii) $\left(\frac{3}{5}x^2y\right) \times \left(\frac{-15}{4}xy^2\right) \times \left(\frac{7}{9}x^2y^2\right)$.

Ans. (i) $(2xy) \times \left(\frac{x^2y}{4}\right) \times (x^2) \times (y^2)$

$$= \frac{1}{2}(x \times x^2 \times x^2) \times (y \times y \times y^2) = \frac{1}{2}x^5y^4 \quad ....(i)$$

Now, putting $x = 2$ and $y = -1$ in (i), we get

$$\frac{1}{2}(2)^5(-1)^4 = 16$$

(ii) $\left(\frac{3}{5}x^2y\right) \times \left(\frac{-15}{4}xy^2\right) \times \left(\frac{7}{9}x^2y^2\right)$

$$= \left[\frac{3}{5} \times \left(\frac{-15}{4}\right) \times \frac{7}{9}\right](x^2 \times x \times x^2) \times (y \times y^2 \times y^2)$$

$$= \frac{-7}{4}x^5y^5 \quad ....(i)$$

Putting $x = 2$ and $y = -1$ in (i), we get

$$\frac{-7}{4}(2^5)(-1)^5 = \frac{7}{4} \times 32 = 56$$

SE. 8

Find the product of $-3y(xy + y^2)$ and find the value for $x = 4$ and $y = 5$.

Ans. $-3y(xy + y^2)$

$= -3y \times xy + (-3) \times y \times y^2$

$= -3y^2x - 3y^3 \quad ....(i)$

Now putting $x = 4$ and $y = 5$ in (i), we get

$= -3(5)^2 \times 4 - 3 \times (5)^3 = -300 - 375 = -675.$

SE. 9

Simplify :

(i) $4ab(a - b) - 6a^2(b - b^2) - 3b^2(2a^2 - a) + 2ab(b - a)$

(ii) $a(b - c) - b(c - a) - c(a - b)$.

Ans. (i) $4ab(a - b) - 6a^2(b - b^2) - 3b^2(2a^2 - a) + 2ab(b - a)$

$= 4a^2b - 4ab^2 - 6a^2b + 6a^2b^2 - 6b^2a^2 + 3ab^2 + 2ab^2 - 2a^2b$

$= 4a^2b - 6a^2b - 2a^2b - 4ab^2 + 3ab^2 + 2ab^2 + 6a^2b^2 - 6a^2b^2$

$= -4a^2b + ab^2$

(ii) $a(b - c) - b(c - a) - c(a - b)$

$= ab - ac - bc + ab - ac + cb$

$= ab + ab - ac - ac - bc + bc = 2ab - 2ac$

SE. 10

Multiply $(3x^2 + y^2)$ by $(x^2 + 2y^2)$.

Ans. We have, $(3x^2 + y^2)(x^2 + 2y^2)$

$= 3x^2 \times (x^2 + 2yz) + y^2 \times (x^2 + 2y^2)$

$= 3x^2 \times x^2 + 3x^2 \times 2y^2 + y^2 \times x^2 + y^2 \times 2y^2$

$= 3x^4 + 6x^2y^2 + x^2y^2 + 2y^4$

$= 3x^4 + 7x^2y^2 + 2y^4$

SE. 11

Multiply $(2x^2 - 3x + 5)$ by $(5x + 2)$.

Ans. We have, $(2x^2 - 3x + 5) \times (5x + 2)$

$= (2x^2 - 3x + 5) \times 5x + (2x^2 - 3x + 5) \times 2$

$= (10x^3 - 15x^2 + 25x) + (4x^2 - 6x + 10)$

$= 10x^3 - 11x^2 + 19x + 10$ (On solving like terms)

SE. 12

Write down the squares of each of the following binomials :

(i) $\left(x + \frac{a}{2}\right)$ (ii) $\left(5b - \frac{1}{2}\right)$

(iii) $\left(y + \frac{y^2}{2}\right)$

Ans. (i) We have, $\left(x + \frac{a}{2}\right)^2 = x^2 + 2 \times x \times \frac{a}{2} \times \left(\frac{a}{2}\right)^2$

[Using $(a + b)^2 = a^2 + 2ab + b^2$] ....(i)

$x^2 + xa + \frac{a^2}{4}$

(ii) We have,

$\left(5b - \frac{1}{2}\right)^2 = (5b)^2 - 2 \times 5b \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$

[Using $(a - b)^2 = a^2 - 2ab + b^2$]

$= 25b^2 - 5b + \frac{1}{4}$

(iii) We have,

$\left(y + \frac{y^2}{2}\right)^2 = y^2 + 2 \times y \times \frac{y^2}{2} + \left(\frac{y^2}{2}\right)^2$

$= y^2 + y^3 + \frac{y^4}{4}$

SE. 13

Find the following products :

(i) $\left(\frac{4}{3}x^2 + 3\right)\left(\frac{4}{3}x^2 + 3\right)$

(ii) $\left(x + \frac{1}{5}\right)(x + 5)$

(iii) $(3x^2 - 4xy)(3x^2 - 3xy)$

(iv) $(p^2 + 16)\left(p^2 - \frac{1}{4}\right)$

Ans. (i) We have, $\left(\frac{4}{3}x^2 + 3\right)\left(\frac{4}{3}x^2 + 3\right) = \left(\frac{4}{3}x^2 + 3\right)^2$

$= \left(\frac{4}{3}x^2\right)^2 + 2 \times \frac{4}{3}x^2 \times 3 + (3)^2$

[Using $(a + b)^2 = a^2 + 2ab + b^2$]

$= \frac{16}{9}x^4 + 8x^2 + 9$

(ii) $\left(x + \frac{1}{5}\right)(x + 5) = x^2 + \left(\frac{1}{5} + 5\right)x + \frac{1}{5} \times 5$

$= x^2 + \frac{26}{5}x + 1$

(iii) $(3x^2 - 4xy)(3x^2 - 3xy)$

$= 3x^2(3x^2 - 3xy) - 4xy(3x^2 - 3xy)$

$= (3 \times 3) \times (x^2 \times x^2) - (3 \times 3)(x^2 \times x) \times y$

$- (4 \times 3)(x \times x^2) \times y + (4 \times 3)(x \times x) \times (y \times y)$

$= 9x^4 - 9x^3y - 12x^3y + 12x^2y^2$

$= 9x^4 - 21x^3y + 12x^2y^2$

(iv) $(p^2 + 16)\left(p^2 - \frac{1}{4}\right) = (p^2 + 16)\left\{p^2 + \left(-\frac{1}{4}\right)\right\}$

$= (p^2)^2 + \left(16 - \frac{1}{4}\right)p^2 + 16 \times \left(-\frac{1}{4}\right)$

$= p^4 + \frac{63}{4}p^2 - 4$

SE. 14

If $x-\frac{1}{x}=9$, find the value of $x^2+\frac{1}{x^2}$.

Ans. We have, $x-\frac{1}{x}=9$

On squaring both sides, we get

$$\left(x-\frac{1}{x}\right)^2=9^2=81$$

$$\Rightarrow x^2-2\times x\times\frac{1}{x}+\left(\frac{1}{x}\right)^2=81$$

$$\Rightarrow x^2-2+\frac{1}{x^2}=81$$

$$\Rightarrow x^2+\frac{1}{x^2}=83$$

SE. 15

Evaluate the following by using the identity :

$(x + a)(x + b) = x^2 + (a + b)x + ab$.

(i) 109×107 (ii) 994×1006

Ans. (i) $109 \times 107 = (100 + 9) \times (100 + 7)$

$= (100)^2 + (9 + 7) \times 100 + 9 \times 7$

$= 10000 + 16 \times 100 + 63$

$= 10000 + 1600 + 63 = 11663$

(ii) $994 \times 1006 = (1000 - 6)(1000 + 6)$

$= \{1000 + (-6)\}(1000 + 6)$

$= (1000)^2 + (-6 + 6) \times 1000 + (-6) \times (6)$

$= 1000000 - 36 = 999964$

Space for Notes :

EXERCISE – 9.1

NS. 1

Identify the terms, their coefficients for each of the following expressions.

(i) $5xyz^2 - 3zy$ (ii) $1 + x + x^2$

(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$ (iv) $3 - pq + qr - rp$

(v) $\frac{x}{2} + \frac{y}{2} - xy$ (vi) $0.3a - 0.6ab + 0.5b$

Ans.

No.	Terms	Coefficients
(i)	$5xyz^2$	5
	$-3zy$	-3
(ii)	1	1
	x	1
	x^2	1
(iii)	$4x^2y^2$	4
	$-4x^2y^2z^2$	-4
	z^2	1
(iv)	3	3
	$-pq$	-1
	qr	1
	$-rp$	-1
(v)	$\frac{x}{2}$	$\frac{1}{2}$
	$\frac{y}{2}$	$\frac{1}{2}$
	$-xy$	-1
(vi)	$0.3a$	0.3
	$-0.6ab$	-0.6
	$0.5b$	0.5

NS. 2

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these categories ?

$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

Ans.

Mono – mials	Bino – mials	Trino – mials	None of these
1000 pqr	$x+y$ $2y-3y^2$ $4z-15z^2$ p^2q+pq^2 $2p+2q$	$7+y+5x$ $2y-3y^2+4y^3$ $5x-4y+3xy$	$x+x^2+x^3+x^4$ $ab+bc+cd+da$

NS. 3

Add the following :

(i) $ab - bc$, $bc - ca$, $ca - ab$

(ii) $a - b + ab$, $b - c + bc$, $c - a + ac$

(iii) $2p^2q^2 - 3pq + 4$, $5 + 7pq - 3p^2q^2$

(iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, $2lm + 2mn + 2nl$.

Ans. (i)

$$\begin{array}{rrr} ab & -bc & \\ & bc & -ca \\ -ab & & +ca \\ \hline 0 & +0 & +0 \end{array}$$

(ii)

$$\begin{array}{rrrrrr} a & -b & & +ab & & \\ & b & -c & & +bc & \\ -a & & +c & & & +ac \\ \hline & & & ab & +bc & +ac \end{array}$$

(iii)

$$\begin{array}{r} 2p^2q^2 - 3pq + 4 \\ -3p^2q^2 + 7pq + 5 \\ \hline -p^2q^2 + 4pq + 9 \end{array}$$

(iv)

$$\begin{array}{l} l^2 + m^2 \\ \quad m^2 + n^2 \\ l^2 \quad + n^2 \\ \qquad 2lm + 2mn + 2nl \\ \hline 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl \end{array}$$

NS. 4

(a) Subtract $4a – 7ab + 3b + 12$ from $12a – 9ab + 5b – 3$

(b) Subtract $3xy + 5yz – 7zx$ from $5xy – 2yz – 2zx + 10xyz$

(c) Subtract $4p^2q – 3pq + 5pq^2 – 8p + 7q – 10$ from $18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q$

Ans. (a)

$$\begin{array}{l} 12a - 9ab + 5b - 3 \\ 4a - 7ab + 3b + 12 \\ (-)\quad (+)\quad (-)\quad (-) \\ \hline 8a - 2ab + 2b - 15 \\ \hline \end{array}$$

(b)

$$\begin{array}{l} 5xy - 2yz - 2zx + 10xyz \\ 3xy + 5yz - 7zx \\ (-)\quad (-)\quad (+) \\ \hline 2xy - 7yz + 5xz + 10xyz \\ \hline \end{array}$$

(c)

$$\begin{array}{l} 5p^2q - 2pq^2 + 5pq - 11q - 3p + 18 \\ 4p^2q + 5pq^2 - 3pq + 7q - 8p - 10 \\ (-)\quad (-)\quad (+)\quad (-)\quad (+)\quad (+) \\ \hline p^2q - 7pq^2 + 8pq - 18q + 5p + 18 \\ \hline \end{array}$$

EXERCISE – 9.2

NS. 5

Find the product of the following pairs of monomials.

(i) $4, 7p$

(ii) $– 4p, 7p$

(iii) $– 4p, 7pq$

(iv) $4p^3, – 3p$

(v) $4p, 0$

Ans. (i) $4 \times 7p = 28p$

(ii) $– 4p \times 7p = – 28p^2$

(iii) $– 4p \times 7pq = – 28p^2q$

(iv) $4p^3 \times – 3p = – 12p^4$

(v) $4p \times 0 = 0$

NS. 6

Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

(p, q); $(10m, 5n)$; $(20x^2, 5y^2)$; $(4x, 3x^2)$; $(3mn, 4np)$

Ans.

Length	Breadth	Area
p	q	pq
$10m$	$5n$	$10m \times 5n = 50mn$
$20x^2$	$5y^2$	$20x^2 \times 5y^2 = 100x^2y^2$
$4x$	$3x^2$	$4x \times 3x^2 = 12x^3$
$3mn$	$4np$	$3mn \times 4np = 12mn^2p$

NS. 7

Complete the table of products.

First monomial→ / Second monomial↓	$2x$	$–5y$	$3x^2$	$–4xy$	$7x^2y$	$–9x^2y^2$
$2x$	$4x^2$	....	....	....	....	....
$–5y$	....	....	$–15x^2y$	....	....	....
$3x^2$	....	....	....	....	....	....
$–4xy$	....	....	....	....	....	....
$7x^2y$	....	....	....	....	....	....
$–9x^2y^2$	....	....	....	....	....	....

Ans.

First monomial→ / Second monomial↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

NS. 8

Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) $5a, 3a^2, 7a^4$ (ii) $2p, 4q, 8r$

(iii) $xy, 2x^2y, 2xy^2$ (iv) $a, 2b, 3c$

Ans.

No.	Length	Breadth	Height	Volume
(i)	$5a$	$3a^2$	$7a^4$	$5a \times 3a^2 \times 7a^4 = 105a^7$
(ii)	$2p$	$4q$	$8r$	$2p \times 4q \times 8r = 64pqr$
(iii)	xy	$2x^2y$	$2xy^2$	$xy \times 2x^2y \times 2xy^2 = 4x^4y^4$
(iv)	a	$2b$	$3c$	$a \times 2b \times 3c = 6abc$

NS. 9

Obtain the product of

(i) xy, yz, zx (ii) $a, -a^2, a^3$

(iii) $2, 4y, 8y^2, 16y^3$ (iv) $a, 2b, 3c, 6abc$

(v) $m, -mn, mnp$

Ans. (i) $xy \times yz \times zx = x^2y^2z^2$

(ii) $a \times -a^2 \times a^3 = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 1024y^6$

(iv) $a \times 2b \times 3c \times 6abc = 36a^2b^2c^2$

(v) $m \times (-mn) \times mnp = -m^3n^2p$

EXERCISE – 9.3

NS. 1

Carry out the multiplication of the expressions in each of the following pairs.

(i) $4p, q + r$ (ii) $ab, a - b$

(iii) $a + b, 7a^2b^2$ (iv) $a^2 - 9, 4a$

(v) $pq + qr + rp, 0$

Ans. (i) $4p \times (q + r) = 4p \times q + 4p \times r = 4pq + 4qr$

(ii) $ab \times (a - b) = a^2b - ab^2$

(iii) $(a + b) \times 7a^2b^2 = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2 - 9) \times (4a) = 4a^3 - 36a$

(v) $(pq + qr + rp) \times 0 = 0$

NS. 2

Complete the table.

No.	First expression	Second expression	Product
(i)	a	$b+c+d$	.....
(ii)	$x+y-5$	$5xy$	.....
(iii)	p	$6p^2-7p+5$	.....
(iv)	$4p^2q^2$	p^2-q^2	.....
(v)	$a+b+c$	abc	.....

Ans.

No.	First expression	Second expression	Product
(i)	a	$b+c+d$	$a\times(b+c+d)$ $=ab+ac+ad$
(ii)	$x+y-5$	$5xy$	$(x+y-5)\times5xy$ $=5x^2y+5xy^2-25xy$
(iii)	p	$6p^2-7p+5$	$p\times(6p^2-7p+5)$ $=6p^3-7p^2+5p$
(iv)	$4p^2q^2$	p^2-q^2	$4p^2q^2(p^2-q^2)$ $=4p^4q^2-4p^2q^4$
(v)	$a+b+c$	abc	$(a+b+c)(abc)$ $=a^2bc+ab^2c+abc^2$

NS. 3

Find the product.

(i) $(a^2)\times(2a^{22})\times(4a^{26})$

(ii) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)$

(iii) $\left(-\frac{10}{3}pq^3\right)\times\left(\frac{6}{5}p^3q\right)$

(iv) $x\times x^2\times x^3\times x^4$

Ans. (i) $(a^2)\times(2a^{22})\times(4a^{26})=8a^{50}$

(ii) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)=\frac{-3}{5}x^3y^3$

(iii) $\left(-\frac{10}{3}pq^3\right)\times\left(\frac{6}{5}p^3q\right)=-4p^4q^4$

(iv) $x\times x^2\times x^3\times x^4=x^{10}$.

NS. 4

(a) Simplify $3x(4x-5)+3$ and find its value for

(i) $x=3$ (ii) $x=\frac{1}{2}$

(b) Simplify $a(a^2+a+1)+5$ and find its value for

(i) $a=0$ (ii) $a=1$

(iii) $a=-1$

Ans. (a) $3x(4x-5)+3=12x^2-15x+3$

(i) For $x=3$, $12(3)^2-15(3)+3=108-45+3$ $=66$.

(ii) For $x=\frac{1}{2}$, $12x^2-15x+3=12\left(\frac{1}{2}\right)^2-15\left(\frac{1}{2}\right)$

$+3=3-\frac{15}{2}+3=\frac{-3}{2}$

(b) $a(a^2+a+1)+5=a^3+a^2+a+5$

(i) For $a=0$, $a^3+a^2+a+5=(0)^3+0^2+0+5$ $=5$

(ii) For $a=1$, $a^3+a^2+a+5=1^3+1^2+1+5=8$.

(iii) For $a=-1$, $(-1)^3+(-1)^2+(-1)+5$

$=-1+1-1+5=4$

NS. 5

(a) Add : $p(p-q)$, $q(q-r)$ and $r(r-p)$

(b) Add : $2x(z-x-y)$ and $2y(z-y-x)$

(c) Subtract : $3l(l-4m+5n)$ from $4l(10n-3m+2l)$

(d) Subtract : $3a(a+b+c)-2b(a-b+c)$ from $4c(-a+b+c)$

Ans. (a) First expression $=p(p-q)=p^2-pq$ ...(i)

Second expression $=q(q-r)=q^2-qr$ ...(ii)

Third expression $=r(r-p)=r^2-pr$ ...(iii)

Adding (i), (ii) and (iii), we get

$p^2-pq+q^2-qr+r^2-pr$

$=p^2+q^2+r^2-pq-qr-pr$.

(b) First expression $=2x(z-x-y)$

$=2xz-2x^2-2yx$

Second expression $=2y(z-y-x)$

$=2yz-2y^2-2xy$

Adding the two expressions,

$$\begin{array}{l} -2x^2 - 2yx + 2xz \\ \quad\ -2xy \quad -2y^2 + 2yz \\ \hline -2x^2 - 4xy + 2xz - 2y^2 + 2yz \end{array}$$

(c) First expression = $3l(l - 4m + 5n)$

$= 3l^2 - 12lm + 15ln$

Second expression = $4l(10n - 3m + 2l)$

$= 40ln - 12lm + 8l^2$

Subtracting the two expressions,

$$\begin{array}{l} 40ln - 12lm + 8l^2 \\ 15ln - 12lm + 3l^2 \\ (-) \quad (+) \quad (-) \\ \hline 25ln \qquad + 5l^2 \end{array}$$

(d) First expression = $3a(a + b + c) - 2b(a - b + c)$

$= 3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc$

$= 3a^2 + ab + 2b^2 + 3ac - 2bc$

Second expression = $4c(-a + b + c)$

$= -4ac + 4bc + 4c^2$

Subtracting the two expressions,

$$\begin{array}{l} -4ac + 4bc + 4c^2 \\ +3ac - 2bc \quad + 3a^2 + 2b^2 + ab \\ (-) \quad (+) \quad (-) \quad (-) \quad (-) \\ \hline -7ac + 6bc + 4c^2 - 3a^2 - 2b^2 - ab \\ \hline \end{array}$$

EXERCISE – 9.4

NS. 1

Multiply the binomials.

(i) $(2x + 5)$ and $(4x - 3)$

(ii) $(y - 8)$ and $(3y - 4)$

(iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$

(iv) $(a + 3b)$ and $(x + 5)$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 - \frac{2}{3}b^2\right)$

Ans. (i) $(2x + 5) \times (4x - 3) = 2x \times (4x - 3) + 5(4x - 3)$

$= 8x^2 - 6x + 20x - 15 = 8x^2 + 14x - 15.$

(ii) $(y - 8) \times (3y - 4) = y(3y - 4) - 8(3y - 4)$

$= 3y^2 - 4y - 24y + 32 = 3y^2 - 28y + 32.$

(iii) $(2.5l - 0.5m) \times (2.5l + 0.5m)$

$= 2.5l\,(2.5l + 0.5m) - 0.5\,m\,(2.5l + 0.5m)$

$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$

$= 6.25l^2 - 0.25m^2.$

(iv) $(a + 3b) \times (x + 5) = a(x + 5) + 3b(x + 5)$

$= ax + 5a + 3bx + 15\,b.$

(v) $(2pq + 3q^2) \times (3pq - 2q^2)$

$= 2pq\,(3pq - 2q^2) + 3q^2\,(3pq - 2q^2)$

$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p^2q^2 + 5pq^3 - 6c^4.$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right) \times 4\left(a^2 - \frac{2}{3}b^2\right)$

$\frac{3}{4}a^2\left(4a^2 - \frac{8}{3}b^2\right) + 3b^2\left(4a^2 - \frac{8}{3}b^2\right)$

$= 3a^4 - 2a^2b^2 + 12a^2b^2 - 8b^4$

$= 3a^4 + 10a^2b^2 - 8b^4$

NS. 2

Find the product.

(i) $(5 - 2x)(3 + x)$

(ii) $(x + 7y)(7x - y)$

(iii) $(a^2 + b)(a + b^2)$

(iv) $(p^2 - q^2)(2p + q)$

Ans. (i) $(5 - 2x) \times (3 + x) = 5(3 + x) - 2x(3 + x)$

$= 15 + 5x - 6x - 2x^2 = 15 - x - 2x^2.$

(ii) $(x + 7y)(7x - y) = x(7x - y) + 7y(7x - y)$

$= 7x^2 - xy + 49xy - 7y^2 = 7x^2 + 48xy - 7y^2.$

(iii) $(a^2 + b)(a + b^2) = a^2(a + b^2) + b(a + b^2)$

$= a^3 + a^2b^2 + ab + b^3$.

(iv) $(p^2 – q^2)(2p + q) = p^2(2p + q) – q^2(2p + q)$

$= 2p^3 + p^2q – 2q^2p – q^3$.

NS. 3

Simplify.

(i) $(x^2 – 5)(x + 5) + 25$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

(iii) $(t + s^2)(t^2 – s)$

(iv) $(a + b)(c – d) + (a – b)(c + d) + 2(ac + bd)$

(v) $(x + y)(2x + y) + (x + 2y)(x – y)$

(vi) $(x + y)(x^2 – xy + y^2)$

(vii) $(1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y$

(viii) $(a + b + c)(a + b – c)$

Ans. (i) $(x^2 – 5)(x + 5) + 25$

$= x^2(x + 5) – 5(x + 5) + 25$

$= x^3 + 5x^2 – 5x – 25 + 25 = x^3 + 5x^2 – 5x$.

(ii) $(a^2 + 5)(b^3 + 3) + 5$

$= a^2(b^3 + 3) + 5(b^3 + 3) + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 20$.

(iii) $(t + s^2)(t^2 – s) = t(t^2 – s) + s^2(t^2 – s)$

$= t^3 – ts + s^2t^2 – s^3$.

(iv) $(a + b)(c – d) + (a – b)(c + d) + 2(ac + bd)$

$= a(c – d) + b(c – d) + a(c + d) – b(c + d) + 2ac + 2bd$

$= ac – ad + bc – bd + ac + ad – bc – bd + 2ac + 2bd = 4ac$.

(v) $(x + y)(2x + y) + (x + 2y)(x – y)$

$= x(2x + y) + y(2x + y) + x(x – y) + 2y(x – y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 – xy + 2xy – 2y^2$

$= 3x^2 + 4xy – y^2$.

(vi) $(x + y)(x^2 – xy + y^2)$

$= x(x^2 – xy + y^2) + y(x^2 – xy + y^2)$

$= x^3 – x^2y + xy^2 + x^2y – xy^2 + y^3 = x^3 + y^3$.

(vii) $(1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y$

$= 1.5x(1.5x + 4y + 3) – 4y(1.5x + 4y + 3) – 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x – 6xy – 16y^2 – 12y – 4.5x + 12y$

$= 2.25x^2 – 16y^2$.

(viii) $(a + b + c)(a + b – c)$

$= a(a + b – c) + b(a + b – c) + c(a + b – c)$

$= a^2 + ab – ac + ab + b^2 – bc + ac + bc – c^2$

$= a^2 + 2ab + b^2 – c^2$.

EXERCISE – 9.5

NS. 1

Use a suitable identity to get each of the following products.

(i) $(x + 3)(x + 3)$

(ii) $(2y + 5)(2y + 5)$

(iii) $(2a – 7)(2a – 7)$

(iv) $(3a – \frac{1}{2})(3a – \frac{1}{2})$

(v) $(1.1m – 0.4)(1.1m + 0.4)$

(vi) $(a^2 + b^2)(– a^2 + b^2)$

(vii) $(6x – 7)(6x + 7)$

(viii) $(– a + c)(– a + c)$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)$

(x) $(7a – 9b)(7a – 9b)$

Ans. (i) $(x + 3)(x + 3) = (x + 3)^2$

$= (x)^2 + 2 . (x)(3) + (3)^2 = x^2 + 6x + 9$

[Using identity : $(a + b)^2 = a^2 + 2ab + b^2$]

(ii) $(2y + 5)(2y + 5) = (2y + 5)^2$

$= (2y)^2 + 2 . (2y)(5) + (5)^2 = 4y^2 + 20y + 25$

[Using identity : $(a + b)^2 = a^2 + 2ab + b^2$]

(iii) $(2a - 7)(2a - 7) = (2a - 7)^2$

$= (2a)^2 - 2(2a)(7) + (7)^2$

$= 4a^2 - 28a + 49$

[Using identity $(a - b)^2 = a^2 - 2ab + b^2$]

(iv) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right) = \left(3a - \frac{1}{2}\right)^2$

$= (3a)^2 - 2(3a)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$

$= 9a^2 - 3a + \frac{1}{4}$

[Using identity $(a - b)^2 = a^2 - 2ab + b^2$]

(v) $(1.1m - 0.4)(1.1m + 0.4) = (1.1m)^2 - (0.4)^2$

[Using identity $a^2 - b^2 = (a + b)(a - b)$]

$= 1.21m^2 - 0.16$

(vi) $(a^2 + b^2)(-a^2 + b^2) = (b^2 + a^2)(b^2 - a^2)$

$= (b^2)^2 - (a^2)^2$

$= b^4 - a^4$ [Using identity $(a^2 - b^2) = (a + b)(a - b)$]

(vii) $(6x - 7)(6x + 7) = (6x)^2 - (7)^2 = 36x^2 - 49.$

[Using identity $(a^2 - b^2) = (a + b)(a - b)$]

(viii) $(-a + c)(-a + c) = (c - a)(c - a) = (c - a)^2$

$= (c)^2 - 2(c)(a) + a^2 = c^2 - 2ac + a^2$

[Using identity $(a - b)^2 = a^2 - 2ab + b^2$]

(ix) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$

$= \left(\frac{x}{2} + \frac{3y}{4}\right)^2 = \left(\frac{x}{2}\right)^2 + 2\cdot\left(\frac{x}{2}\right)\left(\frac{3y}{4}\right) + \left(\frac{3y}{4}\right)^2$

$= \frac{x^2}{4} + \frac{3}{4}xy + \frac{9y^2}{16}$

[Using identity $(a + b)^2 = a^2 + 2ab + b^2$]

(x) $(7a - 9b)(7a - 96) = (7a - 9b)^2$

$= (7a)^2 - 2(7a)(9b) + (9b)^2 = 49a^2 - 126ab + 81b^2$ [Using identity $(a - b)^2 = a^2 - 2ab + b^2$]

NS. 2

Use the identity

$(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.

(i) $(x + 3)(x + 7)$

(ii) $(4x + 5)(4x + 1)$

(iii) $(4x - 5)(4x - 1)$

(iv) $(4x + 5)(4x - 1)$

(v) $(2x + 5y)(2x + 3y)$

(vi) $(2a^2 + 9)(2a^2 + 5)$

(vii) $(xyz - 4)(xyz - 2)$

Ans. (i) $(x + 3)(x + 7) = x^2 + (3 + 7)x + 3 \times 7$

$= x^2 + 10x + 21.$

(ii) $(4x + 5)(4x + 1) = (4x)^2 + (5 + 1)4x + 5 \times 1$

$= 16x^2 + 24x + 5.$

(iii) $(4x - 5)(4x - 1)$

$= [4x + (-5)][4x + (-1)]$

$= (4x)^2 + (-5 - 1)(4x) + (-5) \times (-1)$

$= 16x^2 - 24x + 5.$

(iv) $(4x + 5)(4x - 1)$

$= (4x + 5)[4x + (-1)]$

$= (4x)^2 + (5 - 1)(4x) + (5)\, x\, (-1)$

$= 16x^2 + 16x - 5.$

(v) $(2x + 5y)(2x + 3y)$

$= (2x)^2 + (5 + 3)y(2x) + (5y) \times (3y)$

$= 4x^2 + 16xy + 15y^2.$

(vi) $(2a^2 + 9)(2a^2 + 5)$

$= (2a^2)^2 + (9 + 5)(2a^2) + 9 \times 5$

$= 4a^4 + 28a^2 + 45.$

(vii) $(xyz - 4)(xyz - 2)$

$= [xyz + (-4)][xyz + (-2)]$

$= (xyz)^2 + (-4 - 2)(xyz) + (-4)(-2)$

$= x^2y^2z^2 - 6xyz + 8.$

NS. 3

Find the following squares by using the identities.

(i) $(b-7)^2$ (ii) $(xy+3z)^2$

(iii) $(6x^2-5y)^2$ (iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$

(v) $(0.4p-0.5q)^2$ (vi) $(2xy+5y)^2$

Ans. (i) $(b-7)^2 = (b)^2 - 2(b)(7) + (7)^2 = b^2 - 14b + 49$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

(ii) $(xy+3z)^2 = (xy)^2 + (3z)^2 + 2(xy)(3z)$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= x^2y^2 + 9z^2 + 6xyz$

(iii) $(6x^2-5y)^2 = (6x^2)^2 + (5y)^2 - 2(6x^2)(5y)$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 36x^4 + 25y^2 - 60x^2y$

(iv) $\left(\frac{2m}{3}+\frac{3n}{2}\right)^2$

$= \left(\frac{2m}{3}\right)^2 + \left(\frac{3n}{2}\right)^2 + 2\left(\frac{2m}{3}\right)\left(\frac{3n}{2}\right)$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= \frac{4m^2}{9} + \frac{9n^2}{4} + 2mn$

(v) $(0.4p-0.5q)^2$

$= (0.4p)^2 + (0.5q)^2 - 2(0.4p)(0.5q)$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 0.16p^2 + 0.25q^2 - 0.4pq$

(vi) $(2xy+5y)^2$

$= (2xy)^2 + (5y)^2 + 2(2xy)(5y)$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 4x^2y^2 + 25y^2 + 20xy^2$

NS. 4

Simplify :

(i) $(a^2-b^2)^2$

(ii) $(2x+5)^2 - (2x-5)^2$

(iii) $(7m-8n)^2 + (7m+8n)^2$

(iv) $(4m+5n)^2 + (5m+4n)^2$

(v) $(2.5p-1.5q)^2 - (1.5p-2.5q)^2$

(vi) $(ab+bc)^2 - 2ab^2c$

(vii) $(m^2-n^2m)^2 + 2m^3n^2$

Ans. (i) $(a^2-b^2)^2 = (a^2)^2 - 2a^2b^2 + (b^2)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= a^4 - 2a^2b^2 + b^4$

(ii) $(2x+5)^2 - (2x-5)^2$

$= [2x+5-(2x-5)][2x+5+2x-5]$

[Using identity $a^2 - b^2 = (a+b)(a-b)$]

$= (10)(4x) = 40x.$

(iii) $(7m-8n)^2 + (7m+8n)^2$

$= (7m)^2 - 2(7m)(8n) + (8n)^2 + (7m)^2 + 2(7m)(8n) + (8n)^2$

$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$

$= 98m^2 + 128n^2.$

(iv) $(4m+5n)^2 + (5m+4n)^2$

$= (4m)^2 + 2(4m)(5n) + (5n)^2 + (5m)^2 + 2(5m)(4n) + (4n)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$=16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$

$= 41m^2 + 41n^2 + 80mn.$

(v) $(2.5p-1.5q)^2 - (1.5p-2.5q)^2$

$= (2.5p)^2 - 2(2.5p)(1.5q) + (1.5q)^2$

$- [(1.5p)^2 - 2(1.5q)(2.5q) + (2.5q)^2]$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 6.25p^2 - 7.5pq + 2.25q^2 - [2.25p^2 - 7.5pq + 6.25q^2]$

$= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2$

$= 4p^2 - 4q^2$.

(vi) $(ab + bc)^2 - 2ab^2c$

$= (ab)^2 + 2(ab)(bc) + (bc)^2$

$- 2ab^2c = a^2b^2 + b^2c^2 - 2ab^2c + 2ab^2c$

$= a^2b^2 + b^2c^2$.

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

$= (m^2)^2 - 2(m^2)(n^2m) + (n^2m)^2 + 2m^3n^2$

[Using identity $(a - b)^2 = a^2 + b^2 - 2ab$]

$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$

$= m^4 + n^4m^2$.

NS. 5

Show that.

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$

(ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

(v) $(a - b)(a + b +)(b - c)(b + c)$

$+ (c - a)(c + a) = 0$

Ans. (i) To prove : $(3x + 7)^2 - 84x = (3x - 7)^2$

L.H.S. $= (3x + 7)^2 - 84x$

$= (3x)^2 + 2(3x)(7) + (7)^2 - 84x$

$= 9x^2 + 42x + 49 - 84x = 9x^2 - 42x + 49$

R.H.S. $= (3x - 7)^2 = (3x)^2 - 2(3x)(7) + (7)^2$

$= 9x^2 + 49 - 42x$

$\therefore$ L.H.S. = R.H.S.

(ii) To prove : $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

L.H.S. $= (9p - 5q)^2 + 180pq$

$= (9p)^2 - 2(9p)(5q) + (5q)^2 + 180pq$

$= 81p^2 - 90pq + 25q^2 + 180pq$

$= 81p^2 + 90pq + 25q^2$

R.H.S. $= (9p + 5q)^2 = (9p)^2 + 2(9p)(5q) + (5q)^2$

$= 81p^2 + 90pq + 25q^2$

$\therefore$ L.H.S. = R.H.S.

(iii) To prove :

$\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

L.H.S. $= \left(\frac{4m}{3} - \frac{3n}{4}\right)^2 + 2mn$

$= \left(\frac{4m}{3}\right)^2 - 2\left(\frac{4m}{3}\right)\left(\frac{3n}{4}\right) + \left(\frac{3n}{4}\right)^2 + 2mn$

[Using identity $(a - b)^2 = a^2 + b^2 - 2ab$]

$= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2 =$ R.H.S.

(iv) To prove : $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

L.H.S. $= (4pq + 3q)^2 - (4pq - 3q)^2$

$= (4pq)^2 + 2(4pq)(3q) + (3q)^2$

$- [(4pq)^2 - 2(4pq)(3q) + (3q)^2]$

$= 16p^2q^2 + 24pq^2 + 9q^2 - [16p^2q^2 - 24pq^2 + 9q^2]$

$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$

$= 48pq^2 =$ R.H.S.

(v) To prove : $(a - b)(a + b) + (b - c)(b + c)$

$+ (c - a)(c + a) = 0$

L.H.S. $= (a - b)(a + b) + (b - c)(b + c)$

$+ (c - a)(c + a)$

[Using identity $(a - b)(a + b) = a^2 - b^2$]

$= 0 =$ R.H.S.

NS. 6

Using identities, evaluate.

(i) 71^2	(ii) 99^2
(iii) 102^2	(iv) 998^2
(v) 5.2^2	(vi) 297×303
(vii) 78×82	(viii) 8.9^2
(ix) 1.05×9.5	

Ans. (i) $71^2 = (70+1)^2 = (70)^2 + 2(70)(1) + 1^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 4900 + 140 + 1 = 5041$

(ii) $992 = (100-1)^2 = (100)^2 - 2(100)(1) + 1^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 10000 - 200 + 1 = 9801$

(iii) $(102)^2 = (100+2)^2 = (100)^2 + 2(100)(2) + 2^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 10000 + 400 + 4 = 10404$

(iv) $(998)^2 = (1000-2)^2$

$= (1000)^2 - 2(1000)(2) + (2)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 1000000 - 4000 + 4 = 996004$

(v) $(5.2)^2 = (5+0.2)^2$

$= (5)^2 + 2(5)(0.2) + (0.2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 25 + 0.04 + 2 = 27.04$

(vi) $297 \times 303 = (300-3)(300+3)$

$= (300)^2 - 32$

[Using identity $(a^2 - b^2) = (a+b)(a-b)$]

$= 90000 - 9 = 89991$

(vii) $78 \times 82 = (80-2)(80+2)$

$= (80)^2 - (2)^2$

[Using identity $(a+b)(a-b) = a^2 - b^2$]

$= 6400 - 4 = 6396$

(viii) $(8.9)^2 = (9.0 - 0.1)^2$

$= (9.0)^2 - 2(0.1)(9) + (0.1)^2$

[Using identity $(a-b)^2 = a^2 + b^2 - 2ab$]

$= 81 - 1.8 + 0.01 = 79.21$

(ix) 1.05×9.5

$$= \frac{105}{100} \times \frac{95}{10} = \frac{105 \times 95}{1000} = \frac{(100+5)(100-5)}{1000}$$

$$= \frac{(100)^2 - 5^2}{1000} = \frac{10000 - 25}{1000} = \frac{9975}{1000} = 9.975$$

[Using identity $(a^2 - b^2) = (a+b)(a-b)$]

NS. 7

Using $a^2 - b^2 = (a+b)(a-b)$, find

(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$ (iv) $12.1^2 - 7.9^2$

Ans. (i) $51^2 - 49^2 = (51-49)(51+49)$

$= (2)(100) = 200.$

(ii) $(1.02)^2 - (0.98)^2 = (1.02 - 0.98)(1.02 + 0.98)$

$= (0.04)(2) = 0.08.$

(iii) $153^2 - 147^2 = (153 + 147)(153 - 147)$

$= (300)(6) = 1800.$

(iv) $(12.1)^2 - (7.9)^2 = (12.1 + 7.9)(12.1 - 7.9)$

$= (20.0)(4.2) = 84$

NS. 8

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$, find

(i) 103×104 (ii) 5.1×5.2

(iii) 103×98 (iv) 9.7×9.8

Ans. (i) $103 \times 104 = (100+3)(100+4)$

$= (100)^2 + (3+4)\,100 + 3 \times 4$

$= 10000 + 700 + 12 = 10712$

(ii) $5.1 \times 5.2 = (5+0.1)(5+0.2)$

$= (5)^2 + (0.1+0.2) \times 5 + (0.1)(0.2)$

$= 25 + 1.5 + 0.02 = 26.52$

(iii) $103 \times 98 = (100+3)(100-2)$

$= (100+3)[100 + (-2)]$

$= (100)^2 + (3-2) \times 100 + (3)(-2)$

$= 10000 + 100 - 6 = 10094$

(iv) $9.7 \times 9.8 = (10-0.3)(10-0.2)$

$= [10 + (-0.3)][10 + (-0.2)]$

$= (10)^2 + (-0.3 - 0.2) \times 10 + (-0.3)(-0.2)$

$= 100 - 5 + 0.06 = 95.06$

EXERCISE – I

ONLY ONE CORRECT TYPE

1. If $a-\frac{1}{a}=2$, then $a^2+\frac{1}{a^2}$ will be equal to
(A) 6 (B) 0
(C) 4 (D) 2

2. $a^2-\frac{1}{a^2}$ is same as
(A) $\left(a+\frac{1}{a}\right)(a+1)$ (B) $\left(a-\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$
(C) $\left(a-\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$ (D) $\left(a+\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$

3. The value of $16.1^2 - 8.9^2$ is
(A) 160 (B) 180
(C) 170 (D) 175

4. The value of $(x + 3y)^2 + (x - 3y)^2$ is
(A) $2x^2 + 18y^2$
(B) $2x^2 - 18y^2$
(C) $2x^2 + 18y^2 - 12xy$
(D) $2x^2 + 9y^2$

5. Add $6x + 4y - z + 3$, $2y - 3z + 4$, $y - 7x + 2z - 1$ and $2z - 5x - 6$.
(A) $4x - 6y + 2$ (B) $-3x + 14y - 3z + 2$
(C) $-6x + 7y$ (D) $-6x + 6y + z - 4$

6. The product obtained on multiplying $x^3 + 3x^2 - 5x$ by $(x - 1)$ is a
(A) Monomial (B) Binomial
(C) Trinomial (D) None of these

7. If $ab = 6$ and $a + b = 5$ then $a^2 + b^2 =$
(A) 11 (B) 12
(C) 13 (D) 16

8. What expression must be added to $1 - x + x^2 - 2x^3$ to get x^3?
(A) $x^3 - x^2 + x - 1$
(B) $-1 + x + x^2 - 3x^3$
(C) $3x^3 - x^2 + x - 1$
(D) None of these

9. Multiply $3x^3 - 7x + 8 - 2x^2$ by $4x^2 + 6$.
(A) $12x^5 + 8x^4 - 10x^3 + 20x^2 - 42x + 48$
(B) $12x^5 - 8x^4 - 10x^3 + 20x^2 - 42x + 48$
(C) $12x^5 - 8x^4 - 10x^3 + 20x^2 - 42x + 8$
(D) $12x^5 + 8x^4 + 10x^3 + 20x^2 - 42x + 48$

10. Simplify the expression :
$-[-2x - \{3y - (2x - 3y) + (3x - 2y)\} + 2x]$
(A) $x - 4y$ (B) $x + 4$
(C) $x + 4y$ (D) x

11. The value of $\frac{2m\times 5mn}{24n}$ is
(A) 4m (B) $\frac{1}{2}m^2$
(C) $\frac{5}{12}m^2$ (D) $\frac{1}{2}m$

12. The value of $\frac{24pq}{5p^2}\times\frac{25pr}{12q}$ is
(A) 10 pqr (B) 10r
(C) 10 pr (D) $\frac{10r}{p}$

13. In the term $\frac{-6}{7}a^4b^5c^3$ the coefficient of b^5 is
(A) $\frac{-6}{7}$ (B) $\frac{-6}{7}a^4$
(C) $\frac{-6}{7}a^4b^5c^3$ (D) $\frac{-6}{7}a^4c^3$

14. Which of the following is true ?

(A) $(a - b)^2 = a^2 - b^2 - 2ab$

(B) $(a + b)^2 = a^2 + b^2 - 2ab$

(C) $(x - a)(x + b) = x^2 - (a - b)x - ab$

(D) $(x + a)(x - b) = x^2 - (a + b)x + ab$

15. Find the value of the expression $(9x^2 + 25y^2 - 30xy)$, when $x = \frac{2}{3}$ and $y = \frac{3}{5}$.

(A) 1 (B) – 1

(C) 2 (D) – 2

16. Find the volume of a rectangular box whose length, $l = 2mn$ metre breadth, $b = 6nl$ metre and height, $h = 5lm$ metre.

(A) $30\ m^2n^2l^2$ cu. metre

(B) $60\ mnl$ cu. metre

(C) $120\ m^2n^2l^2$ cu. metre

(D) $60\ m^2n^2l^2$ cu. metre

17. Which of the following is a monomial ?

(A) $\frac{3}{a}$ (B) $5y^{-2}$

(C) $10p^2q^2$ (D) $-3l + \frac{1}{2}$

18. The product of $3xy$ and $-9x^2yz$ is a

(A) Monomial (B) Binomial

(C) Trinomial (D) None of these

19. What must be subtracted from $4p^2 - 2pq - 6q^2 - r + 5$ to get $-p^2 + pq - 8q^2 - 2r + 6$?

(A) $3p^2 - pq - 14q^2 - 3r + 111$

(B) $5p^2 - 3pq + 2q^2 + r - 1$

(C) $-5p^2 + 3pq - 2q^2 - r + 1$

(D) $-3p^2 + pq + 14q^2 + 3r - 11$

20. If $a + b = 11$ and $ab = 30$, then find the value of $(a^2 + b^2)$.

(A) 61 (B) 21

(C) 24 (D) 81

21. Find the value of x, if $4x = 23^2 - 17^2$.

(A) 10 (B) 40

(C) 50 (D) 60

22. Subtract $6x^2 + 7x - 5$ from the sum of $-2x^2 + 3x + 6$ and $4x^2 + 3x - 2$.

(A) $-8x^2 + 2x - 9$

(B) $4x^2 + 3x - 5$

(C) $8x^2 - 2x + 5$

(D) $-4x^2 - x + 9$

23. Evaluate : $\frac{(6732)^2 - (6720)^2}{13452}$

(A) 20 (B) 24

(C) 12 (D) 30

24. Find the number of terms in product of $(2x + 3y)$ and $(4x + 5y)$.

(A) 5 (B) 2

(C) 3 (D) 4

25. The length and the breadth of a rectangle are $(2x + 1)$ units and $(x + y + 1)$ units respectively. Find its area (in sq. units).

(A) $2x^2 + 2xy + 3x + y + 1$

(B) $2x^2 + xy + 2x + y + 1$

(C) $2x^2 + xy + x + 2y + 2$

(D) $2x^2 + 2xy + 2x + y + 2$

PARAGRAPH TYPE

Passage # I

$3l(l - 4m + 5n)$ and $4l(10n - 3m + 2l)$ are algebraic expressions.

26. The sum of two expressions is
(A) $11l^2 - 24lm + 55ln$
(B) $11l^2 + 55ln$
(C) $11l^2 - 24ln - 55lm$
(D) $11l^2 + 24lm + 55ln$

27. Subtracting first expression from second expression, we get
(A) $25lm + 5l^2$ (B) $25ln - 5l^2$
(C) $25ln + 5l^2$ (D) $25lm - 5l^2$

28. When $l = 1$, $m = 2$, $n = 3$ the result of the subtracted expression becomes
(A) 75 (B) 80
(C) 25 (D) 85

Passage # II

$(a - b)^2 = a^2 + b^2 - 2ab$.

29. The square of $(2x - 3y)$ is
(A) $4x^2 - 12xy + 29y^2$
(B) $4x^2 + 12xy + 9y^2$
(C) $4x^2 - 12xy + 9y^2$
(D) $4x^2 - 6xy + 9y^2$

30. If $x - \frac{1}{x} = 3$, then the value of $x^2 + \frac{1}{x^2}$ is
(A) 8 (B) 9
(C) 10 (D) 11

31. If $x^2 + \frac{1}{x^2} = 18$, then the value of $x - \frac{1}{x}$ is
(A) 5 (B) 4
(C) 3 (D) 16

MATCH THE COLUMN TYPE

In this section, each question has two matching lists. Choices for the correct combination of elements from Column I and Column II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Identify the number of terms given in Column – II for each of the following expression given in Column – I and then match the following.

Column – I	Column – II
(P) $4xyz - 3xyz$	(i) 2
(Q) $1 + y + y^3$	(ii) 1
(R) $4 - ab + bc - ca$	(iii) 4
(S) $0.7 + 0.8t$	(iv) 3

(A) (P) → (iv), (Q) → (ii), (R) → (i), (S) → (iii)
(B) (P) → (ii), (Q) → (iv), (R) → (iii), (S) → (i)
(C) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)
(D) (P) → (i), (Q) → (iv), (R) → (iii), (S) → (ii)

33. Match the following polynomials given in Column – I as monomial, binomial, trinomial and quadranomial given in Column – II.

Column – I	Column – II
(P) $y + 3$	(i) Trinomial
(Q) 300	(ii) Binomial
(R) $1 + y^2 + y^3 + y^4$	(iii) Monomial
(S) $8 + x + 5y$	(iv) Quadranomial

(A) (P) → (iv), (Q) → (ii), (R) → (i), (S) → (iii)
(B) (P) → (i), (Q) → (iv), (R) → (iii), (S) → (ii)
(C) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)
(D) (P) → (ii), (Q) → (iv), (R) → (i), (S) → (iii)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Identify the terms, their coefficients for each of the following expressions :

(i) $\frac{a}{2}+\frac{b}{2}-ab$ (ii) $3x^2y^2 - 5x^2y^2z^2 + z^2$

2. Add : $7x^2 - 4x + 5$, $-3x^2 + 2x - 1$ and $5x^2 - x + 9$.

3. Find the product : $(4p^2 + 5p + 7) \times 3p$.

4. Find the volume of the rectangular boxes with following length (l), breadth (b) and height (h) :

	l	b	h
(i)	2ax	3by	5cz
(ii)	m^2n	n^2p	p^2m

5. Add : $5m(3 - m)$ and $6m^2 - 13m$.

6. Subtract $3pq(p - q)$ from $2pq(p + q)$.

7. Multiply : $\{2m + (-n)\}$ by $\{-3m + (-5)\}$

8. If $x^2 + \frac{1}{x^2} = 53$, find the value of $x - \frac{1}{x}$.

9. Find the value of $5p^2 - 4p(-p + q)$.

10. Find the value of $(a + 2b)^2$.

SHORT ANSWER TYPE

1. What should be added to $4p^2 + 5p + 7$ to get $7p^2 + 2p + 9$?

2. Multiply : $\left(4x+\frac{3y}{5}\right)$ and $\left(3x-\frac{4y}{5}\right)$.

3. If $x + y = 14$ and $xy = 16$, find the value of $x^2 + y^2$.

4. Simplify the following using identities :

(i) $\frac{58^2 - 42^2}{16}$

(ii) $1.73 \times 1.73 - 0.27 \times 0.27$

5. Prove that : $(a + c)^2 - 4ac = (a - c)^2$.

LONG ANSWER TYPE

1. Simplify the following :

(i) $\frac{1}{3}(6x^2 + 15y^2)(6x^2 - 15y^2)$

(ii) $9x^4(2x^3 - 5x^4) + 5x^6(x^4 - 3x^2)$

2. Simplify the following :

(i) $(2x + 5)(3x - 2) + (x + 2)(2x - 3)$

(ii) $(3x + 2)(2x + 3) - (4x - 3)(2x - 1)$

3. $x^2 + \frac{1}{x^2} = 25$, find the value of each of the following :

(i) $x + \frac{1}{x}$ (ii) $x - \frac{1}{x}$

4. Find the value of x, if

(i) $6x = 23^2 - 17^2$ (ii) $4x = 98^2 - 88^2$

5. Find the product :

(i) $(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$

(ii) $\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)$

(iii) $\left(x-\frac{y}{5}-1\right)\left(x+\frac{y}{5}+1\right)$

TRUE / FALSE TYPE

1. $5x^2 - x^3 + x^2$ is a trinomial.
2. $x + x^{-1}$ is a polynomial.
3. Coefficient of ab in $-12a^2b$ is $-12a$.
4. Degree of $\frac{8x^2 - 3x}{2x}$ is one.
5. Degree of a polynomial cannot be negative.

NUMERICAL PROBLEMS

1. Multiply $\frac{7}{2}p^2q$ by 2p + 4q and evaluate the product for p = 1, q = – 1.

2. The value of the product $(4a^2 + 3b)(4a^2 + 3b)$ at a = 1, b = 2 is k. The value of k ÷ 2 is :

3. Find the value of $\frac{(997+496)^2-(997-496)^2}{997\times496}$.

4. If a + b = 11 and ab = 30, then find the value of (a – b).

5. The value of $\left(x+\frac{1}{x}\right)^2$ is k, when $x^2+\frac{1}{x^2}=27$.

Find the ones digit of k.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. If we divide $15(y + 3)(y^2 – 16)$ by $5(y^2 – y – 12)$ and multiply the result by 3(y + 4), we get ___ .

(A) $8y^2 + 16y + 124$ (B) $9y^2 + 72y + 144$

(C) $72y^2 + 9y + 144$ (D) $18y^2 + 4y + 164$

2. The value of $\frac{(469+174)^2-(469-174)^2}{469\times174}$ is.

(A) 2 (B) 4

(C) 689 (D) 1023

3. Evaluate : $\frac{-5}{3}x^2-\frac{3}{4}x^2-\frac{4}{3}x^2-\frac{1}{4}x^2+x^2$.

(A) $– 5x^2$ (B) $4x^2$

(C) $– 6x^2$ (D) $– 3x^2$

4. If $x^2 + y^2 = 47$ and $xy = \frac{19}{2}$, then the value of $2(x + y)^2 + (x – y)^2$ is :

(A) 160 (B) 270

(C) 226 (D) 86

5. If $x^2+\frac{1}{x^2}=\frac{17}{4},(x>0)$, then find the value of $\frac{2}{5}\left(x+\frac{1}{x}\right)+\left(x-\frac{1}{x}\right)$.

(A) $\frac{3}{2}$ (B) $\frac{25}{4}$

(C) $\frac{5}{2}$ (D) $\frac{9}{4}$

6. Simplify :

$$\frac{x^2-(y-z)^2}{(x+z)^2-y^2}+\frac{y^2-(x-z)^2}{(x+y)^2-z^2}+\frac{z^2-(x-y)^2}{(y+z)^2-x^2}$$

(A) – 1 (B) 0

(C) 1 (D) 2

7. If the perimeter of a triangle is (4y – 3x +2z) cm and two sides of the triangle measure (4x + 2y + z) cm and (3x + 7y – 2z) cm, find the length of the third side of the triangle.

(A) (– 10x – 5y + 3z) cm

(B) (4x + 13y + z) cm

(C) (10x + 5y – 3z) cm

(D) (4x – 13y – z) cm

8. If $p = x + \frac{1}{x}$, then the value of $p - \frac{1}{p}$ will be :

(A) 3x (B) $\frac{3}{x}$

(C) $\frac{x^4+x^2+1}{x^3+x}$ (D) $\frac{x^4+3x^2+1}{x^3+x}$

9. If x – y = 5, xy = 24 then the value of $x^2 + y^2$ will be :

(A) 23 (B) 73

(C) 65 (D) 74

10. A colony has (2x + y) buildings. Each building has (3x – 5y) flats and each flat has (x + 3y) rooms. How many rooms are there in the colony ?

(A) $6x^3 + 11x^2y - 26xy^2 - 15y^3$

(B) $6x^3 - 18x^2y + 21xy^2 - 5y^3$

(C) $5x^3 + 12x^2y - 8xy^2 - 7y^3$

(D) $8x^3 - 11x^2y - 6xy^2 + 2y^3$

11. Find the value of t if $\frac{1}{t+1}+\frac{1}{t+2}=\frac{7}{12}$.

(A) 0
(B) 1
(C) 2
(D) 3

12. Find the quotient obtained when $10x^3 - 27x^2 + 36x - 27$ is divided by $(2x - 3)$.

(A) $5x^2 - 8x + 9$
(B) $5x^2 + 8x + 9$
(C) $5x^2 - 6x + 9$
(D) $5x^2 + 6x - 9$

Space for Notes :

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	B	A	C	D	C	C	C	C	B	B	D	C	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	B	D	B	C	A	D	C	A	A	A	C	B	C	D
31	32	33												
B	B	C												

EXERCISE II

VERY SHORT ANSWER TYPE

2. $9x^2 - 3x + 13$ **3.** $12p^3 + 15p^2 + 21p$ **4.** (i) 30abcxyz, (ii) $m^3n^3p^3$

5. $2m + m^2$ **6.** $-p^2q + 5pq^2$ **7.** $-6m^2 - 10m + 3mn + 5n$

8. $\sqrt{51}$ **9.** $9p^2 - 4pq$ **10.** $a^2 + 4b^2 + 4ab$

SHORT ANSWER TYPE

1. $3p^2 - 3p + 2$ **4.** $12x^2 - \frac{7xy}{5} - \frac{12y^2}{25}$ **6.** 164 **7.** (i) 100, (ii) 2.92

LONG ANSWER TYPE

1. (i) $12x^4 - 75y^4$, (ii) $18x^7 - 60x^8 + 5x^{10}$ **2.** (i) $8x^2 + 12x - 16$, (ii) $-2x^2 + 23x + 3$

3. (i) $3\sqrt{3}$, (ii) $\sqrt{23}$ **4.** (i) 40, (ii) 465 **5.** (i) $x^8 - 1$, (ii) $x^8 - \frac{1}{x^8}$, (iii) $x^2 - \frac{y^2}{25} - 1 - \frac{2y}{5}$

TRUE / FALSE

1. F **2.** F **3.** T **4.** T **5.** T

NUMERICAL PROBLEMS

3. 7 **4.** 50 **6.** 4 **8.** 1 **10.** 9

ANALYTICAL PROBLEMS & BRAIN TEASER

1. B **2.** B **3.** D **4.** A **5.** C **6.** C

7. A **8.** C **9.** B **10.** A **11.** C **12.** C

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : ALGEBRAIC EXPRESSIONS AND IDENTITIES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Solutions			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

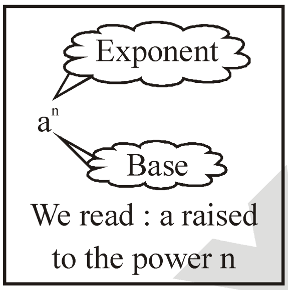
EXPONENTS AND POWERS

9

Concepts

INTRODUCTION

For any non-zero rational number ‘a’ and a natural number ‘n’, the product of its n identical factors, i.e. a × a × a × .... n times can be represented as a^n. It has two parts, a is the base and n is the exponent. It is read as “a raised to the power n”. Exponent and power are similar terms where power is a generic term for exponents.



Note : $a^{p/q}$ is the q^{th} root of a^p if p and q are positive integers. a^{-n} is the reciprocal of a^n.

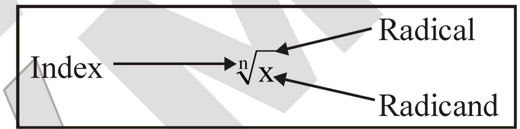
Positive exponents : For any non-zero rational number a and any positive integer n, a^n represents a positive exponent.

Example 1

4^3, where $4 \neq 0$ and $3 > 0$ or $(-6)^8$, where $-6 \neq 0$ and $8 > 0$

Fractional Exponents : A fractional exponent of the form, $x^{\frac{1}{n}}$ means to take n^{th} root of x. It is also called as radical or surd.

$\sqrt[n]{x} = x^{1/n}$ (It indicates a root)



Example 2

$\sqrt{x} = x^{1/2}$; $\sqrt[3]{5} = 5^{1/3}$

Zero exponents : Any non-zero rational number ‘a’ to the power zero is equal to one.

i.e. $a^0 = 1$ $(a \neq 0)$

$1^0 = 1, 3^0 = 1, (-2)^0 = 1$

(0^0 is “an indeterminate form”)

Decimal number system : We have learnt how to express decimal numbers in the expanded form using exponents.

For example, 3478.67 can be expressed in the following form :

$$3478.67 = 3 \times 1000 + 4 \times 100 + 7 \times 10 + 8 \times 1 + \frac{6}{10} + \frac{7}{100}$$

Using exponents, we have

$10000 = 10^4$, $1000 = 10^3$, $100 = 10^2$, $10 = 10^1$

$$1 = 10^0,\ 10^{-1} = \frac{1}{10}, 10^{-2} = \frac{1}{100}, 10^{-3} = \frac{1}{1000}, 10^{-4} = \frac{1}{10000}$$

$\therefore\ 3478.67 = 3 \times 10^3 + 4 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 + 6 \times 10^{-1} + 7 \times 10^{-2}$.

Thus, any decimal number can be written in expanded form by using integral exponents of 10.

1. NUMBERS WITH NEGATIVE EXPONENTS

For any non-zero rational number a and any negative integer n, a^n represents a negative exponent.

$$a^n = a^{-m} = \frac{1}{a^m} \text{ (where, n = – m and m is a positive integer)}$$

So, $a \neq 0$ because if $a = 0$, the term becomes undefined.

Example 3

3^{-2}, where $3 \neq 0$ and $-2 < 0$ or $(-3)^{-4}$, where $-3 \neq 0$ and $-4 < 0$

Example 4

Find the multiplicative inverse of the following.

(i) 3^{-4} (ii) 2^{-5} (iii) 10^{-100}

Solution :

(i) $3^{-4} = \frac{1}{3^4} \left(\because a^{-n} = \frac{1}{a^n}\right)$

$\therefore\ 3^4$ is the multiplicative inverse of 3^{-4}

(ii) $2^{-5} = \frac{1}{2^5} \left(\because a^{-n} = \frac{1}{a^n}\right)$

$\therefore\ 2^5$ is the multiplicative inverse of 2^{-5}

(iii) $10^{-100} = \frac{1}{10^{100}} \left(\because a^{-n} = \frac{1}{a^n}\right)$

$\therefore\ 10^{100}$ is the multiplicative inverse of 10^{-100}

Focus Point

Though positive or negative integral powers of positive or negative numbers can be found, it becomes difficult or impossible to find the value of some power of negative numbers (e.g. $(-2)^{\frac{1}{2}}$) in the set of real numbers. Hence, for a^n we assume $a > 0$, $a \neq 1$ and n is any real number.

2. LAWS OF EXPONENTS

If a and b are non-zero rational numbers, then

(i) $a^m \times a^n = a^{m+n}$

Example 5

Express $2^3 \times 64$ in the simplest form.

Solution :

We know that,

$64 = 2^6$

and according to the law of exponents,

$a^m \times a^n = a^{m+n}$

So, $2^3 \times 2^6 = 2^{3+6}$

$= 2^9$

Focus Point

Exponents never distribute across addition or subtraction. For $m \neq 1$,

(i) $(a \pm b)^m \neq a^m \pm b^m$

(ii) $\sqrt[m]{a \pm b} \neq \sqrt[m]{a} \pm \sqrt[m]{b}$

(iii) $\sqrt[m]{a^m \pm b^m} \neq a \pm b$

(ii) $\dfrac{a^m}{a^n} = a^{m-n}, m > n$

Example 6

Express $\dfrac{5^8}{125}$ in the simplest form.

Solution :

We know that,

$125 = 5^3$

and according to the law of exponents,

$\dfrac{a^m}{a^n} = a^{m-n}, m > n$

So, $\dfrac{5^8}{125} = \dfrac{5^8}{5^3} = 5^{8-3} = 5^5$

(iii) $(a^m)^n = a^{mn} = (a^n)^m$

Example 7

Express 8^2 in the simplest form.

Solution :

We know that,

$8 = 2^3$

and according to the law of exponents,

$(a^m)^n = a^{m \times n}$

So, $8^2 = (2^3)^2 = 2^{3 \times 2} = 2^6$

Focus Point

a^{m^n} means power of a is m^n. Do not confuse it with $(a^m)^n$ where power of a is $m \times n$.

Ex. $2^{2^3} = 2^8$ and $(2^2)^3 = 2^6$

(iv) $(ab)^n = a^n b^n$

Example 8

Express 15^4 in the simplest form.

Solution :

We know that,

$15 = 3 \times 5$

and according to the law of exponents,

$(ab)^n = a^n \times b^n$

So, $(15)^4 = (3 \times 5)^4$

$= 3^4 \times 5^4$

(v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example 9

Express $\left(\frac{6}{7}\right)^2$ in the simplest form.

Solution :

According to the law of exponents,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

So, $\left(\frac{6}{7}\right)^2 = \frac{6^2}{7^2} = \frac{36}{49}$

(vi) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Example 10

Express $\left(\frac{5}{8}\right)^{-3}$ in the simplest form.

Solution :

According to the laws of exponents,

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

So, $\left(\frac{5}{8}\right)^{-3} = \left(\frac{8}{5}\right)^3 = \frac{512}{125}$

(vii) $a^1 = a$ and $a^0 = 1$

Example 11

Find the value of $(3^0 - 2^0) + 3^1$.

Solution :

According to the laws of exponents,

$a^0 = 1$ and $a^1 = a$

So, $3^0 = 1$, $2^0 = 1$ and $3^1 = 3$

$\therefore (3^0 - 2^0) + 3^1 = (1 - 1) + 3$

$= 0 + 3 = 3$

Example 12

Express each of the following in the simplest form :

(i) $\sqrt[4]{5^2}$ (ii) $\sqrt[n]{x^m}$ (iii) $\sqrt[3]{64^{-4}}$

Solution :

(i) $\sqrt[4]{5^2} = \left(5^2\right)^{\frac{1}{4}}$

$= 5^{2\times\frac{1}{4}}$ $[(a^m)^n = a^{mn}]$

$= 5^{\frac{1}{2}}$

(ii) $\sqrt[n]{x^m} = \left(x^m\right)^{\frac{1}{n}}$

$= x^{\frac{m}{n}}$ $[(a^m)^n = a^{mn}]$

(iii) $\sqrt[3]{64^{-4}} = \left(64^{-4}\right)^{\frac{1}{3}}$

$= 64^{-4\times\frac{1}{3}}$ $[(a^m)^n = a^{mn}]$

$= \left(4^3\right)^{\frac{-4}{3}}$

$= 4^{-4}$ $[(a^m)^n = a^{mn}]$

$= \frac{1}{4^4}$ $\left[a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{256}$

Example 13

Express the following as a rational number of the form $\frac{p}{q}$:

(i) 4^{-3} (ii) $\left(\frac{2}{3}\right)^{-3}$ (iii) $\left(\frac{-2}{5}\right)^{-4}$

Solution :

(i) $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$ $\left[a^{-n} = \frac{1}{a^n}\right]$

(ii) $\left(\frac{2}{3}\right)^{-3} = \frac{1}{\left(\frac{2}{3}\right)^3}$ $\left[a^{-n} = \frac{1}{a^n}\right]$

$= \frac{1}{\frac{2^3}{3^3}} = \frac{3^3}{2^3} = \frac{27}{8}$ $\left[\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$

(iii) $\left(\frac{-2}{5}\right)^{-4} = \frac{1}{\left(\frac{-2}{5}\right)^4}$ $\left[a^{-n} = \frac{1}{a^n}\right]$

$= \frac{1}{\frac{(-2)^4}{5^4}} = \frac{5^4}{(-2)^4} = \frac{625}{16}$ $\left[\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$

Example 14

Express each of the following as a power of a rational number with positive exponent :

(i) $\left(\frac{1}{6}\right)^{-3}$ (ii) $4^{-3} \times 4^{-8}$ (iii) $\left(\frac{-1}{2}\right)^{-7} \times \left(\frac{-1}{2}\right)^{-3}$

Solution :

(i) We have,

$\left(\frac{1}{6}\right)^{-3} = \frac{1}{\left(\frac{1}{6}\right)^3}$

$= \frac{1}{\frac{1^3}{6^3}} = \frac{6^3}{1^3} = 6^3$

(ii) We have,

$4^{-3} \times 4^{-8} = 4^{-3+(-8)}$

$= 4^{-11}$

$= \left(\frac{1}{4}\right)^{11}$

(iii) $\left(\frac{-1}{2}\right)^{-7} \times \left(\frac{-1}{2}\right)^{-3}$

$= \left(\frac{-1}{2}\right)^{(-7)+(-3)}$

$= \left(\frac{-1}{2}\right)^{-10} = \left(\frac{2}{-1}\right)^{10}$

$= (-2)^{10} = (-1)^{10}\,(2)^{10} = 2^{10}$

Example 15

Find the value of k in each of the following :

(i) $\left(\sqrt[3]{8}\right)^{-\frac{1}{2}} = 2^k$ (ii) $\sqrt[4]{\sqrt[3]{x^2}} = x^k$ (iii) $\left(\sqrt{9}\right)^{-7}.\left(\sqrt{3}\right)^{-5} = 3^k$

Solution :

(i) $\left(\sqrt[3]{8}\right)^{-\frac{1}{2}} = \left(8^{\frac{1}{3}}\right)^{-\frac{1}{2}} = 8^{\frac{1}{3}\left(-\frac{1}{2}\right)}$

$= 8^{-\frac{1}{6}} = \left(2^3\right)^{-\frac{1}{6}} = 2^{3\left(-\frac{1}{6}\right)} = 2^{-\frac{1}{2}}$

$\therefore 2^k = 2^{-\frac{1}{2}} \Rightarrow k = -\frac{1}{2}$ [If the bases are same, then the exponents can be compared]

(ii) $\sqrt[4]{\sqrt[3]{x^2}} = \left(\sqrt[3]{x^2}\right)^{\frac{1}{4}}$

$\left\{\left(x^2\right)^{\frac{1}{3}}\right\}^{\frac{1}{4}} = \left(x^{2.\frac{1}{3}}\right)^{\frac{1}{4}} = \left(x^{\frac{2}{3}}\right)^{\frac{1}{4}}$

$= x^{\frac{2}{3}.\frac{1}{4}} = x^{\frac{1}{6}}$

$\therefore x^k = x^{\frac{1}{6}} \Rightarrow k = \frac{1}{6}$ [If the bases are same, then the exponents can be compared]

(iii) $\left(\sqrt{9}\right)^{-7}.\left(\sqrt{3}\right)^{-5}=\left(9^{\frac{1}{2}}\right)^{-7}.\left(3^{\frac{1}{2}}\right)^{-5}$

$=9^{\frac{1}{2}(-7)}\cdot 3^{\frac{1}{2}(-5)}=\left(3^2\right)^{-\frac{7}{2}}.(3)^{-\frac{5}{2}}$

$=3^{2\times\left(-\frac{7}{2}\right)}.3^{-\frac{5}{2}}=3^{-7}.3^{-\frac{5}{2}}=3^{-7-\frac{5}{2}}=3^{-\frac{19}{2}}$

$\therefore 3^k=3^{-\frac{19}{2}}\Rightarrow k=-\frac{19}{2}$ [If the bases are same, then the exponents can be compared]

Example 16

Find m such that $\left(\frac{2}{9}\right)^{3}\times\left(\frac{2}{9}\right)^{-6}=\left(\frac{2}{9}\right)^{2m-1}$

Solution :

We have,

$\left(\frac{2}{9}\right)^{3}\times\left(\frac{2}{9}\right)^{-6}=\left(\frac{2}{9}\right)^{2m-1}$

$\Rightarrow\left(\frac{2}{9}\right)^{3-6}=\left(\frac{2}{9}\right)^{2m-1}$

$\Rightarrow\left(\frac{2}{9}\right)^{-3}=\left(\frac{2}{9}\right)^{2m-1}$

$\Rightarrow 2m-1=-3$ [If the bases are same, then the exponents can be compared]

$\Rightarrow 2m=-3+1$

$\Rightarrow 2m=-2$

$\therefore\ m=-1$

3. USE OF EXPONENTS

3.1 TO EXPRESS VERY SMALL AND VERY LARGE NUMBERS IN STANDARD FORM

We can represent very large and very small numbers in standard form with the help of exponents.

For example : 160, 000, 000, 000 = 1.6×10^{11}

Now, let us express 0.000007 in standard form.

$$0.000007=\frac{7}{1000000}=\frac{7}{10^6}=7\times10^{-6}$$

$\therefore\ 0.000007=7\times10^{-6}$

Focus Point

1. The power of 10 is positive integer equal to the number of places the decimal point has been moved left, when the number is greater than 1.

2. The power of 10 is negative integer equal to the number of places the decimal point has been moved right, when the given number is less than 1.

3. When we have to add numbers which are in standard form, we first convert them into numbers with the same exponents.

Example 17

Express the following numbers in standard form :

(i) 0.000000456

(ii) 597300000

Solution :

(i) To express 0.000000456 in standard form , the decimal point is moved 7 places to the right i.e. 4.56×10^{-7}

(ii) $597300000 = 5.973 \times 10^{8}$

Example 18

Express in simplest form.

(i) 4.23×10^{5}

(ii) 6×10^{-5}

Solution :

(i) $4.23 \times 10^{5} = 4.23 \times 100000 = 423000$

(ii) $6 \times 10^{-5} = \dfrac{6}{10^{5}} = \dfrac{6}{100000} = 0.00006$

Example 19

Express the numbers in the following statements in standard form :

(i) The thickness of a normal paper is 0.07 mm

(ii) The diameter of wire on a computer chip is 0.000003 m

Solution :

(i) The thickness of a normal paper is 0.07 mm = 7×10^{-2} mm

(ii) The diameter of wire on a computer chip is 0.000003 m = 3×10^{-6} m

Example 20

The mass of earth is 5.97×10^{24} kg and mass of moon is 7.35×10^{22} kg what is the total mass ?

Solution :

Total mass $= 5.97 \times 10^{24}$ kg $+ 7.35 \times 10^{22}$ kg

$= 5.97 \times 100 \times 10^{22}$ kg $+ 7.35 \times 10^{22}$ kg

$= 597 \times 10^{22}$ kg $+ 7.35 \times 10^{22}$ kg

$= (597 + 7.35) \times 10^{22}$ kg

$= 604.35 \times 10^{22}$ kg $= 6.0435 \times 10^{24}$ kg

3.2 COMPARING VERY LARGE AND VERY SMALL NUMBERS

Example 21

The size of a red blood cell is 0.000007 m and the size of a plant cell is 0.00001276 m. Compare these two cells.

Solution :

Size of a red blood cell = 0.000007 m = 7×10^{-6} m

Size of a plant cell = 0.00001276 = 1.276×10^{-5} m

$$\therefore \frac{\text{Size of the red blood cell}}{\text{Size of the plant cell}} = \frac{7\times10^{-6}}{1.276\times10^{-5}}$$

$$= \frac{7\times10^{-6+5}}{1.276} = \frac{7\times10^{-1}}{1.276} = \frac{0.7}{1.276} \approx \frac{0.7}{1.3} \approx \frac{1}{2}$$

Hence, a red blood cell is approximately half the size of a plant cell.

Example 22

The diameter of the sun and the earth are 1.4×10^{9} metres and 1.275×10^{7} metres respectively. Compare the diameters of the two.

Solution :

$$\frac{\text{Diameter of the sun}}{\text{Diameter of the earth}} = \frac{1.4\times10^{9}}{1.275\times10^{7}} = \frac{1.4\times10^{2}\times10^{7}}{1.275\times10^{7}}$$

$$= \frac{1.4}{1.275}\times100 \approx \frac{1.4}{1.3}\times100 \approx 100 \qquad \left[1.275 \approx 1.3 \text{ and } \frac{1.4}{1.3} \approx 1\right]$$

So, the diameter of the sun is approximately 100 times the diameter of the earth.

SOLVED EXAMPLES

SE. 1

Express each of the following as a rational number of the form $\frac{p}{q}$.

(i) $\left(\frac{3}{8}\right)^{-2}\times\left(\frac{4}{5}\right)^{-3}$ (ii) $\left(\frac{-2}{7}\right)^{-4}\times\left(\frac{-7}{5}\right)^{2}$

Ans. (i) We have,

$$\left(\frac{3}{8}\right)^{-2}\times\left(\frac{4}{5}\right)^{-3}=\frac{1}{\left(\frac{3}{8}\right)^{2}}\times\frac{1}{\left(\frac{4}{5}\right)^{3}}\left[\because a^{-n}=\frac{1}{a^{n}}\right]$$

$$=\frac{1}{\frac{3^2}{8^2}}\times\frac{1}{\frac{4^3}{5^3}}\qquad\left[\because\left(\frac{a}{b}\right)^{n}=\left(\frac{a^n}{b^n}\right)\right]$$

$$=\frac{1}{\frac{9}{64}}\times\frac{1}{\frac{64}{125}}=\frac{64}{9}\times\frac{125}{64}=\frac{125}{9}$$

(ii) $\left(\frac{-2}{7}\right)^{-4}\times\left(\frac{-7}{5}\right)^{2}$

$$=\frac{1}{\left(\frac{-2}{7}\right)^{4}}\times\left(\frac{-7}{5}\right)^{2}=\frac{1}{\frac{(-2)^4}{7^4}}\times\frac{(-7)^2}{5^2}$$

$$=\frac{7^4}{(-2)^4}\times\frac{(-7)^2}{5^2}=\frac{7\times7\times7\times7}{16}\times\frac{(-7)\times(-7)}{25}$$

$$=\frac{7^6\times(-1)^2}{16\times25}=\frac{7^6}{400}=\frac{117649}{400}$$

SE. 2

Find the absolute value of :

(i) $\left(\frac{2}{-3}\right)^{4}$ (ii) $\left(\frac{-2}{7}\right)^{3}$

Ans. (i) $\left(\frac{2}{-3}\right)^{4}=\left|\left(\frac{2}{-3}\right)^{4}\right|=\left|\frac{2^4}{(-3)^4}\right|=\left|\frac{16}{81}\right|=\frac{16}{81}$

(ii) $\left(\frac{-2}{7}\right)^{3}=\left|\left(\frac{-2}{7}\right)^{3}\right|=\left|\frac{(-2)^3}{(7)^3}\right|=\left|\frac{-8}{343}\right|=\frac{8}{343}$

SE. 3

Find the reciprocal of :

(i) 3^4 (ii) $\left(\frac{2}{3}\right)^{6}$

Ans. (i) Reciprocal of $3^4=\frac{1}{3^4}=\frac{1}{81}$

(ii) Reciprocal of $\left(\frac{2}{3}\right)^{6}=\left(\frac{3}{2}\right)^{6}=\frac{729}{64}$

SE. 4

Evaluate : $\left\{\left(\frac{-3}{2}\right)^{2}\right\}^{-3}$

Ans. We have $\left\{\left(\frac{-3}{2}\right)^{2}\right\}^{-3}=\left(\frac{-3}{2}\right)^{2\times(-3)}$

$$=\left(\frac{-3}{2}\right)^{-6}=\left(\frac{2}{-3}\right)^{6}=\frac{2^6}{(-3)^6}=\frac{2^6}{(-1\times3)^6}$$

$$=\frac{2^6}{(-1)^6\times3^6}=\frac{2^6}{3^6}=\frac{64}{729}$$

SE. 5

Express each of the following as a rational number of the form $\frac{p}{q}$.

(i) $\left(2^{-1}+3^{-1}\right)^{2}$

(ii) $\left(2^{-1}-4^{-1}\right)^{2}$

(iii) $\left\{\left(\frac{4}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

Ans. We know that for any positive integer n and any rational number a, $a^{-n}=\frac{1}{a^n}$. Thus, we have

(i) $\left(2^{-1}+3^{-1}\right)^2=\left(\frac{1}{2}+\frac{1}{3}\right)^2=\left(\frac{3+2}{6}\right)^2$

$=\left(\frac{5}{6}\right)^2=\frac{5^2}{6^2}=\frac{25}{36}$

(ii) $\left(2^{-1}-4^{-1}\right)^2=\left(\frac{1}{2}-\frac{1}{4}\right)^2=\left(\frac{2-1}{4}\right)^2$

$=\left(\frac{1}{4}\right)^2=\frac{1^2}{4^2}=\frac{1}{16}$

(iii) $\left\{\left(\frac{4}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}=\left(\frac{1}{\frac{4}{3}}-\frac{1}{\frac{1}{4}}\right)^{-1}$

$\left(\frac{3}{4}-\frac{4}{1}\right)^{-1}=\left(\frac{3-16}{4}\right)^{-1}=\left(\frac{-13}{4}\right)^{-1}=\frac{-4}{13}$

SE. 6

Simplify :

(i) $\left(2^{-1}\times 5^{-1}\right)^{-1}\div 4^{-1}$

(ii) $\left(4^{-1}+8^{-1}\right)\div\left(\frac{2}{3}\right)^{-1}$

Ans. (i) $\left(2^{-1}\times 5^{-1}\right)^{-1}\div 4^{-1}$

$=\left(\frac{1}{2}\times\frac{1}{5}\right)^{-1}\div\left(\frac{4}{1}\right)^{-1}$

$=\left(\frac{1}{10}\right)^{-1}\div\left(\frac{1}{4}\right)=\left(10\div\frac{1}{4}\right)=(10\times 4)=40$

(ii) $\left(4^{-1}+8^{-1}\right)\div\left(\frac{2}{3}\right)^{-1}$

$=\left(\frac{1}{4}+\frac{1}{8}\right)\div\left(\frac{3}{2}\right)$

$=\frac{(2+1)}{8}\div\frac{3}{2}=\left(\frac{3}{8}\div\frac{3}{2}\right)=\left(\frac{3}{8}\times\frac{2}{3}\right)=\frac{1}{4}$

SE. 7

If $\frac{2^x}{1+2^x}=\frac{1}{4}$, then find the value of $\frac{8^x}{1+8^x}$.

Ans. $\frac{2^x}{1+2^x}=\frac{1}{4}$

$\Rightarrow 4.2^x = 1 + 2^x$ [By cross multiplication]

$\Rightarrow 4.2^x - 2^x = 1 \Rightarrow 2^x(4-1)=1 \Rightarrow 3.2^x = 1$

$\Rightarrow 2^x=\frac{1}{3}$ [Dividing by 3 on both sides]

$\therefore (2^x)^3=\left(\frac{1}{3}\right)^3=\frac{1}{27}$

$\therefore \frac{8^x}{1+8^x}=\frac{(2^3)^x}{1+(2^3)^x}=\frac{\frac{1}{27}}{1+\frac{1}{27}}=\frac{1}{28}$

SE. 8

Arrange in descending order of magnitude $\sqrt[3]{2},\sqrt[6]{3},\sqrt[9]{4}$.

Ans. L.C.M of 3, 6, 9 is 18.

$\therefore \sqrt[3]{2}=\sqrt[3\times 6]{2^6}=\sqrt[18]{64}$

[For comparison either make base same or exponent same]

$\sqrt[6]{3}=\sqrt[6\times 3]{3^3}=\sqrt[18]{27}$; $\sqrt[9]{4}=\sqrt[9\times 2]{4^2}=\sqrt[18]{16}$

Since, $\sqrt[18]{64}>\sqrt[18]{27}>\sqrt[18]{16}$

$\therefore \sqrt[3]{2}>\sqrt[6]{3}>\sqrt[9]{4}$

SE. 9

Which is greater of the two : 2^{300} or 3^{200} ?

Ans. For comparing two numbers of different base and different exponents, it is better that either the base or the exponent is made same for both numbers.

Hence, $2^{300} = (2^3)^{100} = 8^{100}$ (exponent is 100)

and $3^{200} = (3^2)^{100} = 9^{100}$ (exponent is 100)

From the above, it is evident that

$9^{100} > 8^{100} \Rightarrow 3^{200} > 2^{300}$

SE. 10

Solve for y if

$$\frac{\left(\frac{1}{9}\right)^{2y-1} \cdot (0.0081)^{1/3}}{\sqrt{243}} = \left(\frac{1}{3}\right)^{2y-5} \sqrt[3]{\frac{27^{y-1}}{10000}}$$

Ans. The given equation can be written as

$$\frac{(3^{-2})^{2y-1}.(3^4 \times 10^{-4})^{\frac{1}{3}}}{3^{\frac{5}{2}}} = \frac{3^{-(2y-5)}.3^{3\left(\frac{y-1}{3}\right)}}{10^{\frac{4}{3}}}$$

$$\frac{3^{-4y+2+\frac{4}{3}-\frac{5}{2}}}{10^{\frac{4}{3}}} = \frac{3^{-2y+5+y-1}}{10^{\frac{4}{3}}}$$

$$\Rightarrow 3^{-4y+\frac{5}{6}} = 3^{-y+4} \Rightarrow -4y + \frac{5}{6} = -y + 4$$

$$\Rightarrow -4y + y = 4 - \frac{5}{6} = \frac{24-5}{6} = \frac{19}{6}$$

$$\Rightarrow -3y = \frac{19}{6} \Rightarrow y = -\frac{19}{18}$$

SE. 11

By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplied so that the product is $\left(\frac{-5}{4}\right)^{-1}$?

Ans. Let the required number be x. Then,

$$x \times \left(\frac{1}{2}\right)^{-1} = \left(\frac{-5}{4}\right)^{-1} \Rightarrow x \times \frac{2}{1} = \frac{4}{-5}$$

$$\Rightarrow 2x = \frac{-4}{5} \Rightarrow x = \left(\frac{1}{2} \times \frac{-4}{5}\right) = \frac{-2}{5}$$

Hence, the required number is $\frac{-2}{5}$

SE. 12

Find the value of m for $5^{2m} \div 5^{-1} = 5^5$.

Ans. $5^{2m} \div 5^{-1} = 5^5 \Rightarrow 5^{2m-(-1)} = 5^5$

$\Rightarrow 5^{2m+1} = 5^5$

Since, both sides have the same base, therefore their exponents must be equal.

So, $2m + 1 = 5 \Rightarrow 2m = 5 - 1 \Rightarrow 2m = 4 \Rightarrow m = 2$

SE. 13

Simplify :

(i) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$ (ii) $\left(\frac{-2}{3}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3}$

(ii) $\left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{2}\right)^{-3}$ (ii) $\left(\frac{3}{7}\right)^{-2} \times \left(\frac{7}{6}\right)^{-3}$

Ans. (i) We have, $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-5}}{5^{-5}}$

$$= 5^{-7+5} \times 8^{-5+7} = 5^{-2} \times 8^2 = \frac{8^2}{5^2} = \frac{64}{25}$$

(ii) We have, $\left(\frac{-2}{3}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3} = \frac{(-2)^{-2}}{3^{-2}} \times \frac{4^{-3}}{5^{-3}}$

$$= \frac{3^2}{(-2)^2} \times \frac{5^3}{4^3} = \frac{9}{4} \times \frac{125}{64} = \frac{1125}{256}$$

(iii) We have, $\left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{2}\right)^{-3}$

$$=\left(\frac{3}{4}\right)^{-4} \times \frac{1}{\left(\frac{3}{2}\right)^{-3}} = \left(\frac{3}{4}\right)^{-4} \times \left(\frac{3}{2}\right)^{3} = \frac{3^{-4}}{4^{-4}} \times \frac{3^3}{2^3}$$

$$= \frac{3^{-4} \times 3^3}{(2^2)^{-4} \times 2^3} = \frac{3^{-4} \times 3^3}{(2)^{-8} \times 2^3}$$

$$= \frac{3^{-4+3}}{(2)^{-8+3}} = \frac{3^{-1}}{2^{-5}} = \frac{2^5}{3^1} = \frac{32}{3}$$

(iv) We have,

$$\left(\frac{3}{7}\right)^{-2} \times \left(\frac{7}{6}\right)^{-3} = \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{6^{-3}} = \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{(2 \times 3)^{-3}}$$

$$= \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{2^{-3} \times 3^{-3}} = \frac{3^{-2}}{3^{-3}} \times \frac{7^{-3}}{7^{-2}} \times \frac{1}{2^{-3}}$$

$= 3^{-2+3} \times 7^{-3+2} \times 2^3 = 3 \times 7^{-1} \times 2^3$

$$= 3 \times \frac{1}{7} \times 8 = \frac{24}{7}$$

SE. 14

Write the following numbers are in standard form.

(i) 0.4579 (ii) 0.000007

(iii) 0.0000021 (iv) 216000000

(v) 0.0000529×10^4 (vi) 9573×10^{-4}

Ans. (i) To express 0.4579 in standard form, the decimal point is moved through one place only to the right so that there is just one digit on the left of the decimal point.

$\therefore$ $0.4579 = 4.579 \times 10^{-1}$ is in standard form.

(ii) $0.000007 = 7 \times 10^{-6}$ [$\because$ The decimal point is moved 6 place to the right]

(iii) $0.0000021 = 2.1 \times 10^{-6}$ [$\because$ The decimal point is moved 6 places to the right]

(iv) $216000000 = 2.1 \times 10^8$ [$\because$ The decimal point is moved 8 places to the left]

(v) $0.0000529 \times 10^4 = 5.29 \times 10^{-5} \times 10^4$

[$\because$ The decimal point is moved 5 places to the right]

$= 5.29 \times 10^{-5+4} = 5.29 \times 10^{-1}$

(vi) $9573 \times 10^{-4} = 9.573 \times 10^3 \times 10^{-4}$ [$\because$ The decimal point is moved 3 places to the left]

$= 9.573 \times 10^{3+(-4)} = 9.573 \times 10^{-1}$

SE. 15

Write the following numbers in usual form.

(i) 1.785×10^7 (ii) 5.1×10^{-7}

Ans. (i) $1.785 \times 10^7 = 1.785 \times 10000000$

$= 17850000$

(ii) $5.1 \times 10^{-7} = \frac{5.1}{10^7} = \frac{5.1}{10000000} = 0.00000051$

SE. 16

Evaluate : (i) $\left(13^2 - 5^2\right)^{\frac{3}{2}}$

(ii) $\left(1^3 + 2^3 + 3^3 + 4^3\right)^{\frac{-3}{2}}$

Ans. (i) $\left(13^2 - 5^2\right)^{\frac{3}{2}} = [(13+5) \times (13-5)]^{\frac{3}{2}}$

$$= [18 \times 8]^{\frac{3}{2}} = [3 \times 3 \times 2 \times 2 \times 2 \times 2]^{\frac{3}{2}}$$

$$= \left[3^2 \times 2^4\right]^{\frac{3}{2}} = \left[3^2\right]^{\frac{3}{2}} \times \left[2^4\right]^{\frac{3}{2}}$$

$$= 3^{2 \times \frac{3}{2}} \times 2^{4 \times \frac{3}{2}} = 3^3 \times 2^6 = 1728$$

(ii) $\left(1^3 + 2^3 + 3^3 + 4^3\right)^{\frac{-3}{2}}$

$$= (1 + 8 + 27 + 64)^{\frac{-3}{2}}$$

$$= (100)^{-3/2} = \left(10^2\right)^{-3/2} = 10^{-3} = \frac{1}{1000}$$

EXERCISE – 12.1

NS. 1

Evaluate : (i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

Ans. (i) We have,

$$3^{-2}=\frac{1}{3^2}=\frac{1}{3\times3}=\frac{1}{9} \quad \left[\because a^{-m}=\frac{1}{a^m}\right]$$

(ii) We have,

$$(-4)^{-2}=\frac{1}{(-4)^2}=\frac{1}{(-4)\times(-4)}=\frac{1}{16}$$

$$\left[\because a^{-m}=\frac{1}{a^m}\right]$$

(iii) We have, $\left(\frac{1}{2}\right)^{-5}=\left(\frac{2}{1}\right)^5=\frac{2\times2\times2\times2\times2}{1\times1\times1\times1\times1}$

$$=\frac{32}{1}=32. \quad \left[\because \left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^m\right]$$

NS. 2

Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4\times\left(\frac{5}{3}\right)^4$ (iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Ans. (i) We have, $(-4)^5 \div (-4)^8$

$$= (-4)^{5-8} = (-4)^{-3} = \frac{1}{(-4)^3}$$

$$\left[\because a^m \div a^n = a^{m-n}, a^{-m}=\frac{1}{a^m}\right]$$

(ii) We have, $\left(\frac{1}{2^3}\right)^2=\frac{1}{\left(2^3\right)^2}=\frac{1}{2^6}$

$$\left[\because \left(a^m\right)^n=a^{mn}\right]$$

(iii) We have,

$$(-3)^4\times\left(\frac{5}{3}\right)^4=(-1)^4(3)^4\times\frac{(5)^4}{(3)^4}=\frac{3^4}{3^4}\cdot5^4$$

$= 3^{4-4}.5^4 = 3^0 \times 5^4 = 1 \times 5^4 = (5)^4$

$$\left[\because \frac{a^m}{a^n}=a^{m-n}\right]$$

(iv) We have, $(3^{-7} \div 3^{-10}) \times 3^{-5}$

$= [3^{-7-(-10)}] \times 3^{-5}$ $[\because a^m \div a^n = a^{m-n}]$

$= [3^{-7+10}] \times 3^{-5} = 3^3 \times 3^{-5} = 3^{3+(-5)}$

$[\because a^m \times a^n = a^{m+n}]$

$= 3^{3-5} = 3^{-2} = \frac{1}{3^2}$

(v) We have, $2^{-3} \times (-7)^{-3}$

$$=\frac{1}{2^3}\times\frac{1}{(-7)^3}=\frac{-1}{2^3\cdot7^3}=\frac{-1}{(14)^3}=\frac{1}{(-14)^3}$$

$[\because a^m \cdot b^m = (ab)^m]$

NS. 3

Find the value of

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$.

Ans. (i) We have, $(3^0 + 4^{-1}) \times 2^2 = \left(1+\frac{1}{4}\right)\times 2^2$

$$= \left(1+\frac{1}{4}\right)\times 4 = \left(\frac{4+1}{4}\right)\times 4 = \frac{5}{4}\times 4 = 5$$

(ii) We have, $(2^{-1} \times 4^{-1}) \div 2^{-2} = [2^{-1} \times (2^2)^{-1}] \div 2^{-2}$

$= [2^{-1} \times 2^{-2}] \div 2^{-2}$ $[\because (a^m)^n = (a^{mn})]$

$= [2^{-1+(-2)}] \div 2^{-2}$ $[\because (a^m \times a^n = (a^{m+n})]$

$= [2^{-1-2}] \div 2^{-2} = 2^{-3-(-2)}$ $[\because (a^m \div a^n) = a^{m-n}]$

$= 2^{-3+2} = 2^{-1} = \frac{1}{2}$

(iii) We have, $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

$= (2)^2 + (3)^2 + (4)^2 = 4 + 9 + 16 = 29$

$\left[\because (a)^{-m} = \frac{1}{a^m}\right]$

(iv) We have, $(3^{-1} + 4^{-1} + 5^{-1})^0$

$\left[\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right]^0$ $\left[\because (a)^{-m} = \frac{1}{a^m}\right]$

$= \left[\frac{20+15+12}{60}\right]^0 = \left[\frac{47}{60}\right]^0 = 1$ $[\because a^0 = 1]$

(v) We have,

$\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2 = \left\{\left(-\frac{3}{2}\right)^2\right\}^2$ $\left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right]$

$= \left\{\left(\frac{-3}{2}\right)\times\left(\frac{-3}{2}\right)\right\}^2 = \left\{\frac{9}{4}\right\}^2 = \frac{9}{4}\times\frac{9}{4} = \frac{81}{16}$

NS. 4

Evaluate :

(i) $\frac{8^{-1}\times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Ans. (i) We have, $\frac{8^{-1}\times 5^3}{2^{-4}}$

$= \frac{(2^3)^{-1}\times 5^3}{(2)^{-4}} = \frac{(2)^{-3}\times 5^3}{2^{-4}}$ $[\because (a^m)^n = a^{mn}]$

$= (2)^{-3-(-4)} \times 5^3 = (2)^{-3+4} \times 5^3 = (2)^1 \times 5^3$

$\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= 2 \times 5 \times 5 \times 5 = 250$

(ii) We have, $(5^{-1} \times 2^{-1}) \times 6^{-1}$

$= \left[\frac{1}{5}\times\frac{1}{2}\right]\times\frac{1}{6} = \left[\frac{1}{10}\right]\times\frac{1}{6} = \frac{1}{60}.$ $\left[\because (a)^{-m} = \frac{1}{a^m}\right]$

NS. 5

Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Ans. We have, $5^m \div 5^{-3} = 5^5$

$\Rightarrow 5^{m-(-3)} = 5^5 \Rightarrow 5^{m+3} = 5^5$ $[\because a^m \div a^n = a^{m-n}]$

$\Rightarrow m + 3 = 5 \Rightarrow m = 5 - 3 = 2.$ $[\because a^m = a^n \Rightarrow m = n]$

NS. 6

Evaluate :

(i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Ans. (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \left\{\left(\frac{3}{1}\right) - \left(\frac{4}{1}\right)\right\}^{-1}$

$\left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right]$

$= (3 - 4)^{-1} = (-1)^{-1} = -1$

(ii) We have, $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

$= \left(\frac{8}{5}\right)^7 \times \left(\frac{5}{8}\right)^4$ $\left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right]$

$\frac{(8)^7}{(5)^7} \times \frac{(5)^4}{(8)^4}$ $\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

$= (8)^{7-4} \times (5)^{4-7}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= (8)^3 \times (5)^{-3} = \frac{(8)^3}{(5)^3} = \frac{8\times8\times8}{5\times5\times5} = \frac{512}{125}$

NS. 7

Simplify :

(i) $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}} (t \neq 0)$ (ii) $\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}$

Ans. (i) We have, $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}$

$= \frac{(5)^2\times t^{-4}}{(5)^{-3}\times(5)^1\times(2)^1\times t^{-8}}$

$= \frac{(5)^2\times t^{-4}}{(5)^{-3+1}\times(2)^1\times t^{-8}}$ $[\because a^m . a^n = a^{m+n}]$

$= \frac{(5)^2\times t^{-4}}{(5)^{-2}\times(2)^1\times t^{-8}} = \frac{(5)^{2+2}\times t^{-4+8}}{2}$

$\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= \frac{(5)^4\times t^4}{2} = \frac{625\times t^4}{2} = \frac{625t^4}{2}$

(ii) We have, $\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}$

$= \frac{(3)^{-5}\times(2\times5)^{-5}(5)^3}{(5)^{-7}\times(2\times3)^{-5}}$

$= \frac{(3)^{-5}\times(2)^{-5}\times(5)^{-5}\times(5)^3}{(5)^{-7}\times(2)^{-5}\times(3)^{-5}}$

$[\because (a.b)^m = a^m . b^m]$

$= (3)^{-5+5} \times (2)^{-5+5} \times (5)^{-5+7+3}$

$\left[\because \frac{a^m}{a^n} = a^{m-n}, a^m.a^n = a^{m+n}\right]$

$= (3)^0 \times (2)^0 \times (5)^5 = 1 \times 1 \times 3125 = 3125$

$[\because a^0 = 1]$

EXERCISE – 12.2

NS. 1

Express the following numbers in standard form.

(i) 0.0000000000085

(ii) 0.00000000000942

(iii) 6020000000000000

(iv) 0. 00000000837

(v) 37860000000

Ans. (i) $0.0000000000085 = 8.5 \times 10^{-12}$

(ii) $0.00000000000942 = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 8.37 \times 10^{-9}$

(v) $37860000000 = 3.786 \times 10^{10}$

NS. 2

Express the following numbers in usual form.

(i) 3.02×10^{-6} (ii) 4.5×10^4

(iii) 3×10^{-8} (iv) 1.0001×10^9

(v) 5.8×10^{12} (vi) 3.61492×10^6

Ans. (i) We have, $3.02 \times 10^{-6} = 0.00000302$

(ii) We have, $4.5 \times 10^4 = 45000$

(iii) We have, $3 \times 10^{-8} = 0.0000003$

(iv) We have, $1.0001 \times 10^9 = 1000100000$

(v) We have, $5.8 \times 10^{12} = 5800000000000$

(vi) We have, $3.61492 \times 10^6 = 3614920$

NS. 3

Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}$ m

(ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.

(iii) Size of a bacteria is 0.0000005 m.

(iv) Size of a plant cell is 0.00001275 m.

(v) Thickness of a thick paper is 0.07 mm.

Ans. (i) Standard form of given statement = 1×10^{-6}

(ii) Standard form of given statement = 1.6×10^{-19}

(iii) Standard form of given statement = 5×10^{-7}

(iv) Standard form of given statement = 1.275×10^{-5}

(v) Standard form of given statement = 7×10^{-2}.

NS. 4

In a stack, there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016mm. What is the total thickness of the stack ?

Ans. Total number of books = 5

Thickness of each book = 20 mm

$\therefore$ Thickness of 5 books = (5×20) mm = 100 mm

Thickness of each paper sheet = 0.016 mm

$\therefore$ Thickness of 5 paper sheet = (5×0.016)mm

= 0.08 mm

Hence, total thickness of stack

= 100 mm + 0.08 mm

= 100.08 mm = 1.0008×10^{2}mm

Space for Notes :

EXERCISE – I

ONLY ONE CORRECT TYPE

1. Multiplicative inverse of 10^{-100} is –

(A) $(10)^{-100}$ (B) $\frac{1}{(10)^{-100}}$

(C) $(10)^{-10}$ (D) $(10^{-50})^3$

2. The value of $(3^2)^3 + \left(\frac{2}{3}\right)^0 + 3^5$ is –

(A) 930 (B) 973

(C) 932 (D) 950

3. Which of the following values are equal ?

I. 1^4 II. 4^0

III. 0^4 IV. 4^1

(A) I and II (B) II and III

(C) I and III (D) I and IV

4. The standard form of 15240000 is –

(A) 1.524×10^7 (B) 1.524×10^6

(C) 15.24×10^7 (D) 1.524×10^8

5. $\left(\frac{16}{81}\right)^{3/4} = ?$

(A) $\frac{9}{2}$ (B) $\frac{2}{9}$

(C) $\frac{8}{27}$ (D) $\frac{27}{8}$

6. Usual form of 7.54×10^{-6} is –

(A) 0.000000754 (B) 0.000754

(C) 0.0000754 (D) 0.00000754

7. The value of $(512)^{-2/9}$ is –

(A) $\frac{1}{2}$ (B) 2

(C) 4 (D) $\frac{1}{4}$

8. The value of $\left(\frac{243}{32}\right)^{-\frac{4}{5}}$ is –

(A) $\frac{81}{16}$ (B) $\frac{16}{81}$

(C) $\frac{4}{9}$ (D) $\frac{9}{4}$

9. Given that $2^h \times 2^3 = 2^9$, find the value of h.

(A) 3 (B) 6

(C) 8 (D) 12

10. The value of expression $(8^0 - 3^0) \times (8^0 + 3^0)$ is –

(A) 0 (B) 1

(C) 2 (D) 3

11. When simplified, $(x^{-1} + y^{-1})^{-1}$, (where $x \neq 0$, $y \neq 0$ and $x + y \neq 0$) is equal to

(A) $x + y$ (B) $\frac{xy}{x+y}$

(C) xy (D) $\frac{1}{xy}$

12. $16^{\frac{5}{2}} \div 16^{\frac{1}{2}} = ?$

(A) 250 (B) 256

(C) 255 (D) 200

13. $(32)^{\frac{-2}{5}} \div (125)^{\frac{-2}{3}} = ?$

(A) $\frac{4}{25}$ (B) $\frac{25}{4}$

(C) $\frac{2}{5}$ (D) $\frac{5}{2}$

14. $4^{-3/2} + 8^{2/3}$ is equal to

(A) $2\frac{1}{4}$ (B) $4\frac{1}{8}$

(C) $4\frac{1}{4}$ (D) $8\frac{1}{4}$

15. $(512)^{\frac{-2}{3}} \times \left(\frac{1}{4}\right)^{-3}$ is equal to

(A) 4 (B) $\frac{1}{4}$

(C) 1 (D) 16

16. The value of expression

$\left(\frac{1}{3}\right)^3 \times \left(\frac{-2}{5}\right)^2 \times \left(\frac{-3}{2}\right)^3$ is–

(A) $\frac{1}{50}$ (B) $\frac{-1}{50}$

(C) 1 (D) 0

17. What is the value of x, where $3^{3x-5} = \frac{1}{9^x}$?

(A) $\frac{5}{2}$ (B) 5

(C) 1 (D) $\frac{7}{3}$

18. Find the value of p so that

$\left(\frac{4}{5}\right)^3 \div \left(\frac{4}{5}\right)^{-3} = \left(\frac{4}{5}\right)^{3p}$.

(A) 3 (B) 0

(C) 2 (D) 1

19. If $\frac{10}{3} \times 3^x - 3^{x-1} = 81$, then the value of x is –

(A) 2 (B) 1

(C) 3 (D) 0

20. $8^{4/3} \times 2^{-1}$ =?

(A) 4 (B) 8

(C) 16 (D) 32

21. $9x^6y^2 \div 3x^3y = ?$

(A) $3x^2y^2$ (B) $3x^3y$

(C) $3x^3y^2$ (D) $6x^3y$

22. The value of $\frac{2^{2001} + 2^{1999}}{2^{2000} - 2^{1998}}$ is –

(A) 2 (B) 10/3

(C) $2^{1000} + 1$ (D) 10

23. The value of $[x + x(x^x)]$, when x = 2 is –

(A) 10 (B) 16

(C) 18 (D) 36

24. The largest number among the followings is –

(A) $3^{2^{2^2}}$ (B) $\{(3^2)^2\}^2$

(C) $3^2 \times 3^2 \times 3^2$ (D) 3^{222}

25. For a non–zero rational number a, $a^7 \div a^{-12}$ is–

(A) a^5 (B) a^{-19}

(C) a^{-1} (D) a^{19}

PARAGRAPH TYPE

Passage # I

$a^{-m} \times a^{-n} = a^{-m+(-n)} = a^{-(m+n)}$,

$a^{-m} \div a^{-n} = a^{(-m+n)}$, $a^{-m} = \frac{1}{a^m}$

26. Evaluate : $\frac{9^{-1} \times 5^3}{3^{-3}}$

(A) 370 (B) 315

(C) 375 (D) 400

27. Simplify : $\frac{25 \times a^9}{5^{-3} \times 10 \times a^{-18}}$

(A) $\frac{625a^{27}}{12}$ (B) $\frac{625a^5}{20}$

(C) 625a (D) $\frac{625a^{27}}{2}$

28. Find the value of m for which $(-3)^{m+1} \times (-3)^5 = (-3)^7$.

(A) 1 (B) –1

(C) 0 (D) 4

Passage # II

$$a^m \times a^n \times a^{-p} = \frac{a^m \times a^n}{a^p}$$

29. The value of $6^2 \times 6^{-4} \times 6^8$ is

(A) 6^6 (B) 6^{-6}

(C) 6^{12} (D) 6^{-10}

30. Evaluate :

$$\left(\frac{1}{2}\right)^{-4} \times \left(\frac{1}{2}\right)^{-8} \times \left(\frac{1}{2}\right)^{2} + \left(\frac{1}{4}\right)^{2} \times \left(\frac{1}{4}\right)^{-6} \times \left(\frac{1}{4}\right)^{2}$$

(A) $\left(\frac{1}{2}\right)^{-10} + \left(\frac{1}{4}\right)^{-2}$ (B) $\left(\frac{1}{2}\right)^{12} \times \left(\frac{1}{4}\right)^{-1}$

(C) $\left(\frac{1}{2}\right)^{-14} + \left(\frac{1}{4}\right)^{0}$ (D) $\left(\frac{1}{2}\right)^{12}$

31. Find the value of x if $4^{2x-3} = 4^2 \times 2^3 \times 4$.

(A) 0 (B) 4

(C) $\frac{15}{4}$ (D) $\frac{-9}{8}$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column–I and Column–II are given as options (A), (B), (C) and (D) out of which one is correct.

32.

Column-I	Column-II
(P) $(3^2 + 2^2) \times \left(\frac{1}{2}\right)^3$	(1) $\frac{19}{64}$
(Q) $(3^2 + 2^2) \times \left(\frac{2}{3}\right)^{-3}$	(2) $-\frac{4}{3}$
(R) $\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left[\frac{1}{4}\right]^{-3}$	(3) $\frac{13}{8}$
(S) $(2^2 + 3^2 - 4^2) \div \left(\frac{3}{2}\right)^2$	(4) $\frac{351}{8}$

(A) P–1, Q–2, R–3, S–4

(B) P–4, Q–1, R–2, S–3

(C) P–3, Q–4, R–1, S–2

(D) P–4, Q–2, R–3, S–1

33. Solve the following word problems.

Column-I	Column-II
(P) By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?	(1) $\frac{1}{3}$
(Q) By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?	(2) $\frac{-2}{3}$
(R) By what number should $(-6)^{-1}$ be multiped so that product becomes 9^{-1} ?	(3) $\frac{-8}{729}$
(S) By what number should $\left(\frac{-2}{3}\right)^{-3}$ be divided so that the quotient may be equal to $\left(\frac{4}{27}\right)^{-3}$?	(4) $\frac{-5}{7}$

(A) P–1, Q–3, R–2, S–4

(B) P–3, Q–1, R–2, S–4

(C) P–3, Q–2, R–1, S–4

(D) P–4, Q–1, R–2, S–3

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Express the following as a rational number.

   (i) $\left(\frac{-3}{5}\right)^3$ (ii) $\left(\frac{21}{89}\right)^2$

2. Express the following in power notation.

   (i) $\frac{-125}{343}$ (ii) $\frac{1}{2401}$

3. Find the product of square of $\frac{-1}{2}$ and the cube of $\frac{-2}{3}$.

4. Simplify : $\left\{6^{-1}+\left(\frac{3}{2}\right)^{-1}\right\}^{-1}$.

5. Simplify : $\left(x^{2^{n-1}}+y^{2^{n-1}}\right)\left(x^{2^{n-1}}-y^{2^{n-1}}\right)$.

6. If $x^{x\sqrt{x}}=\left(x^{3/2}\right)^{x}$, then find x.

7. Find the value of $(-4)^2 \div (2)^5$.

8. Using the laws of exponents, simplify each of the following and express in power notation.

   (i) $3^7 \times 3^{-2}$ (ii) $2^{-7} \div 2^{-3}$

   (ii) $(5^2)^{-3}$ (iv) $2^{-3} \times (-7)^{-3}$

   (v) $\frac{3^{-5}}{4^{-5}}$

9. Simplify : $\left(\frac{x^a}{x^b}\right)^{a+b} \div \left(\frac{x^a}{x^{a-b}}\right)^{a^2/b}$.

10. Evaluate : $(3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-2}$.

SHORT ANSWER TYPE

1. Evaluate : $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{2}$.

2. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then find the values of x and y.

3. Evaluate : $\left(\frac{-1}{4}\right)^{-3} \times \left(\frac{-1}{4}\right)^{-2}$.

4. If $2^{x-2} = 5^{2-x}$, then find the value of x .

5. By what number should $(-24)^{-1}$ be divided so that the quotient may be 3^{-1} ?

6. Simplify : $\dfrac{(xyz)^4}{(x^{-2}y^3)^{-3}\left(\frac{1}{z^2}\right)^6}$ $(x \neq 0, y \neq 0, z \neq 0)$.

7. By what number should $(-4)^{-2}$ be multiplied so that the product may be equal to 10^{-2} ?

8. Find the value of m so that $\left(\frac{2}{9}\right)^3 \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2m-1}$.

9. Write the following numbers using scientific notation:

   (i) Two crore fifty three lakh

   (ii) 98000000000

   (iii) 0.00000000015

10. The size of a red blood cell is 0.000007 m and the size of the plant cell is 0.00001275 m. Compare their size.

LONG ANSWER TYPE

1. Find the largest among $\sqrt[4]{8}, \sqrt{2}, \sqrt[3]{6}$.
2. Express each of the following as power of a rational number with positive exponent :
(i) $5^{-3} \times 5^{-6}$
(ii) $\left(\frac{-1}{4}\right)^{-5} \times \left(\frac{-1}{4}\right)^{-7}$
(iii) $\left\{\left(\frac{3}{4}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$
3. Given that $\sqrt[3]{3^x} = 5^{1/4}$ and $\sqrt[4]{5^y} = \sqrt{3}$, then find the value of 2xy.
4. Simplify :
(i) $(2^{-1} \div 5^{-1})^2 \times \left(\frac{-5}{8}\right)^{-1}$
(ii) $(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1}$
(iii) $(5^{-1} \times 3^{-1})^{-1} \div 6^{-1}$
5. If $\frac{10}{3} \times 3 - 3^{x-1} = 81$. Find the value of x.

TRUE / FALSE TYPE

1. $(a^{-1} + b^{-1})^{-1} = \frac{ab}{a+b}$
2. $x^y = 121$, where y is even, then $\sqrt{x-y}$ is 3.
3. The standard form of 0.00000356 is 3.56×10^{-7}.
4. $\left(\frac{1}{2}\right)^m \times \left(2^{-n}\right) = 2^{m-n}$
5. $x^5 + x^2 = x^7$
6. The multiplicative inverse of $(-4)^{-2}$ is $(4)^{-2}$.
7. $329.25 = 3 \times 10^2 + 2 \times 10^1 + 9 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$.
8. $(-4)^{-4} \times (4)^{-1} = (4)^5$.
9. $75(-2)^0 = 75$.
10. The expression for 4^{-3} as a power with the base 2 is 2^6.

NUMERICAL PROBLEMS

1. The value of $(3^0 - 4^0) \times 5^2$ is ______.
2. The value of x in exponential equation $2^{x-14} = 1$ is ______.
3. If $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^m = 1$, then least positive integral value of m is ______.
4. If $2^{x-1} + 2^{x+1} = 320$, then the value of x is______.
5. $(64)^{-2/3} \times (1/4)^{-3}$ is equal to______.
6. $(5(8^{1/3} + 27^{1/3}))^{1/2}$ =______.
7. If $\left(\frac{a}{b}\right)^{x-10} = \left(\frac{b}{a}\right)^{x-16}$, then x is ______.
8. $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ = ______.
9. $\frac{1}{1+a^{m-n}} + \frac{1}{1+a^{n-m}}$ is equal to ______.
10. Given that $4^{n-9} = 256$, then the value of n is______.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Simplify :
(i) $\left(\frac{125}{64}\right)^{2/3} \div \left(\frac{1}{(625/256)^{-1/4}}\right) + \left[\left(\frac{\sqrt{36}}{\sqrt[3]{64}}\right)^0\right]^{1/2}$
(ii) $\frac{\sqrt[6]{2}\left[(625)^{3/5} \times (1024)^{-6/5} \div (25)^{3/5}\right]^{1/2}}{\left[\left(\sqrt[3]{128}\right)^{-5/2}\right] \times (125)^{1/5}}$

(iii) $\left\{\dfrac{4^{m+\frac{1}{4}}\times\sqrt{2.2^{m}}}{2\sqrt{2^{-m}}}\right\}^{\frac{1}{m}}$

(iv) $\dfrac{9^{x}(9^{x-1})^{x}}{9^{x+1}\cdot 3^{2x-2}}\left\{\dfrac{729^{\frac{x}{3}}}{81}\right\}^{-x}\div\dfrac{3^{a}-2^{3}\cdot 3^{a-2}}{3^{a}-3^{a-1}}$

2. If $\dfrac{9^{n}\times 3^{2}\times\left[3^{\frac{-n}{2}}\right]^{-2}-(27)^{n}}{3^{3m}2^{3}}=\dfrac{1}{27}$, then find the relationship between m and n.

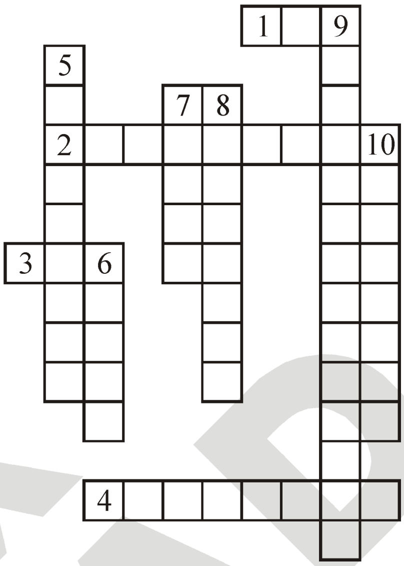
3. If $\left(\sqrt[3]{2}\right)^{12}\times\left(\sqrt{5}\right)^{8}=\left[(2\times 5)^{2}\right]^{x}$, then find the value of x.

4. If $\dfrac{a^{2b-3}\times\left(a^{2}\right)^{b+1}}{\left(a^{4}\right)^{-3}}=(a^{3})^{3}\div(a^{6})^{-3}$, then find the value of 2b.

5. If $x = y^{z}$, $y = z^{x}$ and $z = x^{y}$ then find the relation between x, y and z.

CROSS WORD PUZZLE

1. Solve the given crossword and then fill up the given boxes. Clues are given below for across as well as downward filling. Also, for across and down clues, clue number is written at the corner of the boxes. Answers of clues have to be filled up in their respective boxes.



Clues Across

1. In $x^{m}\times x^{n}=x^{p}$, p is the________ of m and n.
2. Very large numbers like 6,250,000,000 can be conveniently written using ________.
3. The value of a^{n} if n = 0.
4. Very small numbers can be expressed in standard form using _______ exponents.

Clues Down

5. The value of 3^{-2}.
6. The value of $\dfrac{1}{2^{-3}}$.
7. 5^{7} is read as 5 raised to the ______ of 7.
8. As the exponent of base 10 decreases by 1, the value becomes _______ of the previous value.
9. a^{-m} is the ________ inverse of a^{m}.
10. 1.24×10^{-4} is known as the ______ form of 0.000124.

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	B	A	A	C	D	D	B	B	A	B	D	B	B	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	C	C	C	B	B	B	A	D	D	C	D	A	A	A
31	32	33												
C	C	D												

EXERCISE II

VERY SHORT ANSWER TYPE

1. (i) $\frac{-27}{125}$, (ii) $\frac{441}{7921}$ 2. (i) $\left(\frac{-5}{7}\right)^3$, (ii) $\left(\frac{1}{7}\right)^4$ 3. $\frac{-2}{27}$. 4. $\frac{6}{5}$

5. $x^{2^n} - y^{2^n}$ 6. $\frac{9}{4}$ 7. $\frac{1}{2}$ 8. (i) 3^5, (ii) 2^{-4}, (iii) 5^{-6}, (iv) $(-14)^{-3}$, (v) $\left(\frac{3}{4}\right)^{-5}$

9. x^{-b^2} 10. $\frac{45}{4}$

SHORT ANSWER TYPE

1. $\frac{1225}{16}$ 2. $x = \frac{17}{8}$ and $y = \frac{15}{8}$ 3. –1024 4. 2 5. $-\frac{1}{8}$

6. $\frac{y^{13}z^{16}}{x^2}$ 7. $\frac{4}{25}$ 8. –1 9. (i) 2.53×10^7, (ii) 9.8×10^{10}, (iii) 1.5×10^{-10}

10. A red blood cell is approximately half of a plant cell in size.

LONG ANSWER TYPE

1. $\sqrt[3]{6}$ 2. (i) $\frac{1}{5^9}$, (ii) 4^{12}, (iii) $-\frac{3}{8}$ 3. 3 4. (i) –10, (ii) 30, (iii) 90 5. 3

TRUE / FALSE

1. T 2. T 3. F 4. F 5. F 6. F
7. T 8. F 9. T 10. F

CROSSWORD PUZZLE

1. Sum 2. Exponents 3. 1 4. Negative 5. 1/9
6. 8 7. Power 8. One tenth 9. Multiplicative 10. Standard

NUMERICAL PROBLEMS

1. 0 2. 14 3. 2 4. 7 5. 4 6. 5
7. 13 8. 1 9. 1 10. 13

ANALYTICAL PROBLEMS & BRAIN TEASER

1. (i) $\frac{9}{4}$, (ii) 1, (iii) 2^3, (iv) 6 2. m – n = 1 3. 2 4. 8 5. xyz = 1

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : EXPONENTS AND POWER)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Solutions			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

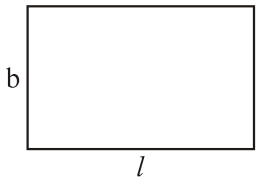
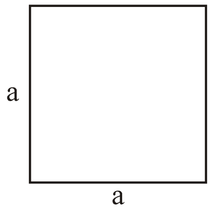
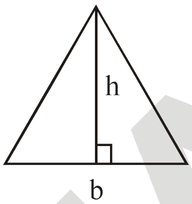
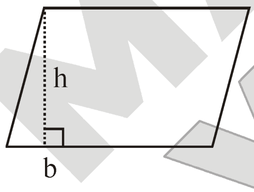
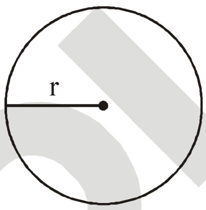
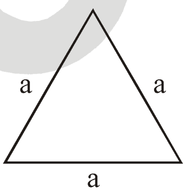
Space for Notes :

MENSURATION 10

Concepts

INTRODUCTION

Plane figures : A plane figure occupies the surface of the plane. For example : Rectangles, Triangles, Quadrilaterals, etc. For a closed plane figure, the perimeter is the distance around its boundary and its area is the region covered by it. The area and perimeter of various plane figures are given below :

S. No.	FIGURE	SHAPE	AREA	PERIMETER
1.		Rectangle	$l \times b$	$2(l + b)$
2.		Square	$a \times a$	4a
3.		Triangle	$\frac{1}{2} \times b \times h$	Sum of all sides
4.		Parallelogram	$b \times h$	2 × (sum of two adjacent sides)
5.		Circle	πr^2	$2\pi r$
6.		Equilateral triangle	$\frac{\sqrt{3}}{4}(a)^2$	3a

Example 1

The area of a square is 42.25 m^2 :

(i) Find the side of the square.

(ii) If the tiles measuring 13 cm × 13 cm are paved on the square area, find how many tiles are used for paving it ?

Solution :

Given : Area of the square = 42.25 m^2

(i) Area of the square = $side^2$

$\Rightarrow$ side = $\sqrt{42.25}$ m

$\Rightarrow$ side = 6.5 m = 6.5 × 100 cm = 650 cm

(ii) Number of tiles required = $\frac{\text{Area of the square}}{\text{Area of one tile}} = \frac{650\times650}{13\times13} = 2500$

Example 2

Find the area of the given figure.

Solution :

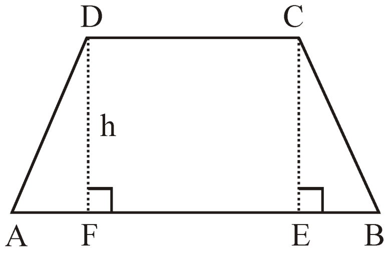
Area of semi-circle $= \frac{\pi r^2}{2} = \frac{1}{2}\times\frac{22}{7}\times6\times6 = 56.57\,cm^2$

1. AREA OF TRAPEZIUM

Definition : A trapezium is a quadrilateral having a pair of parallel opposite sides. Let ABCD be a trapezium in which AB || DC, CE ⊥ AB, DF ⊥ AB and CE = DF = h, where h is the height of trapezium ABCD.

From the given figure, area of quadrilateral ABCD

= Area of ΔAFD + Area of rectangle DFEC + Area of ΔCEB



$= \left(\frac{1}{2}\times AF\times DF\right) + (FE\times DF) + \left(\frac{1}{2}\times EB\times CE\right)$

$= \left(\frac{1}{2}\times AF\times h\right) + (FE\times h) + \left(\frac{1}{2}\times EB\times h\right)$

$= \frac{1}{2}\times h\times(AF+2FE+EB) = \frac{1}{2}\times h\times(AF+FE+EB+FE)$

$= \frac{1}{2} \times h \times (AB + FE) = \frac{1}{2} \times h \times (AB + DC)$ [Since AF + FE + EB = AB and FE = CD]

Hence, area of trapezium = $\frac{1}{2}$ × (sum of parallel sides) × height

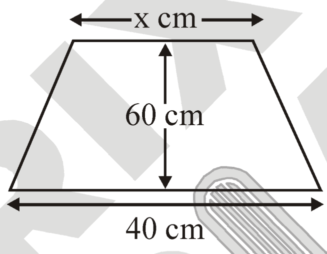
Example 3

A 5100 square cm trapezium has the perpendicular distance between the two parallel sides is 60 cm. If one of the parallel sides be 40 cm then, find the length of the other parallel side.

Solution :

Let the length of the unknown parallel side be 'x' cm.

Area of trapezium = $\frac{1}{2}$ (40 + x) × 60



$\Rightarrow 5100 = \frac{1}{2}(40 + x) \times 60$

$\Rightarrow 170 = 40 + x$

$\Rightarrow x = 170 - 40 = 130$ cm

Example 4

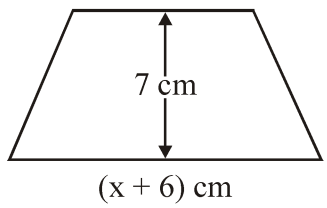
The area of the trapezium is 105 cm^2 and its height is 7 cm. If one of the parallel sides is longer than the other by 6 cm, find the length of parallel sides.

Solution :

Let the length of the smaller parallel side be x cm. Then, the length of other side be (x + 6) cm. We have, height of the trapezium = 7 cm and area of the trapezium = 105 cm^2.

Now, area of the trapezium = $\frac{1}{2}$ × (Sum of the parallel sides) × Height

$\Rightarrow \frac{1}{2} \times (x + 6 + x) \times 7 = 105$



$\Rightarrow \frac{1}{2} \times (2x + 6) \times 7 = 105$

$\Rightarrow 2x + 6 = \frac{105 \times 2}{7}$

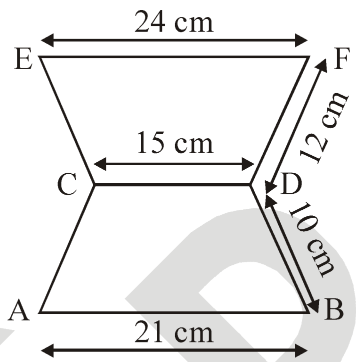
$\Rightarrow 2x + 6 = 30$ $\Rightarrow 2x = 24$ $\Rightarrow x = 12$

Hence, the lengths of parallel sides are 12 cm and (12 + 6) cm = 18 cm

Example 5

Find the area of the given figure.

Solution :



We have,

Area of the given figure

= Area of trapezium ABDC + Area of trapezium CDFE

$= \left\{\frac{1}{2} \times (21 + 15) \times 10\right\} cm^2 + \left\{\frac{1}{2} \times (15 + 24) \times 12\right\} cm^2$

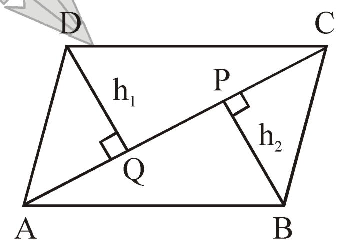
$= \left\{\frac{1}{2} \times 36 \times 10\right\} cm^2 + \left\{\frac{1}{2} \times 39 \times 12\right\} cm^2$

$= (180 + 234) cm^2 = 414\ cm^2$

2. AREA OF QUADRILATERAL

Let ABCD be a quadrilateral with AC as one of its diagonals. Let BP and DQ be the perpendiculars drawn from the vertices B and D on diagonal AC. From the figure, Area of quadrilateral ABCD

= Area of ΔABC + Area of ΔADC



$= \frac{1}{2} \times AC \times BP + \frac{1}{2} \times AC \times DQ$

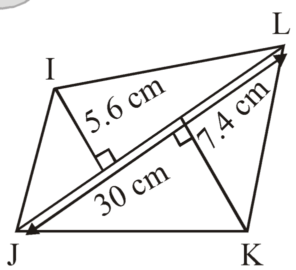
$= \frac{1}{2} \times AC \times (BP + DQ) = \frac{1}{2} \times d \times (h_1 + h_2)$,

where d denotes length of diagonal AC.

$= \frac{1}{2} \times$ diagonal × (sum of perpendiculars on the diagonal from the opposite vertices).

Example 6

Find the area of quadrilateral IJKL.



Solution :

Here, d = 30 cm, h_1 = 5.6 cm, h_2 = 7.4 cm

∴ Area of quadrilateral, IJKL $= \frac{1}{2} \times d \times (h_1 + h_2)$

$= \frac{1}{2} \times 30 \times (5.6 + 7.4)\ cm^2$

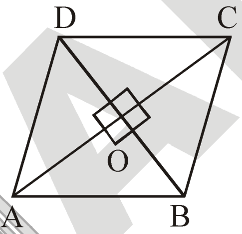
$= \frac{1}{2} \times 30 \times 13\ cm^2 = 195\ cm^2$

3. AREA OF RHOMBUS

Let ABCD be a rhombus and AC and BD be its diagonals which intersect at O. We know that the diagonals of a rhombus bisect each other at right angles and divides the rhombus into four congruent right angled triangle.

Area of rhombus = 4 × Area of ΔAOB = $4 \times \frac{1}{2} \times AO \times OB = 4 \times \frac{1}{2} \times \left(\frac{1}{2}AC\right) \times \left(\frac{1}{2} \times BD\right) = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times d_1 \times d_2$, where AC = d_1 and BD = d_2



∴ Area of rhombus $= \frac{1}{2}$ × product of its diagonals

Note : Side of rhombus $= \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

Example 7

The length of a side of a square field is 4 m. What will be the altitude of the rhombus if the area of the rhombus is equal to the area of a square field and one of its diagonal is 2 m ?

Solution :

Area of the square field = $(4)^2\ m^2 = 16\ m^2$. Area of rhombus = side × altitude ......(i)

(Since, rhombus is a parallelogram whose all sides are equal. So, area of rhombus = area of parallelogram)

∴ Area of rhombus = $16\ m^2$

[∵ Area of rhombus = Area of a square field] Length of the diagonal = 2 m [Given]

∴ Area of the rhombus $= \frac{1}{2} \times d_1 \times d_2$

$\Rightarrow 16 = \frac{1}{2} \times 2 \times d_2 \Rightarrow d_2 = 16m$

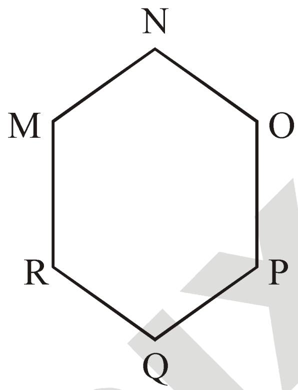
Now, we know that Side of the rhombus $= \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

$= \frac{1}{2}\sqrt{(16)^2 + (2)^2} = \frac{1}{2}\sqrt{256 + 4} = \frac{1}{2}\sqrt{260} = \sqrt{65}$

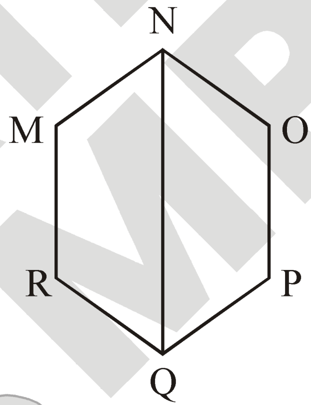
Altitude of the rhombus $= \frac{\text{Area of rhombus}}{\text{Side}} = \frac{16}{\sqrt{65}}$ m [From (i)]

4. AREA OF POLYGON

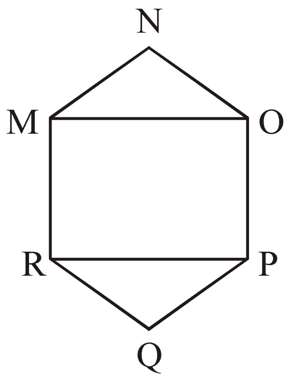
We can calculate the area of irregular polygon by dividing them into triangles or quadrilateral or a combination of the two. Suppose we have to find tha area of polygon MNOPQR as below :



We can do it by two ways :
(i) By dividing the polygon MNOPQR into two trapeziums.

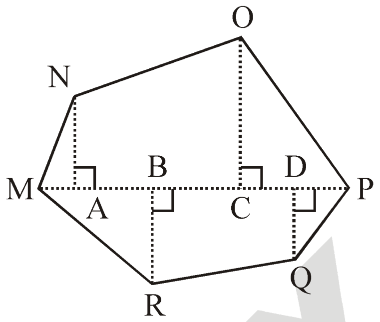


(ii) By dividing the polygon MNOPQR into ΔMNO, rectangle MOPR and ΔPQR.



Example 8

Find the area of the hexagon in the adjoining figure, if MP = 9 cm, MD = 7 cm, MC = 6 cm, MB = 4 cm, MA = 2 cm, AN = 3 cm, OC = 5 cm, DQ = 2 cm and BR = 4 cm.



Solution :

We have, area of hexagon MNOPQR = Area of ΔMAN + Area of trapezium CONA + Area of ΔPCO + Area of ΔMBR + Area of trapezium BDQR + Area of ΔPDQ

$$=\frac{1}{2}(MA\times NA)+\frac{1}{2}(AN+OC)\times AC+\frac{1}{2}(PC\times CO)+\frac{1}{2}(MB\times RB)+\frac{1}{2}(BR+DQ)\times BD+\frac{1}{2}(DP\times DQ)$$

$$=\left[\frac{1}{2}(2\times 3)+\frac{1}{2}(3+5)\times(6-2)+\frac{1}{2}\{(9-6)\times 5\}+\frac{1}{2}(4\times 4)+\frac{1}{2}(4+2)\times(7-4)+\frac{1}{2}(9-7)\times 2\right]cm^2$$

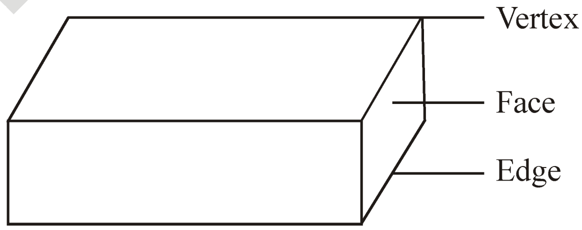
$$=\left(3+16+\frac{15}{2}+8+9+2\right)cm^2=45.5\,cm^2$$

5. SOLID SHAPES

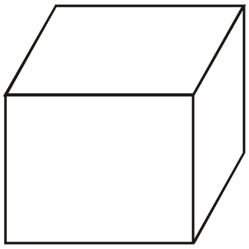
A solid shape is a shape which occupies space. For example : cube, cuboid and cylinder. The diagrams are given below :

Cuboid : A solid bounded by six rectangular plane faces with opposite faces are identical is called a cuboid. A chalkbox, a matchbox, a tea packet, a book etc., are all examples of a cuboid.

It has 6 rectangular faces, 12 edges and 8 vertices. Any face of a cuboid many be called its base. The four faces which meet the base are called the lateral faces of the cuboid. (It has three pair of identical faces)

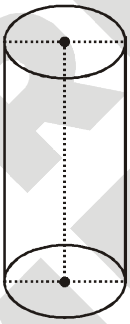


Cube : A cuboid whose length, breadth and height are all equal is called a cube. Ice cubes, sugar cubes, dice etc., are all examples of cubes. A cube has 6 square faces, 12 edges and 8 vertices. (It has all six faces identical)



Cylinder : In our day to day life, we come across several solids like measuring jars, circular pillars, circular pipes, a garden roller, gas cylinder etc. These solids have a curved (lateral) surfaces with congruent cicular ends. Such solids are right circular cylinders.

A right circular cylinder has two plane ends. Each plane end is circular in shape, and these circular ends are parallel; that is, they lie in parallel planes.



Cylinder

Base : Each of the circular ends on which the cylinder rests is called its bases.

Axis : The line segment joining the centres of two circular bases is called the axis of the cylinder.

The axis is always perpendicular to the bases of a right circular cylinder.

Radius : The radius of the circular bases is called the radius of the cylinder.

Height : The length of the axis of the cylinder is called the height of the cylinder.

In other words, the perpendicular distance between the two parallel circular ends or the altitude to either base from a point on the other is called the height of the cylinder.

Note : The cylinders have congruent circular faces that are parallel to each other. The line segment joining the centre of circular faces is perpendicular to the circular faces. Such cylinders are called right circular cylinders.

6. SURFACE AREA

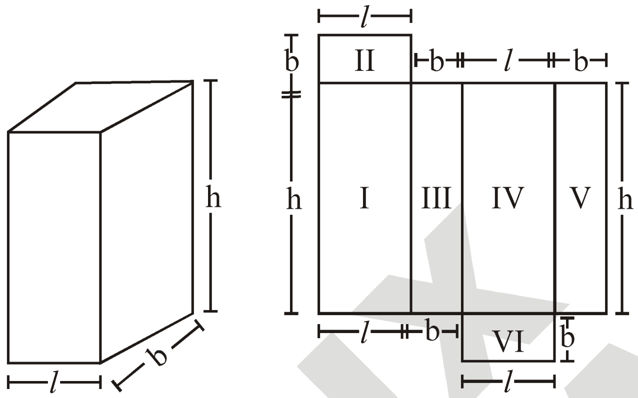
The total surface area of a solid is the sum of the areas of its faces.

Surface area of a cuboid : The net of a cuboid is shown below :

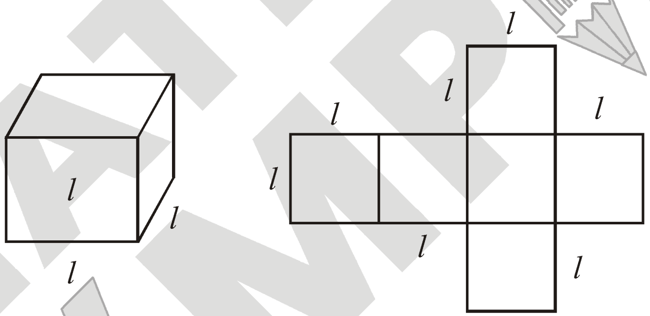
The total surface area of a cuboid = area I + area II + area III + area IV + area V + area VI

$= h \times l + b \times l + b \times h + l \times h + b \times h + l \times b$

So, total surface area = $2(h \times l + b \times h + b \times l) = 2(lb + bh + hl)$ where h, l and b are the height, length and width of the cuboid respectively.



Surface area of a cube : In this case, its length, breadth and height are the same. So, surface area of cube = 2 (side × side + side × side + side × side) = 2 [3 (side)^2] = $6a^2$, where 'a' is the side of the cube.



Example 9

Find the surface area of a chalk box, whose length, breadth and height are 16 cm, 8 cm and 6 cm respectively.

Solution :

Surface area of a cuboid = $2(lb + bh + hl) = 2(16 \times 8 + 8 \times 6 + 6 \times 16)\ cm^2 = 2(128 + 48 + 96)\ cm^2 = 544\ cm^2$.

7. LATERAL SURFACE AREA

Lateral surface area of a cuboid : The side walls (the faces excluding the top and bottom) is the lateral surface area of a cuboid.

∴ Lateral surface area = $2(h \times l + b \times h) = 2h(l + b)$

Lateral surface area of a cube : Lateral surface area of a cube = $2(a + a) \times a = 4a^2$, where 'a' is the side of the cube.

Example 10

A swimming pool is 20 m long, 15 m broad and 4 m deep. Find the cost of cementing its floor and the walls at the rate of Rs. 12 per square metre.

Solution :

Area of swimming pool to be cemented = Area of four walls + Area of a floor = $2(l + b) \times h + l \times b$

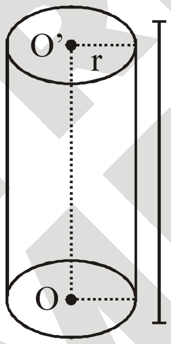
$= [2(20 + 15) \times 4 + 20 \times 15]m^2 = (8 \times 35 + 300)m^2 = (280 + 300)m^2 = 580\ m^2$

Cost of cementing 1 m^2 = Rs. 12. Cost of cementing 580 m^2 = Rs. (12×580) = Rs. 6960

Hence, the total cost of cementing is Rs. 6960.

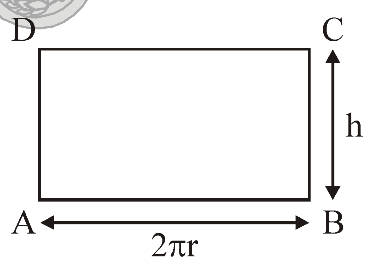
8. CURVED (LATERAL) SURFACE AREA OF A CYLINDER

The curved surface joining the two bases of a right circular cylinder is called its lateral surface. Let us consider a right circular cylinder of radius r and height h.



Cut it along a line on the surface parallel to axis (OO') of the cylinder and flatten this surface on a plane surface. We will obtain a rectangle, whose length is equal to the circumference of the base of the right circular cylinder and breadth is equal to the height of the cylinder.

Curved surface Area = Area of rectangle = Circumference of base × height = $2\pi r \times h = 2\pi rh$.



Total surface area of a right circular cylinder : The total surface area of a right circular cylinder consists of the area of the curved surface and the areas of two circular ends.

Total surface area = Curved surface area + 2 × Area of one circular end = $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$

Example 11

The diameter of a garden roller is 1.4 m and it is 2 m long. How much area will it cover in 5 revolutions ?

Solution :

We have, Area covered = Curved surface × Number of revolutions

Now, radius, $r = \frac{1.4}{2} = 0.7$ m, height, h = 2 m

$\therefore$ Curved surface area = 2πrh

$\Rightarrow A = 2 \times \frac{22}{7} \times 0.7 \times 2m^2 \Rightarrow A = 8.8m^2$

$\therefore$ Area covered by garden roller = A × 5 = (8.8 × 5) m^2 = 44 m^2

Example 12

The ratio between the curved surface area and the total surface area of a right circular cylinder is 1 : 2. Find the ratio between the height and radius of the cylinder.

Solution :

Let h be the height and r be the radius of the cylinder. Then, $\frac{2\pi rh}{2\pi rh + 2\pi r^2} = \frac{1}{2}$

$\Rightarrow \frac{2\pi rh}{2\pi r(h+r)} = \frac{1}{2} \Rightarrow \frac{h}{h+r} = \frac{1}{2}$

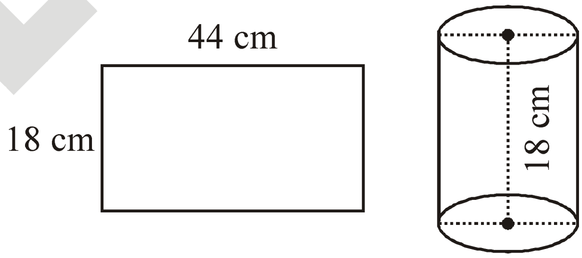
$\Rightarrow 2h = h + r \Rightarrow h = r \Rightarrow h : r = 1 : 1$

Example 13

A rectangular sheet of paper 44 cm × 18 cm is rolled along its length and a cylinder is formed. Find the radius of the cylinder.

Solution :

When the rectangular sheet is rolled along its length, we find that the length of the sheet forms the circumference of its base and breadth of the sheet becomes the height of the cylinder. Let r cm be the radius of the base and h cm be the height. Then, h = 18 cm



Now, Circumference of the base = Length of the sheet

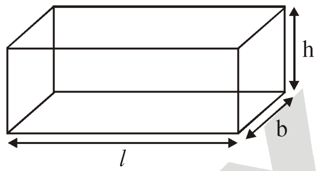
$\Rightarrow 2\pi r = 44 \Rightarrow 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 7cm$

Hence, radius of the cylinder is 7 cm.

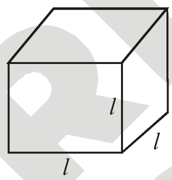
9. VOLUME OF SOLID FIGURES

The amount of space occupied by a three dimensional object is called its volume. All solid figures are 3-dimensional i.e., they have length, breadth and height. These are measured in metres, centimetres, etc., Volume of a solid figure is expressed in cubic metres (m^3), cubic centimetres (cm^3), etc. One cubic centimetre (1 cm^3) means the space occupied by a cube of side 1 cm and same for cubic metre etc.

Volume of a cuboid : Volume of a cuboid = Area of the base × height = $l \times b \times h$



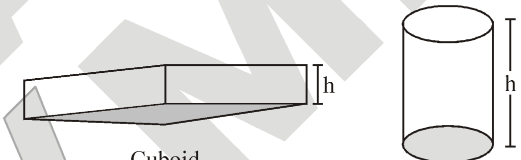
Volume of a cube : A cube is a special type of cuboid whose length, breadth and height are equal. So, volume of a cube = $l \times l \times l = l^3$



Volume of a cylinder : Just like cuboid, cylinder has got a top and a base which are congruent and parallel to each other. Its lateral surface is also perpendicular to the base, just like cuboid.

Volume of a cuboid = Area of base × height = $l \times b \times h = lbh$

Volume of a cylinder = Area of base × height = $\pi r^2 \times h = \pi r^2 h$



Volume and capacity : There is not much difference between these two words.

(a) Volume refers to the amount of space occupied by an object.

(b) Capacity refers to the quantity that a container holds.

Note : If a water tin hold 100 cm^3 of water then the capacity of the water tin is 100 cm^3. Capacity is also measured in terms of litres. The relation between litre and cm^3 is, 1 mL = 1 cm^3, 1 L = 1000 cm^3. Thus, 1 m^3 = 1000000 cm^3 = 1000 L.

Example 14

Find the surface area of a cube whose volume is 512 cm^3.

Solution :

Volume of a cube of side a cm = a^3 cm^3 Here, the given volume = 512 cm^3 So, we have $a^3 = 512 = 8^3$.

$\therefore$ a = 8 cm Now, surface area of a cube = 6 × side^2 = 6 × (8 cm)^2 = 6 × 64 cm^2 = 384 cm^2

Example 15

A rectangular block of ice measures 40 cm by 25 cm by 15 cm. Calculate its weight in kg, if ice weights $\frac{9}{10}$ of the weight of the same volume of water and 1 cm^3 of water weight 1 gm.

Solution :

We have, volume of the rectangular block of ice = $(40 \times 25 \times 15)$ cm^3 = 15000 cm^3.

Now, weight of 1 cm^3 of water = 1 gm. and weight of 1 cm^3 of ice = $\left(\frac{9}{10}\right)^{th}$ of the weight of 1 cm^3 of water.

$\therefore$ Weight of 1 cm^3 of ice = $\left(\frac{9}{10}\right)$ gm.

$\therefore$ Weight of the rectangular block of ice = $\frac{9}{10} \times 15000$ gm = 13500 gm = 13.5 kg.

Example 16

The length of a cold storage is double its breadth. Its height is 3 metres. The area of its four walls (including doors) is 108 m^2. Find its volume.

Solution :

Let length, breadth and height of the cold storage be l metres, b metres and h metres respectively. Then, $l = 2b$ metres (given) and h = 3 metres.

Now, area of four walls = 108 m^2

$\Rightarrow 2(l + b)h = 108$ m $\Rightarrow 2(2b + b) \times 3 = 108$ m

$\Rightarrow 18b = 108$ m $\Rightarrow b = 6$ metres

$\therefore l = (2 \times 6)$ m $\Rightarrow l = 12$ metres

Hence, volume of the cold storage = $(l \times b \times h)$ m^3 = $(12 \times 6 \times 3)$ m^3 = 216 m^3

Example 17

An agricultural field is in the form of a rectangle of length 20 m and width 14 m. A pit 6 m long, 3 m wide and 2.5 m deep is dug in a corner of the field and the earth taken out of the pit is spread uniformly over the remaining area of the field. Find the extent to which the level of the field has been raised.

Solution :

Volume of the earth dugout = $(6 \times 3 \times 2.5)$ m^3 = 45 m^3 ....(i)

Area of the remaining part of the field

= Area of the field – Area of a pit = $(20 \times 14 - 6 \times 3)$ m^2 = 262 m^2

The earth taken out of the pit is spread uniformly over the remaining area of the field. Let h metres be the level raised over the field uniformly. Clearly, the earth taken out forms a cuboid of base area 262 m^2 and height h.

Volume of the earth dugout = $(262 \times h)$ m^3

$\Rightarrow 262\ h = 45$ [From (i)]

$\Rightarrow\ h = \frac{45}{262} = 0.1718\ m = 17.18\ cm$

Example 18

The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rates of Rs. 8 and Rs. 9.50 per m^2 is Rs. 1248. Find the dimensions of the box.

Solution :

Let $l = 2x$, $b = 3x$ and $h = 4x$

Surface area of a cuboid $= 2(lb + bh + hl) = 2\ (2x \times 3x + 3x \times 4x + 4x \times 2x)\ m^2 = 52x^2\ m^2$

Difference of costs at the two rates = Rs. $\{52x^2 \times 9.5 - 52x^2 \times 8\}$ = Rs. $78x^2$

$\therefore\ 78x^2 = 1248 \Rightarrow x^2 = 16 \Rightarrow x = 4$

($\because$ x can't be negative)

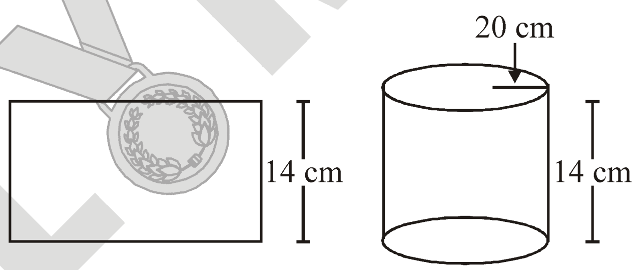
So, $l = (2 \times 4)\ m = 8m$, $b = (3 \times 4)\ m = 12m$ and $h = (4 \times 4)\ m = 16\ m$

Example 19

A rectangular paper of width 14 cm is rolled along its length and a cylinder of radius 20 cm is formed. Find the volume of the cylinder. $\left(\text{Take } \pi = \frac{22}{7}\right)$.

Solution :

A cylinder is formed by rolling a rectangle about its length. Hence the width of the paper becomes height and radius of the cylinder is 20 cm.



Height of the cylinder = h = 14 cm

Radius = r = 20 cm

Volume of the cylinder = $V = \pi r^2 h$

$\Rightarrow V = \frac{22}{7} \times 20 \times 20 \times 14\ cm^3 = 17600\ cm^3$

Hence, the volume of the cylinder is $17600\ cm^3$.

Example 20

The volume of a cylinder is 448π cm^3 and height is 7 cm. Find its lateral surface area and total surface area.

Solution :

Let the radius of the base and height of the cylinder be r cm and h cm respectively. Then, h = 7 cm (Given)

Now, volume = 448π cm^3

$\Rightarrow \pi r^2 h = 448\pi$

$\Rightarrow \pi \times r^2 \times 7 = 448\pi$ [$\because$ h = 7 cm]

$\Rightarrow r^2 = \frac{448}{7} = 64 \Rightarrow r = 8$ cm

($\because$ r can't be negative)

$\therefore$ Lateral surface area = $2\pi rh$ cm^2

$= 2 \times \frac{22}{7} \times 8 \times 7$ cm^2 = 352 cm^2.

Total surface area = $2\pi r(h + r)$ cm^2

$= 2 \times \frac{22}{7} \times 8(7+8)\,\text{cm}^2 = \frac{5280}{7}\,\text{cm}^2 = 754.28\,\text{cm}^2$

Example 21

Find the number of coins, 1.5 cm in diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Solution :

Clearly each coin is cylinder of radius = r = 0.75 cm and height = h = 0.2 cm.

Volume of a coin = $\{\pi \times (0.75)^2 \times 0.2\}$ cm^3

Also, radius of cylinder = R = $\frac{4.5}{2}$ cm = 2.25 cm, H = 10 cm

Volume of the cylinder = $\{\pi \times (2.25)^2 \times 10\}$ cm^3

$\therefore$ Number of coins = $\frac{\text{Volume of the cylinder}}{\text{Volume of a coin}}$

$= \frac{\pi \times (2.25)^2 \times 10}{\pi \times (0.75)^2 \times 0.2}$

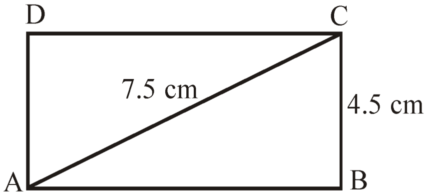
$= \frac{2.25 \times 2.25 \times 10}{0.75 \times 0.75 \times 0.2} = 3 \times 3 \times 50 = 450$

SE. 1

The diagonal of the floor of a rectangular closet is $7\frac{1}{2}$ cm. The shorter side of the closet is $4\frac{1}{2}$ cm. What is the area of the closet in square centimetres?

Ans. In ΔABC, $AB^2 = AC^2 - BC^2 = (7.5)^2 - (4.5)^2 = 36$

$\therefore$ AB = 6 cm



Area of rectangle ABCD = AB × BC = 6 × 4.5 sq.cm = 27 sq. cm

SE. 2

In the adjoining figure, AB | | DC and DA is perpendicular to AB. Further, DC = 7 cm, CB = 10 cm and AB = 13 cm. Find the area of the quadrilateral ABCD.

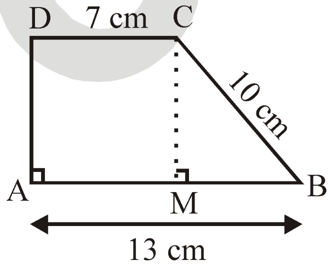
Ans. Draw CM perpendicular from C to AB. Clearly, AMCD is a rectangle.

$\therefore$ AM = DC

$\Rightarrow$ AM = 7 cm $\qquad\Rightarrow$ AB – BM = AM

$\Rightarrow$ 13 cm – BM = 7 cm

$\Rightarrow$ BM = (13 – 7) cm = 6 cm



Applying Pythagoras theorem, in right angled triangle BMC, we have

$BC^2 = BM^2 + CM^2 \Rightarrow CM^2 = 10^2 - 6^2$

$\Rightarrow CM^2 = (100 - 36)\ cm^2 = 64\ cm^2$

$\Rightarrow$ CM = 8 cm

Hence, area of trapezium

ABCD = $\frac{1}{2}$ (AB + DC) × CM

$= \frac{1}{2} \times (13 + 7) \times 8\ cm^2 = 80\ cm^2$

SE. 3

Find the area of the four walls of the room whose length is 6m, breadth is 5m and height is 4 m. Also find the cost of white washing of four walls, if the rate of white washing is Rs. 5 per square metre.

Ans. We have, length (l) = 6m, breadth (b) = 5 m and height (h) = 4 m

$\therefore$ Surface area of four walls = $2h(l + b)$

$= [2 \times 4(6 + 5)]\ m^2$

$= [8 \times 11]\ m^2 = 88\ m^2$

Cost of white washing per square metre = Rs. 5

$\therefore$ Cost of white washing of four walls

= Rs. 5 × 88 = Rs. 440

Thus, the cost of white washing the four walls is Rs. 440.

SE. 4

How many 5 cm cubes can be obtained (cut – offs) from a cube whose edge is 20 cm ?

Ans. The volume of a cube (V) = $(side)^3$

$\therefore V = (20)^3\ cm^3 = 8000\ cm^3$

Also, the side of the smaller cube = 5 cm

$\therefore$ Volume of the smaller cube = V’

$= (5)^3 \text{ cm}^3 = 125 \text{ cm}^3$

Hence, the number of cubes cut–off

$$= \frac{V}{V'} = \frac{8000}{125} = 64$$

SE. 5

A cuboidal oil tin is 30 cm × 40 cm × 50 cm. Find the cost of the tin required for making 20 such tins if the cost of tin sheet is Rs. 20 per square metere.

Ans. The cost of tins depend upon their total surface area. It is given that a tin is in the shape of a cuboid such that, $l = 30$ cm, b = 40 cm and h = 50 cm

∴ Surface area of one tin = $2(lb + bh + lh)$

$= 2(30 \times 40 + 40 \times 50 + 30 \times 50) \text{ cm}^2$

$= 2(1200 + 2000 + 1500) \text{ cm}^2$

$= (2 \times 4700) \text{ cm}^2 = 9400 \text{ cm}^2$

Surface area of 20 such tins = $(20 \times 9400) \text{ cm}^2$

$= 188000 \text{ cm}^2$

$$= \frac{188000}{1000} \text{ m}^2 = 18.8 \text{ m}^2 \quad [\because 1000 \text{ m}^2 = 1\text{m}^2]$$

Now, cost of 1 square metre of tin sheet = Rs. 20

∴ Cost of 18.8m^2 of tin sheet = Rs. (20×18.8)

= Rs. 376

Hence, the cost of making 20 tins = Rs. 376

SE. 6

The paint in a certain container is sufficient to paint an area equal to 9.375 m². How many bricks measuring 22.5 cm by 10 cm by 7.5 cm can be painted out this container ?

Ans. We have, l = length of a brick = 22.5 cm,

b = breadth of a brick = 10 cm

h = height of a brick = 7.5 cm

∴ Surface area of a brick = $2(lb + bh + lh)$

$= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5) \text{ cm}^2$

$= 2(225 + 75 + 168.75) \text{ cm}^2 = 937.5 \text{ cm}^2$

The paint in the container is sufficient to paint an area = $9.375 \text{ m}^2 = 9.375 \times 10000 \text{ cm}^2$ $= 93750 \text{ cm}^2$

∴ Number of bricks that can be painted out of the paint in the container $= \frac{93750}{937.5} = 100$

SE. 7

A cube of 9 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of the base are 15 cm and 12 cm, find the rise of water level in the vessel.

Ans. We have, edge of the given cube = 9 cm

∴ Volume of the cube = $(9^3) \text{ cm}^3 = 729 \text{ cm}^3$

If the cube is immersed in the vessel, then the water level rises.

Let the rise of water level be x cm.

Clearly, Volume of the cube = Volume of the water replaced by it.

$\Rightarrow 729 = 15 \times 12 \times x$

$$\Rightarrow x = \frac{729}{15 \times 12} \Rightarrow x = \frac{81}{10} = 4.05$$

∴ The rise of water level in the vessel = 4.05 cm

SE. 8

How many soap cakes measuring 7 cm × 5 cm × 2.5 cm can be placed in a box of size 56 cm × 0.4 m × 0.25 m?

Ans. Volume of the box = 56 cm × 0.4 m × 0.25 m = 56 cm × 40 cm × 25 cm = 56000 cm³

Volume of each soap cake = 7cm × 5cm × 2.5 cm = 87.5 cm³

Number of soap cakes that can be placed in the box

$$= \frac{\text{Volume of the box}}{\text{Volume of each soap cake}} = \frac{56000 cm^3}{87.cm^3} = 640$$

Hence, 640 soap cakes can be placed in the box.

SE. 9

In a temple there are 25 cylindrical pilars. The radius of each pillar is 28 cm and height is 4m. Find the total cost of painting the curved surface area of pillars at the rate of Rs. 8 per m^2. $\left[\text{Take } \pi = \frac{22}{7}\right]$

Ans. We have, r = Radius of a cylindrical pillar = 28 cm $= \frac{28}{100}$ m

h = Height of a cylindrical pillar = 4 m

∴ Curved surface area of 25 pillars

$$= 2 \times \frac{22}{7} \times \frac{28}{100} \times 4 \times 25 \ m^2 = 176 \ m^2$$

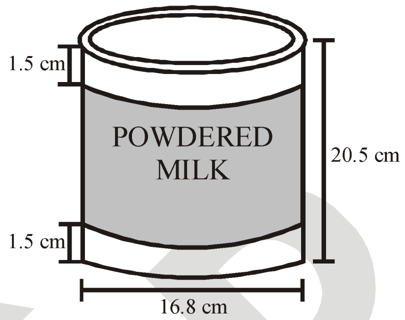
Hence, total cost of painting at the rate of Rs. 8 per m^2 = Rs. (176 × 8) = Rs. 1408.

SE. 10

A company packages its milk powder in cylindrical containers whose base has a diameter of 16.8cm and height 20.5 cm. Company places a label around the curved surface of the container. If the lable is place 1.5 cm from the top and the bottom, what is the surface area of the label ? $\left[\text{Take } \pi = \frac{22}{7}\right]$.

Ans. Clearly, surface area of the label is equal to the curved surface area of a cylinder of base radius

(r) $= \frac{16.8}{2}$ cm = 8.4 cm



and height (h) = (20.5 – 1.5 – 1.5) cm = 17.5 cm.

∴ Surface area of the label = 2πrh

$$= 2 \times \frac{22}{7} \times 8.4 \times 17.5 \ cm^2$$

$$= 2 \times 22 \times 1.2 \times 17.5 \ cm^2 = 924 \ cm^2.$$

SE. 11

A circular well of radius 3.5 m is dug 20 m deep and the earth so dug is spread on a rectangular plot of length 14 m and breadth 11 m. Find :

(i) Volume of the earth dug – out.

(ii) Height of the platform formed by spreading the earth on the rectangular plot. $\left[\text{Take } \pi = \frac{22}{7}\right]$

Ans. (i) We have, r = Radius of the cylindrical well = 3.5 m, h = depth of the cylindrical well = 20 m

∴ Volume of the earth dug–out = $\pi r^2 h$

$$= \frac{22}{7} \times (3.5)^2 \times 20 \ m^3$$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 20 \ m^3 = 770 \ m^3.$$

(ii) Let h metres be the height of the platform formed by spreading the earth dug–out from the well. Clearly, the platform is in the shape of a cuboid of length 14 metres, breadth 11 metres and height h metres.

Volume of the earth in platform = Volume of the earth dug–out

$\Rightarrow 14 \times 11 \times h = 770$

$\Rightarrow h = \frac{770}{14 \times 11} = 5$

$\therefore$ Height of the platform = 5 m.

SE. 12

A rectangular sheet of paper 44 cm long and 33 cm broad is rolled in two different ways to form two different cylinders. Find the volumes of the cylinders in each case. $\left[\text{Take } \pi = \frac{22}{7}\right]$

Ans. When rolled along its length :

Let r_1 be the radius in this case.

$\therefore 2\pi r_1 = 44$ cm

$\Rightarrow 2 \times \frac{22}{7} \times r_1 = 44 \text{ cm} \Rightarrow r_1 = 7 \text{ cm}$

$\therefore$ Volume $= \pi r_1^2 h = \left(\frac{22}{7} \times 7 \times 7 \times 33\right) \text{cm}^3$

$= 5082 \text{ cm}^3$

when rolled along its breadth.

Let r_2 be the radius in this case.

$2\pi r_2 = 33$ cm

$2 \times \frac{22}{7} \times r_2 = 33$ cm

$\Rightarrow r_2 = \left(\frac{33 \times 7}{2 \times 22}\right) \text{cm} = 5.25 \text{ cm}$

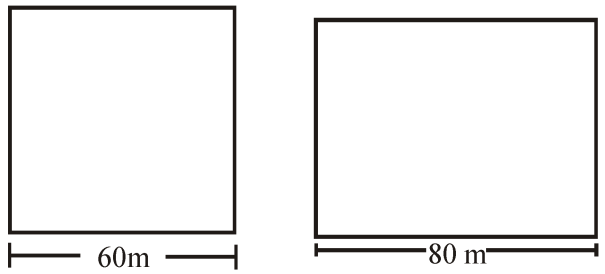
$\therefore$ Volume $= \pi r_2^2 h$

$= \left(\frac{22}{7} \times 5.25 \times 5.25 \times 44\right) \text{cm}^3 = 3811.5 \text{ cm}^3.$

EXERCISE – 11.1

NS. 1

A square and a rectangular field with measurements as given in the figure have the same perimeter. Which field has a larger area ?



(a) (b)

Ans. We have, a square of side 60 m

i.e., s = 60 m and a rectangle of length a = 80 m

Perimeter of square = 4 × s = 4 × 60 m = 240 m

As given, the perimeter of square and rectangle are equal.

Let, b be the other side of a rectangle.

$\therefore$ 2 × a + 2 × b = 240 m

$\Rightarrow$ 2 × 80 + 2 × b = 240

$\Rightarrow$ 2 × b = 240 – 160 $\Rightarrow$ 2 × b = 80 m

$\Rightarrow$ b = 40 m

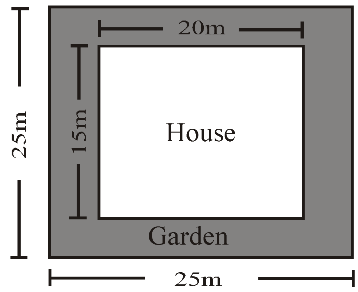
Hence, area of a square = s^2 = 60 m × 60 m = 360 sq. m

Area of rectangle = a × b = 80 m × 40 m = 3200 sq. m

Hence, area of a square is larger than that of the rectangle.

NS. 2

Mrs. Kaushik has a square plot with the measurement as shown in figure. She wants to construct a house in the middle of the plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of Rs. 55 per m^2.



Ans. The dimensions of the plot and house are as shown.

$\therefore$ The area of plot = 25 × 25 = 625 sq. m

and the are of house = 20 × 15 = 300 sq. m

We know, Area of plot = Area of house + Area of garden

$\therefore$ Area of garden = Area of plot – Area of house

= 625 – 300 = 325 sq. m

We also know,

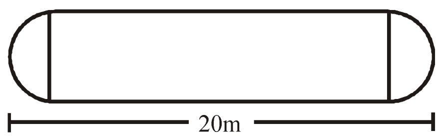
Rate of developing 1 sq. m garden = Rs. 55

$\therefore$ Amount for developing 325 sq. m garden

= 325 × 55 = Rs. 17875.

NS. 3

The shape of a garden is rectangular in the middle and semi circular at the ends as shown in the diagram. find the area and perimeter of this garden (Length of rectangle) is 20 – (3.5 + 3.5) metres.



Ans. The dimensions of a garden are :

Length of rectangle = 20 – radii of semicircles

= (20 – (3.5 + 3.5)) m = 13 m.

Hence area of garden = Area of rectangle + Area of 2 semi-circles

$$= \left\{13 \times 7 + 2\frac{\left(\pi \times (3.5)^2\right)}{2}\right\} m^2 = 129.5 m^2$$

Perimeter of garden = πr + 2 + (l) + πr

= 2(l + πr)

= 2(13 + π × 3.5) m = 48 m.

NS. 4

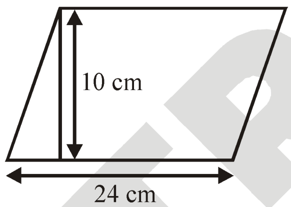
A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m^2 ?
(If required you can split the tiles in whatever way you want to fill up the corners).

Ans. The dimensions are b = 24 cm, h = 10 cm

$\therefore$ Area of 1 tile = 24 × 10 sq. cm

= 240 sq. cm

Thus, to cover an area of 1080 m^2, we need number of tiles

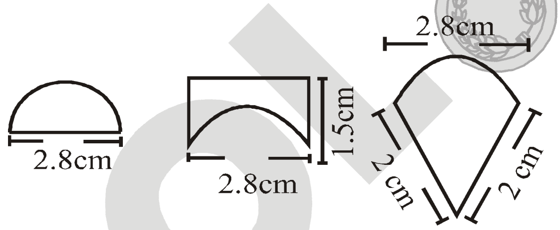


$$= \frac{1080 m^2}{240 cm^2}$$

$$= \frac{1080 \times 10^4}{240} = 45000 \text{ tiles.}$$

NS. 5

An ant is moving around a few food pieces of different shapes scattered on the floor. For which food–piece would the ant have to take a longer round ? Remember, circumference of a circle can be obtained by using the expression c = 2πr, where r is the radius of the circle.



(a) (b) (c)

Ans. The circumference of a circle is given by 2πr and perimeter that of a semicircle by πr + 2r

(a) $r = \frac{2.8}{2}$ cm = 1.4 cm

Hence, perimeter = πr + 2r

= (2 + π) (1.4) cm = 7.2 cm

(b) $r = \frac{2.8}{2}$ = 1.4 cm, l = 1.5 cm

Perimeter = 2r + 2l + πr

= [2 × 1.5 + (π + 2) 1.4] cm = (3 + 7.2) cm

= 10.2 cm

(c) $r = \frac{2.8}{2} = 1.4$ cm, l = 2 cm

Perimeter = πr + 2l = [π × 1.4 + 2 × 2] cm

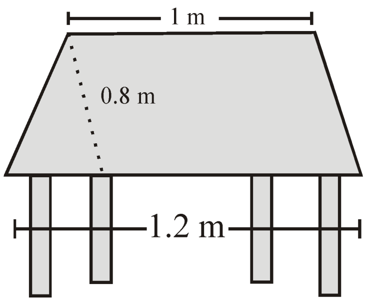
= 8.4 cm

Hence, the ant has to take the longest round around the piece of figure (b).

EXERCISE – 11.2

NS. 1

The shape of the top surface of a table is a trapezium. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.



Ans. Area of trapezium

$= \frac{1}{2} \times$ (sum of the parallel sides) × Distance between parallel sides.

$$= \frac{1}{2} \times (1 + 1.2) \times (0.8) \text{ m}^2$$

$$= \frac{1}{2} (2.2) \times (0.8) \text{ m}^2 = 0.88 \text{ m}^2.$$

NS. 2

The area of a trapezium is 34 cm² and the length of one of the parallel sides is 10 cm and its height is 4 cm. Find the length of the other parallel side.

Ans. Let x be the length of other parallel side. Area of trapezium.

$= \frac{1}{2} \times (\text{Sum of } || \text{ sides}) \times \text{Height}]$

$= \frac{1}{2} \times (10 + x) \times 4$

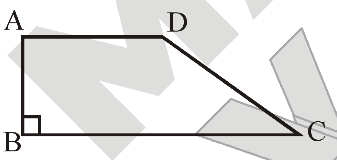
$\Rightarrow 34 = \frac{4}{2} \times (10 + x) \Rightarrow 10 + x = \frac{34 \times 2}{4} = 17$

$\Rightarrow x = 17 - 10 = 7.$

$\therefore$ The length of the other parallel side is 7 cm.

NS. 3

Length of the fence of a trapezium shaped field ABCD is 120 m. If BC = 48 m, CD = 17 m and AD = 40 m, find the area of this field. Side AB is perpendicular to the parallel sides AD and BC.



Ans. Draw a perpendicular DE from a point D which meets BC at E.

$\therefore$ AB = DE

EC = BC – BE = (48 – 40) m = 8 m

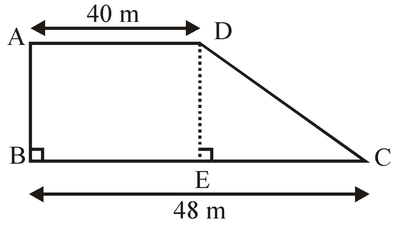
In ΔDEC, $(DE)^2 + (EC)^2 = (DC)^2$

$\Rightarrow (DE)^2 + (8)^2 = (17)^2$

$\Rightarrow (DE)^2 = (17)^2 - (8)^2 = (289 - 64) = 225 \text{ m}^2$

$\Rightarrow DE = \sqrt{225} \text{ m} = 15 \text{ m}$

$\Rightarrow AB = 15 \text{ m}$



Area of trapezium, ABCD

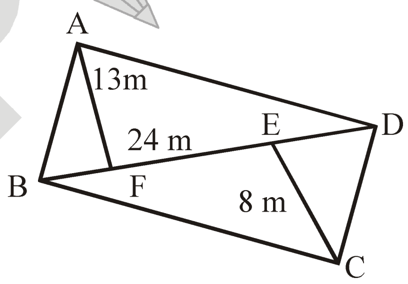
$= \frac{1}{2} \times (\text{Sum of parallel sides}) \times \text{Height}$

$= \frac{1}{2} (AD + BC) \times AB = \frac{1}{2} (40 + 48) \times 15$

$= 660 \text{ m}^2$

NS. 4

The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.



Ans. AF = 13 m, BD = 24 m, CE = 8 m (given)

Area of field = (ΔABD) + area (ΔDBC)

$= \frac{1}{2} \times BD \times AF + \frac{1}{2} BD \times CE$

$\left[\therefore \text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}\right]$

$= \left\{\frac{1}{2} \times 24 \times 13 + \frac{1}{2} \times 24 \times 8\right\} \text{m}^2$

$= \left\{\frac{1}{2} \times 24(13 + 8)\right\} \text{m}^2 = \frac{1}{2} \times 24(21) \text{m}^2$

$= 12 \times 21 \text{ m}^2 = 252 \text{ m}^2$

NS. 5

The diagonals of rhombus are 7.5 cm and 12 cm. Find its area.

Ans. We know that,

The area of a rhombus = $\frac{1}{2}$ (Product of its diagonals)

$= \left(\frac{1}{2}\times 7.5\times 12\right)\text{cm}^2 = 45\text{ cm}^2$

NS. 6

Find the area of a rhombus whose side is 6 cm and whose altitude is 4 cm. If one its diagonals is 8 cm long, find the length of the other diagonal.

Ans. Since, rhombus is a parallelogram whose all sides are equal.

So, area of rhombus = area of a parallelogram

= side × altitude

= (6 × 4) cm^2 = 24 cm^2

Also, area of a rhombus = $\frac{1}{2}$ × (Product of its diagonals)

$\Rightarrow$ 24 cm^2 = $\frac{1}{2}$ (8 × d) cm, where d is the length of another diagonal.

$\Rightarrow \frac{48\text{cm}^2}{8\text{cm}} = d \Rightarrow 6\text{ cm} = d$

$\therefore$ The length of the other diagonal be 6 cm.

NS. 7

The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is Rs. 4.

Ans. Let d_1 and d_2 be the diagonals of a rhombus, where d_1 = 45 cm and d_2 = 30 cm

Area of a rhombus shaped tile = $\frac{1}{2}(d_1 \times d_2)$

$= \frac{1}{2}(45 \times 30)\text{ cm}^2 = 675\text{ cm}^2$

$= 675 \times \frac{1}{10000}\text{ m}^2$

$\because$ Number of tiles = 3000

$\therefore$ Area of 3000 tiles = $\frac{675}{10000}\times 3000\text{ m}^2$

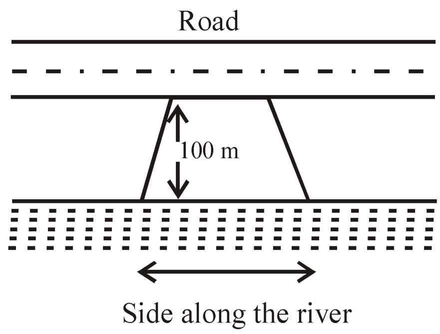
$\because$ Cost of polishing 1 m^2 = Rs. 4

$\therefore$ Cost of polishing the floor

= Rs. $\frac{675\times 3000}{10000}$ × 4 = Rs. 810.

NS. 8

Mohan wants to buy a trapezium shaped field. Its sides along the river is parallel to and twice the side along the road. If the area of this field is 10500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.



Ans. Let the opposite parallel sides of a trapezium be x and 2x.

$\because$ Area of trapezium = $\frac{1}{2}$ (sum of the parallel sides × height)

$\Rightarrow 10500\ m^2 = \frac{1}{2}\ (x + 2x) \times 100\ m$

$\Rightarrow \frac{10500 \times 2}{100} = 3x$

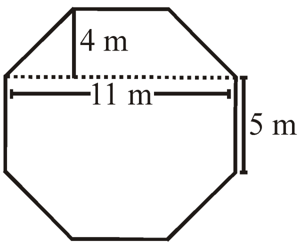
$\Rightarrow \frac{210}{3} = x \Rightarrow x = 70\ m$

$\therefore$ The length of the side along the river

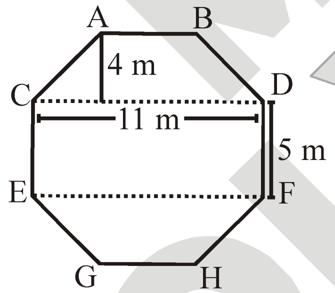
$= 2 \times 70\ m = 140\ m$

NS. 9

Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.



Ans. Area of octagon = Area of  (ABCD) + Area of  (CDEF) + Area of  (EFHG)



$= \left\{\frac{1}{2}((5+11)\times 4) + (11 \times 5) + \frac{1}{2}((5+11)\times 4)\right\} m^2$

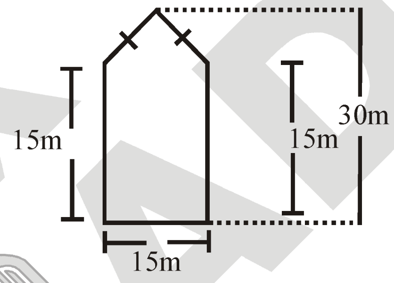
$= \left\{\frac{1}{2}(16 \times 4) + 55 + \frac{1}{2}(16 \times 4)\right\} m^2$

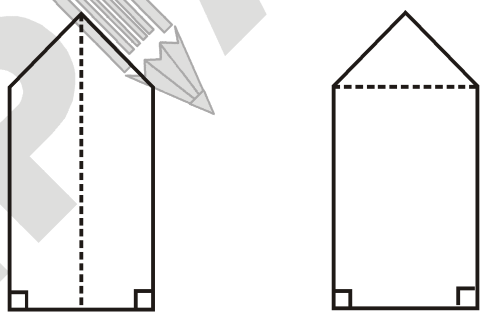
$= \left\{\frac{1}{2}(64) + 55 + \frac{1}{2}(64)\right\} m^2$

$= \{32 + 55 + 32\} m^2 = 119 m^2$

NS. 10

There is a pentagonal shaped park as shown in the figure. For finding its area Jyoti and Kavita divided it in two different ways. Find the area of this park using both ways. Can you suggest some other way of finding its area ?





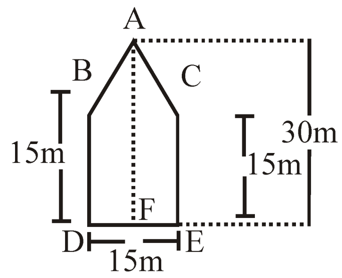
Jyoti's diagram Kavita's diagram

Ans. Jyot's way,

Area of field ABDEC = 2Area of  (ABDF)

$= 2 \times \frac{1}{2}\ [(BD + AF) \times DF]$

Jyoti's diagram



$= \left[(15 \times 30) \times \frac{15}{2}\right] m^2$

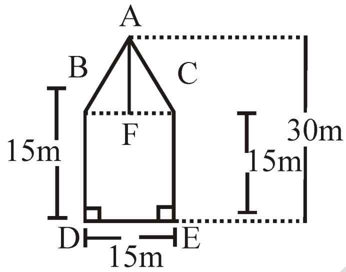
$= \frac{45 \times 15}{2} = 337.5\ m^2$

Kavita's way,

Area of field ABCDE

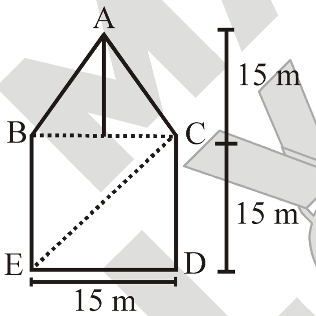
= Area of ΔABE + Area of BEDC

Kavita's diagram



$= \frac{1}{2}(BE \times AF) + (BE)^2 = \left\{\frac{1}{2}(15 \times 15) + (15)^2\right\} m^2$

Yes, we have also some other way of finding its area.



Area of field ABEDC

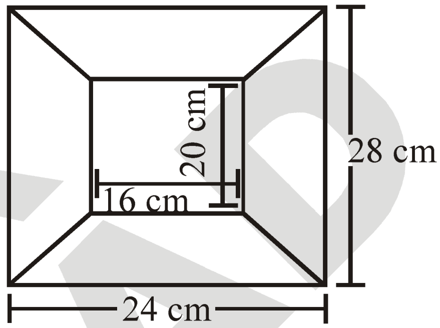
= Area of (ΔABC + ΔCDE + ΔBCE)

$= \left(\frac{1}{2} AF \times BC + \frac{1}{2} \times CD \times ED + \frac{1}{2} \times BC \times BE\right)$

$= 3\left(\frac{1}{2} \times 15 \times 15\right) m^2 = 337.5 m^2$

NS. 11

Diagram of the adjacent picture frame has outer dimensions = 24 cm × 28 cm and inner dimensions = 16 cm × 20 cm. Find the area of each section of the frame, if the width of each section is same.



Ans. Area of ⏢ ABFC

$= \frac{1}{2}$ (AB + EF) × 4 cm^2

$= \frac{1}{2}$ (16 + 24) × 4 cm^2

$= \frac{1}{2}$ × 40 × 4 cm^2 = 80 cm^2

Also, Area of ⏢ GHDC = Area of ⏢ ABFE = 80 cm^2

Area of ⏢ AEGC = $\frac{1}{2}$(20 + 28) × 4 cm^2 = $\frac{1}{2}$ (48 × 4) cm^2 = 36 cm^2

Also, Area of ⏢ AEGC = ⏢ BFHD = 96 cm^2

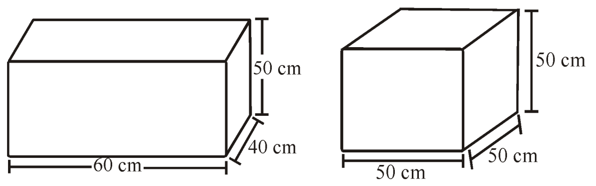
Area of ▭ EFGH = HG × HF

= 16 × 20 cm^2 = 320 cm^2

EXERCISE – 11.3

NS. 1

There are two cuboidal boxes as shown in the adjoining figure. Which box requires the lesser amount of material to make ?



(a) (b)

Ans. Srface area of box (a) = 2 ($lb + bh + hl$)

$= 2(60 \times 40 + 40 \times 50 + 50 \times 60)$

$= 2(2400 + 2000 + 3000)$

$= 2 \times 7400$

$= 14800 \text{ cm}^2$

Surface area of box (b) = $6 \times (\text{side}) = 6 \times 50^2$

$= 15000 \text{ cm}^2$

Hence, box (a) required less amount of material than box (b) to make.

NS. 2

A suitcase with measures 80 cm × 48 cm × 24 cm is to be covered with a trapauling cloth. How many metres of trapaulin of width 96 cm is required to cover 100 such suitcases ?

Ans. Let l, b, h be the length, breadth and height of the suitcase.

$\Rightarrow l = 80 \text{ cm, b} = 48, \text{h} = 24 \text{ cm}$

Total surface area of suitcase = 2 ($lb + gh + hl$)

$= 2(80 \times 48 + 48 \times 24 + 24 \times 80)$

$= 2(3840 + 1152 + 1920) \text{ cm}^2 = 2\ (6912) \text{ cm}^2$

$= 13824 \text{ cm}^2$

Area of cloth required for 1 suitcase

$\Rightarrow l \times 96 = 13824$

$\Rightarrow l = \frac{13824}{96} \Rightarrow l = 144 \text{ cm} = \frac{144}{100} \text{m}$

$\therefore$ Length required for 100 suitcases

$= \frac{144}{100} \times 100 = 144 \text{ m.}$

NS. 3

Find the side of a cube whose surface area is 600 cm².

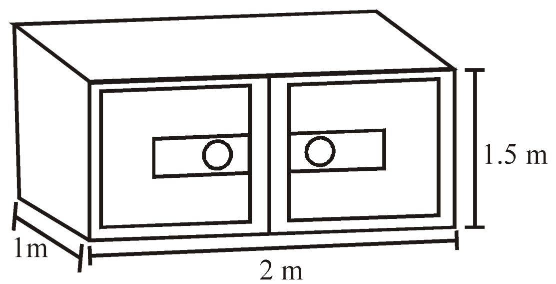
Ans. Surface area of cube = $6 \times (\text{sede})^2$

$\Rightarrow (\text{side})^2 = \frac{600}{6} \text{cm}^2 = 100 \text{ cm}^2$

$\Rightarrow \text{side} = \sqrt{100} \text{ cm} = 10 \text{ cm.}$

NS. 4

Rukhsar painted the outside of the cabinet of measure 1 m × 2m × 1.5 m. How much surface area did she cover if she painted all except the bottom of the cabinet ?



Ans. Rukhsar painted all the cabinet except the bottom means she painted 4 walls and 1 top. Let $l = 1$ m, b = 2m, h = 1.5 m be the length, breadth and height of cabinet.

$\therefore$ Area of painted cabinet = ($lb + bh + bh + lh + lh$)

$= [1 \times 2 + 2 \times 1.5 + 2 \times 1.5 + 1 \times 1.5 \times 1 \times 1.5]\text{m}^2$

$= [2 + 3.0 + 3.0 + 1.5 + 1.5] \text{ m}^2 = 11 \text{ m}^2$

NS. 5

Daniel is painting the walls and ceiling of a cuboidal hall with length, breadth and height of 15 m, 10m and 7 m respectively. From each can of paint 100 m^2 of area is painted. How many cans of paints will she need to paint the room ?

Ans. For painting the walls and ceiling means Daniel is painting 4 walls & 1 ceiling.

$\therefore$ Total painted area = Area of 4 walls + Area of ceiling

Let $l = 15$ m, b = 10 m and h = 7 m be the length, breadth and height of hall.

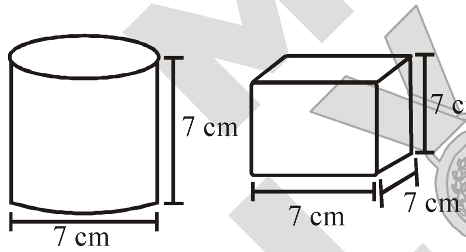
$= 2 \times h(l + b) + lb = \{2 \times 7(15 + 10) + (15 \times 10)\}m^2$

$= \{14 (25) + 150\}\ m^2 = \{350 + 150\}\ m^2 = 500\ m^2$

If 100 m^2 of area is painted with $\frac{500}{100}$ cans = 5 cans.

NS. 6

Describe how the two figures at the right are alike and how they are different. Which box has larger lateral surface area ?



Ans. Alike : They have the same dimensions.

Different : Their shapes are different.

Let r and h be the radius and height of the cylinder.

$r = \frac{7}{2}$ cm and h = 7 cm

$\therefore$ Lateral surface area = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 7\ cm^2 = 154\ cm^2$

Let the side of cube be a = 7 cm

$\Rightarrow$ Lateral surface area = $4(a)^2 = 4(7)^2\ cm^2$

$= 196\ cm^2$

$\therefore$ Cube has larger lateral surface area.

NS. 7

A closed cylindrical tank of radius 7 m and height 3 m is made from a sheet of metal. How much sheet of metal is required ?

Ans. Let r and h be the radius and height of the closed cylinder.

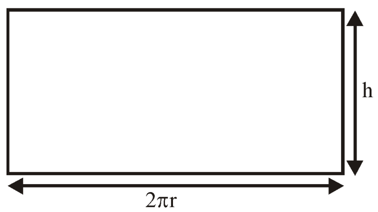
Total surface area = $2\pi r (r + h)$

$= 2 \times \frac{22}{7} \times 7(7 + 3) = 44 \times 10 = 440\ m^2.$

NS. 8

The lateral surface area of a hollow cylinder is 4224 cm^2 . It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of rectangular sheet.

Ans. Let r and h be the radius and height of the hollow cylinder and l be its lateral surface area.



$\therefore\ l = 2\pi rh \Rightarrow 4224\ cm^2 = 2\pi rh$

$\Rightarrow 2\pi r = \frac{4224 cm^2}{33^2} = 128$ cm

Now, perimeter = $2(2\pi r + h) = 2\ (128 + 33)$ cm

$= 2\ (161)$ cm = 322 cm

NS. 9

A road roller takes 750 complete revolutions to move once over to levle a road. Find the area of the road if the diameter of a road roller is 84 cm and length is 1 m.

Ans. Area covered in 1 revolution

= curved surface area of cylinder

$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{42}{100} \times 1\ m^2 = 2.64\ m^2$

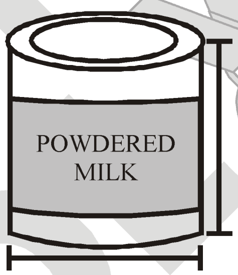
$\therefore$ Area covered in 750 revolutions

$= 2.64 \times 750\ m^2$

$= 1980\ m^2$

NS. 10

A company packages its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in figure). If the label is placed 2 cm from top and bottom, what is the area of the label ?



Ans. Radius $= \frac{14}{2}$ cm = 7 cm and

Height = (20 – 2 – 2) cm = (20 – 4) cm = 16 cm

Curved surface area of the label

$= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 16\ cm^2 = 704\ cm^2$

EXERCISE – 11.4

NS. 1

Given a cylindrical tank, in which situatin will you find surface area and in which situation volume.

(a) To find how much it can hold.

(b) Number of cement bags required to plaster it.

(c) To find the number of smaller tanks that can be filled with water from it .

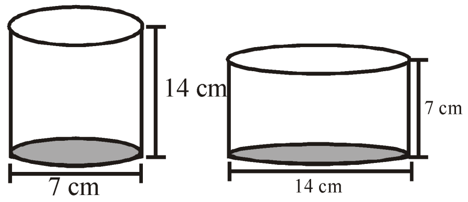
Ans. (a) To find how much a cylinder can hold we need to find the volume of the cylindrical tank.

(b) To find the number of cement bags required to plaster the tank, we need to find the surface area of the clindrical tank.

(c) To find the number of smaller tanksthat can be filled with water from the bigger tank, we need to find the volume of big cylindrical tank and one small tank.

NS. 2

Diameter of cylinder A is 7 cm, and the height is 14 cm. Diameter of cylinder B is 14 cm and height is 7 cm. Without doing any calculations can you suggest whose volume is greater ? Verify it by finding the volume of both the cylinder. Check whether the cylinder with greater volume also has greater surface area ?



Ans. Yes, its cylinder B whose volume is more than that of A, since the diameter of cylinder B is greater than the diameter of cylinder A.

For a cylinder, Volume = $\pi r^2 h$ & surface area $= 2\pi r(h + r)$

$\therefore$ For cylinder A,

$$\text{Volume} = \pi \times \left(\frac{7}{2}\right)^2 \times 14 \text{ cm}^3 = 539 \text{ cm}^3$$

$$\text{Surface area} = 2\pi\left(\frac{7}{2}\right)\left(14+\frac{7}{2}\right) \text{ cm}^2 = 385 \text{ cm}^2$$

And for cylinder B,

$$\text{Volume} - \pi\left(\frac{14}{2}\right)^2 \times 7 \text{ cm}^3 = 1078 \text{ cm}^3$$

$$\text{Surface area} = 2\pi\left(\frac{14}{2}\right)\left(7+\frac{14}{2}\right) \text{ cm}^2 = 616 \text{ cm}^2$$

Hence, cylinder B has both, greater volume and greater surface area.

NS. 3

Find the height of cuboid whose base area is 180 cm^2 and volume is 900 cm^3.

Ans. We have, base area = 180 cm^2,

Volume = 900 cm^3

To find, height of a cuboid = h cm, say

We know, Volume = Base area × Height of a cuboid

$\Rightarrow 900 = 180 \times h \Rightarrow h = 5$ cm.

NS. 4

A cuboid is of dimensions 60 cm × 54 cm × 30 cm. How many small cubes with side 6 cm can be placed in the given cuboid ?

Ans. We have,

Big cuboid dimensions = 60 cm × 54 cm × 30 cm

side of a small cube = 6 cm

Number of cubes that can be placed in the given cuboid

$$= \frac{\text{Volume of a cubiod}}{\text{Volume of cube}} = \frac{60\times54\times30\text{cm}^2}{6\times6\times6\text{cm}^3} = 450.$$

NS. 5

Find the height of the cylinder whose volume is 1.54 m^3 and diameter of the base is 140 cm.

Ans. We have, volume of the cylinder = 1.54 m^3

$= 1.54 \times 10^6 \text{ cm}^3$

Diameter = 140 cm, Radius = 140 ÷ 2 = 70 cm

$\because$ Volume of the cylinder = $\pi r^2 h$

$$\Rightarrow 1.54 \times 10^6 = \frac{22}{7} \times (70)^2 \times h$$

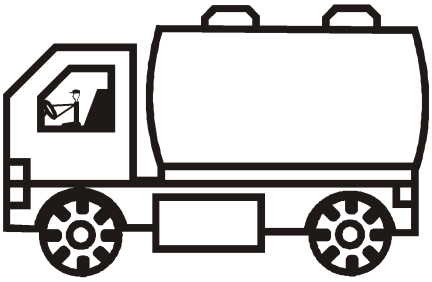
$$\Rightarrow \frac{1.54\times10^6\times7}{22\times70\times70} = h$$

$\Rightarrow$ 100 cm = h

$\therefore$ Height of the cylinder is 100 cm.

NS. 6

A milk tank is in the form of cylinder whose radius is 1.5 m and length is 7m. Find thequantity of milk in litres that can be stored in the tank .



Ans. We have, radius of the cylindrical tank

= 1.5 m and length = 7 m

$\therefore$ Volume of tank = $\pi r^2 h = \pi (1.5)^2 \times 7$

= 49.5 m^3 = 49500 litres. [$\because$ 1 m^3 = 1000 L]

NS. 7

If each edge of a cube is doubled,

(i) how many times will its surface area increase ?

(ii) how many times will its volume increase ?

Ans. Let the edge of the cube be a cm.

After doubling the length, the edge becomes 2a cm.

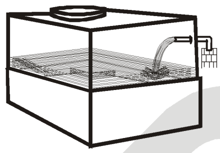
$\therefore$ Surface area of old cube = $6a^2$ and volume of old cube = a^3.

Surface area of new cube = $6(2a)^2$ = 24 a^2 and volume of new cobe = $(2a)^3 = 8a^3$

Hence, surface area increases 4 times and volume increases 8 times if the edge of a cube is doubled.

NS. 8

Water is pouring into a coboidal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill the reservoir.



Ans. We have,

Volume of reservoir = 108 m^3 = 108×10^3 L

Rate of pouring water = 60 L/minute

$\therefore$ Time to fill the reservoir

$$= \frac{\text{Volume of reservoir}}{\text{Rate of pouring water}} = \frac{108 \times 10^3}{60}$$

= 1.8×10^3 min. = 1800 min. = 30 hrs.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. The length of a rectangle is 18 cm and its breadth is 10 cm. When the length is increased to 25 cm, what will be the breadth of the rectangle if the area remains the same ?
(A) 7 cm (B) 7.1 cm
(C) 7.2 cm (D) 7.3 cm

2. A rectangular plot measuring 90 metres by 50 metres is to be enclosed by wire fencing. If the poles of the fence are kept 5 metres apart, how many poles will be needed ?
(A) 55 (B) 56
(C) 57 (D) 58

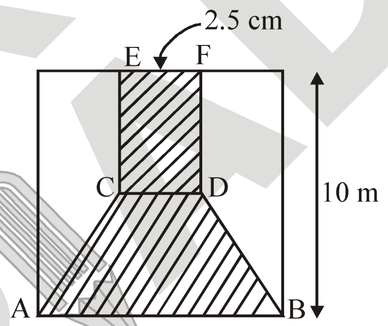
3. A rectangular parking space is marked out by painting three of its sides, If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet ?
(A) 46 sq. ft (B) 81 sq. ft
(C) 126 sq. ft (D) 252 sq. ft

4. The difference between the length and breadth of a rectangle is 23 m. If its perimeter is 206 m, then its area is
(A) 1520 m^2 (B) 2420 m^2
(C) 2480 m^2 (D) 2520 m^2

5. The length of a rectangular plot is 20 metres more than its breadth. If the cost of fencing the plot at the rate of Rs. 26.50 per metre is Rs. 5300, what is the length of the plot in metres ?
(A) 40 (B) 50
(C) 120 (D) None of these

6. The area of a rhombus is 840 cm^2 and one of its diagonals is 14 cm, find the other diagonal.
(A) 100 m (B) 140 m
(C) 120 m (D) 210 m

7. In the given figure, the side of the square is 10 cm. EF = 2.5 cm and C and D are half way between the top and bottom sides of the figure. The area of the shaded portion of the figure is



(A) 43.75 cm^2 (B) 56.25 cm^2
(C) 55.25 cm^2 (D) 50.25 cm^2

8. The perimeter of a rhombus is 146 cm and one of its diagonal is 55 cm. Find the other diagonal and the area of the rhombus.
(A) 24 cm, 660 cm^2 (B) 24 cm, 330 cm^2
(C) 48 cm, 660 cm^2 (D) 48 cm, 1320 cm^2

9. If the perimeter of a rhombus is 4a and the length of the diagonals are x and y, then its area is
(A) a(x + y) (B) $x^2 + y^2$
(C) xy (D) $\frac{1}{2}$xy

10. The maximum length of a pencil that can be kept in a rectangular box of dimensions 12 cm × 9 cm × 8cm, is
(A) 13 cm (B) 17 cm
(C) 18 cm (D) 19 cm

11. The volume of a cube is 2744 cm^3. Its surface area is
(A) 196 cm^2 (B) 588 cm^2
(C) 784 cm^2 (D) 1176 cm^2

12. How many cubes of 10 cm edge can be put in a cubical box of 1 m edge ?
(A) 10 (B) 100
(C) 1000 (D) 7200

13. A metallic sheet is a rectangular shape with dimensions 48 m × 36 m. From each of its corners, a square is cut off so as to make an open box. If the length of each square is 8 m, then the volume of the box is
(A) 4830 m^3 (B) 5120 m^3
(C) 6420 m^3 (D) 8960 m^3

14. Three cubes of iron whose edges are 6 cm, 8 cm, and 10 cm respectively are melted and formed into a single cube. The edge of the new cube formed is
(A) 12 cm (B) 14 cm
(C) 16 cm (D) 18 cm

15. Five equal cubes, each of edge 5 cm, are placed adjacent to each other. The volume of the new solid formed will be
(A) 125 cm^3 (B) 375 cm^3
(C) 525 cm^3 (D) 625 cm^3

16. A circular well with a diameter of 2 metres, is dug to a depth of 14 metres. What is the volume of the dug out ?
(A) 32 m^3 (B) 36 m^3
(C) 40 m^3 (D) 44 m^3

17. If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 14 m, the depth of the tank is
(A) 8 m (B) 12 m
(C) 16 m (D) 18 m

18. The number of coins, each of radius 0.75 cm and thickness 0.2 cm to be melted to make a right circular cylinder of height 8 cm and base radius 3 cm is
(A) 460 (B) 500
(C) 600 (D) 640

19. The length of a room is 5.5 m and width is 3.75 m. Find the cost of paving the floor by slabs at the rate of Rs. 800 per sq. metre.
(A) Rs. 15000 (B) Rs. 15550
(C) Rs. 15600 (D) Rs. 16500

20. The capacity of a tank of dimensions (8m × 6m × 2.5 m) is
(A) 120 litres (B) 1200 litres
(C) 12000 litres (D) 120000 litres

21. The diagonal of a cube is $6\sqrt{3}$ cm. Find its volume.
(A) 612 cm^3 (B) 216 cm^3
(C) 226 cm^3 (D) 136 cm^3

22. Find the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.
(A) 1540 cm^2 (B) 880 cm^2
(C) 957 cm^2 (D) 415 cm^2

23. An open cylindrical tank is of radius 2.8 m and height 3.5 m. What is the capacity of the tank ?
(A) 96.24 m^3 (B) 84.26 m^3
(C) 86.24 m^3 (D) 82.64 m^3

24. The length of the longest rod that can be put in a box of dimensions 10 cm by 10 cm by 5 cm is
(A) 8 cm (B) 9 cm
(C) 12 cm (D) 15 cm

25. What is the area of the region of the circle which is situated outside the inscribed square of side x ?
(A) $(\pi - 2)x^2$ (B) $(\pi - 2)x^2/2$
(C) $2(\pi - 2)x^2$ (D) $(\pi - 2)x^2/4$

PARAGRAPH TYPE

Passage # I

The area of a rhombus is $\frac{1}{2}(d_1 \times d_2)$ and perimeter = $2\sqrt{(d_1^2 + d_2^2)}$, where d_1 and d_2 are the diagonals of the rhombus.

26. The area of a rhombus, each side of which measure 20 cm and one of whose diagonals is 24 cm is
(A) 380 cm^2 (B) 384 cm^2
(C) 384 cm^2 (D) 38 cm^2

27. The area of the field in the form of rhombus if the length of each side be 14 cm and the altitude be 16 cm is :
(A) 224 cm^2
(B) 210 cm^2
(C) 148 cm^2
(D) 228 cm^2

28. The area of a rhombus is 84 m^2. If its perimeter is 40 m, its altitude is
(A) 4.8 m (B) 8.4 m
(C) 6.8 m (D) 4.9 m

Passage # II

The area of a trapezium equals half the sum of parallel sides multiplied by its height.

29. The altitude of a trapezium when, the sum of the lengths of whose parallel sides is 6.5 cm and area is 26 cm^2 is
(A) 20 m (B) 4 cm
(C) 6 cm (D) 8 cm

30. The sum of the lengths of parallel sides of a trapezium whose altitude is 11 cm and area is 0.55 m^2 is
(A) 10 m (B) 10 cm
(C) 1000 m (D) 40 cm

31. If the perimeter of a trapezium is 52 cm, its non parallel sides are equal to 10 cm each and its altitude is 8 cm. The area of trapezium is
(A) 138 m^2 (B) 128 m^2
(C) 130 cm^2 (D) 128 cm^2

MATCH THE COLUMN TYPE

In this section, each question has two matching lists. Choices for the correct combination of elements from Column I and Column II are given as options (a), (b), (c) and (d) out of which one is correct.

32.

Column – I	Column – II
(P) The perimeter of a rhombus is	(1) $\frac{\sqrt{3}}{2}$(side)
(Q) The circumference of a circle is	(2) $2\sqrt{d_1^2 + d_2^2}$
(R) The altitude of an equilateral triangle is	(3) $2\pi r$
(S) The diagonal of a rectangle is	(4) $\sqrt{l^2 + b^2}$

(A) P–2, Q–3, R–4, S–1
(B) P–2, Q–3, R–1, S–4
(C) P–2, Q–1, R–3, S–4
(D) P–1 , Q–3, R–4, S–2

33. **Column – I** **Column – II**

(P) The area of triangle (in sq. cm) whose base is 24 cm and whose altitude is 1.5 dm is — (1) 11

(Q) The diameter of a wheel is 1.26 m. How far will it travel in 500 revolutions in metres ? — (2) 34650

(R) The height (in cm) of a cuboid where volume is 275 cm^3 and base area is 25 cm^2 is — (3) 180

(S) The circumference of the base of the cylinder is 132 cm and its height is 25 cm. The volume of the cylinder (in cm^3) is — (4) 1980

(A) P–3, Q–4, R–1, S–2
(B) P–4, Q–1, R–3, S–2
(C) P–4, Q–1, R–2, S–3
(D) P–3, Q–4, R–2, S–1

Space for Notes :

EXERCISE – II

VERY SHORT ANSWER TYPE

1. How many metres of the carpet 75 cm wide will be required to cover the floor of a room which is 20 metres long and 12 metres broad ?

2. How many paving stones each measuring 2.5m × 2m are required to pave the rectangular courtyard 30 m long and 16.5 m wide ?

3. A rectangular grassy plot is 112 m by 78 m. It has a gravel path 2.5 m wide all round it on the inside. Find the area of the path and the cost of constructing it at Rs. 2 per square metre.

4. What will be the ratio of the circumference to the diameter of the circle if its original radius is tripled ?

5. There are two 2 m wide cross roads in a lawn 150 m by 120 m dimensions. One of the roads is parallel to the length and the other is parallel to the breadth. If it costs Rs. 2 per sq. metre for levelling the road, what would be the cost involved ?

6. A cuboidal vessel is 10 cm long and 8 cm wide. How high must it be made to hold 480 cubic centimetres of a liquid ?

7. Find the volume in cu. dm of the cube whose side is 1.2 m.

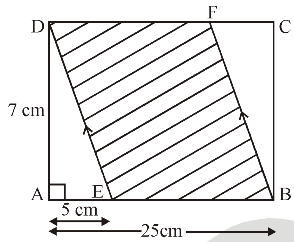
8. A cuboidal wooden box has length = 1.5 m, breadth = 25 cm and height = 15 cm. Find its volume.

9. The area of the base of a right cylinder is 154 cm^2 and its height is 15 cm. Find its volume.

10. A solid cube is cut into two cuboids of equal volumes. Find the ratio of the total surface area of the given cube to one of the cuboids.

SHORT ANSWER TYPE

1. Find the area of the shaded regions.



2. Eight identical cuboidal wooden blocks are stacked one on top of the order. The total volume of the solid so formed is 128 cm^3. If the height of each block is 1 cm and the base is a square, find the dimensions of each block.

3. A rectangular water reservoir contains 42000 litres of water. Find the depth of the water in the reservoir if its base measures 6 m by 3.5 m

4. What is the weight of a cubical block of ice 50 cm in length, if one cubic metre of ice weighs 900 kilograms ?

5. The radius of the base of a cylindrical waterdrum open at the top is 35 cm and the height is 1.3 m. Find the inner surface area of the waterdrum.

LONG ANSWER TYPE

1. The parallel sides of a trapezium are 20 cm and 10 cm. Its non–parallel sides are both equal, each being 13 cm. Find the area of the trapezium.

2. The length of a room is half more than its breadth. The cost of carpeting the room at Rs. 3.25 per m^2 is Rs. 175.50 and the cost of papering the walls at Rs. 1.40 per m^2 is Rs. 240.80. If 1 door and 2 windows occupy 8 m^2, find the dimensions of the room.

3. Theexternal length, breadth and height f a closed rectangular wooden box are 18 cm, 10 cm and 6 cm respectively and thickness o wood is $\frac{1}{2}$ cm. When the box is empty, it weighs 100 kg. Find the weight of the 1 cubic cm of wood and 1 cubic cm of sand.

4. An open rectangular cistern when measured from outside is 1.35 m long, 1.08 m broad and 90 cm deep and is made of iron which is 2.5 cm thick. Find the capacity of thecistern andthe volume of the iron used.

5. A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm andthickness 5 cm. Find the length of the pipe.

TRUE / FALSE TYPE

1. Area of a rectangle = Product of Adjacent sides
2. The area of an equilateral triangle with side 2a cm is $\frac{\sqrt{3}}{2}a^2\ cm^2$
3. The area of a sector with sector angle 60º is $\frac{1}{5}$th of the area of circle.
4. Area of an isosceles right triangle with hypotenuse $\sqrt{2}$ a is $\frac{1}{2}a^2$.
5. Base of a triangle = $\frac{2 \times \text{Area}}{\text{height}}$

NUMERICAL PROBLEMS

1. The cost of carpeting a room 18 m long with a carpet 75 cm wide at Rs. 4.50 per metre is Rs. 810. The breadth of the room is (k – 0.5)m. The value of k is.

2. The area of a rhombus whose diagonals are 10 cm and 12 cm is x cm^2 is x cm^2. The value of x is

3. The area of a parallelogram with base 14 cm and altitude 8 cm and 12 cm is x cm^2. The value of x is

5. The area of a triangular garden is 9520 m^2. If its base is 340 m, the altitude is a m. The value of $\frac{a}{8}$ is

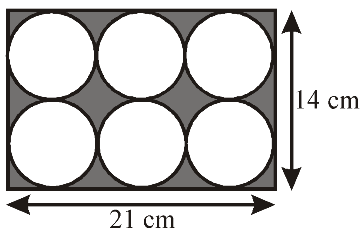
8. Unit digit of the surface area of a chalk box, whose length, breadth and height are 16 cm, 8 cm and 6 cm respectively, is

ANALYTICAL PROBLEMS & BRAIN TEASER

1. A swimming pool is 24 m long and 15 m broad, when a number of men dive into the pool, the height of the water rises by 1 cm. If the average amount of water displaced by one of the men be 0.1 cu. m, how many men are there in the pool ?

(A) 42 (B) 46
(C) 32 (D) 36

2. Sam cut out 6 identical circles from a rectangular piece of paper shown in the figure. Find the shaded area. $\left(\text{Take}\ \pi = \frac{22}{7}\right)$



(A) 62 cm^2 (B) 294 cm^2
(C) 63 cm^2 (D) 98 cm^2

3. The perimeter of the rectangular field is 406 m. What will be its area if its length is 43 m more than its breadth ?

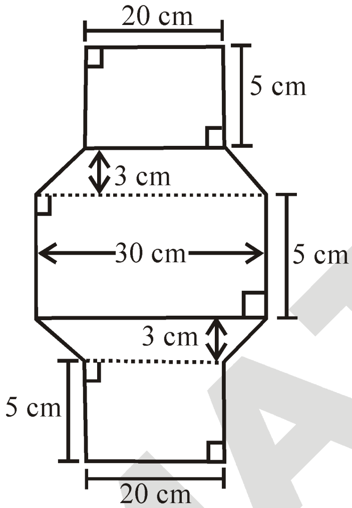
(A) 1520 m^2
(B) 9840 m^2
(C) 2480 m^2
(D) 8240 m^2

4. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with side 15 cm. The area (in cm^2) of the octagon is

(A) $\frac{30}{\sqrt{2}+1}$ (B) $\frac{450}{\sqrt{2}-1}$

(C) $\frac{30}{1-\sqrt{2}}$ (D) $\frac{450}{\sqrt{2}+1}$

5. Find the area of the given figure (not drawn to scale).



(A) 650 cm^2 (B) 500 cm^2
(C) 575 cm^2 (D) 525 cm^2

6. If the height of cylinder becomes $\frac{1}{2}$ of the original height and the radius is doubled, then volume of cylinder becomes _______ of its original volume.

(A) 2 times (B) $\frac{1}{2}$ times

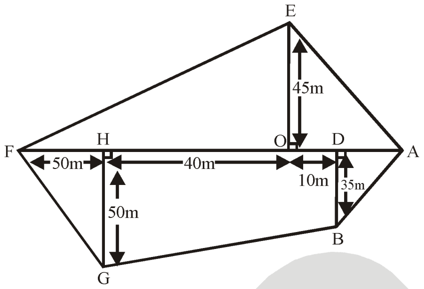
(C) $\frac{1}{4}$ times (D) 3 times

7. If the area of three adjacent faces of a cuboid are x, y and z respectively, then the volume of a cuboid is

(A) $\sqrt{xyz}$ (B) x + y + z

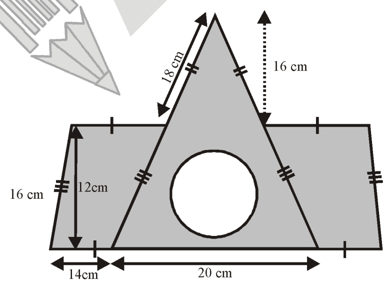
(C) x^2yz (D) xy + z

8. The area of the field ABGFEA (not drawn to scale) is



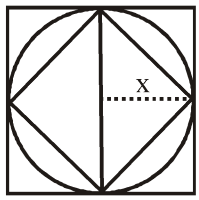
(A) 7225 m^2 (B) 7230 m^2
(C) 7235 m^2 (D) 7240 m^2

9. If radius of circle is 7 cm, then the perimeter of the figure and area shaded portion of the given figure respectively is



(A) 144 cm, 462 cm^2
(B) 156 cm, 462 cm^2
(C) 122 cm, 294 cm^2
(D) 144 cm, 394 cm^2

10. The difference of the area of the circumscribed andthe inscribed square of a circle is 35 sq. cm. Find the area of the circle.



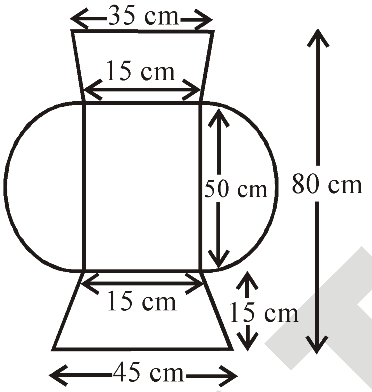
(A) 55 sq. m (B) 70 sq. m
(C) 55 sq. cm (D) 70 sq. m

11. A rectangular block of wood has dimensions 24 cm by 9 cm by 7 cm. It is cut into bricks. Each brick is a cube of side 3 cm. Find the largest number of bricks that can be cut from the block.

(A) 48 (B) 56

(C) 49 (D) 52

12. Find the area of the given figure (not drawn to scale).



(A) 3339.29 cm^2 (B) 3539.29 cm^2

(C) 4506.75 cm^2 (D) 5967.47 cm^2

13. Sum of the lengths of all edges of a cube is x metres. If the surface area of the cube is x sq. metres, then its volume (in cubic metres) is

(A) x^3 (B) 8

(C) x (D) 2

14. If one of the diagonals of a rhombus is equal to its side, then the diagonals of the rhombus are in the ratio

(A) $\sqrt{3}:1$ (B) $\sqrt{2}:1$

(C) 3 : 1 (D) 2 : 1

15. Three cubes with sides in the ratio 3 : 4 : 5 are melted to form a single cube whose diagonal is $12\sqrt{3}$ cm. The sides of the cube respectively are

(A) 6 cm, 8 cm, 10 cm

(B) 3 cm, 4 cm, 5 cm

(C) 9 cm, 12 cm, 15 cm

(D) 12 cm, 16 cm, 20 cm

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	B	C	D	D	C	A	D	D	B	D	C	B	A	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
D	B	C	D	D	B	C	C	D	B	B	A	B	D	A
31	32	33												
D	B	A												

EXERCISE II

VERY SHORT ANSWER TYPE

1. 320 m **2.** 99 **3.** 925 m^2, Rs. 1850 **4.** π **5.** Rs. 1072

6. 6 cm **7.** 1728 dm^3 **8.** 56250 cm^3 **9.** 2310 cm^3 **10.** 3 : 2

SHORT ANSWER TYPE

2. 140 cm^2 **3.** 4 cm × 4 cm × 1 cm **5.** 2 m **6.** 112.5 kg

8. 32450 cm^2

LONG ANSWER TYPE

1. 180 cm^2 **2.** L = 9 m, B = 6 m, H = 6 m **3.** $\frac{1}{9}$ kg **4.** 140575 cm^3

5. 112 m

TRUE / FALSE

1. T **2.** F **3.** F **4.** T **5.** T

NUMERICAL PROBLEMS

1. 8 **2.** 60 **3.** 2 **5.** 7 **8.** 4

ANALYTICAL PROBLEMS & BRAIN TEASER

1. D **2.** C **3.** B **4.** D **5.** B **6.** A **7.** A

8. A **9.** A **10.** D **11.** A **12.** B **13.** B **14.** A

15 . A

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : MENSURATION)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Solutions			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.
2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :