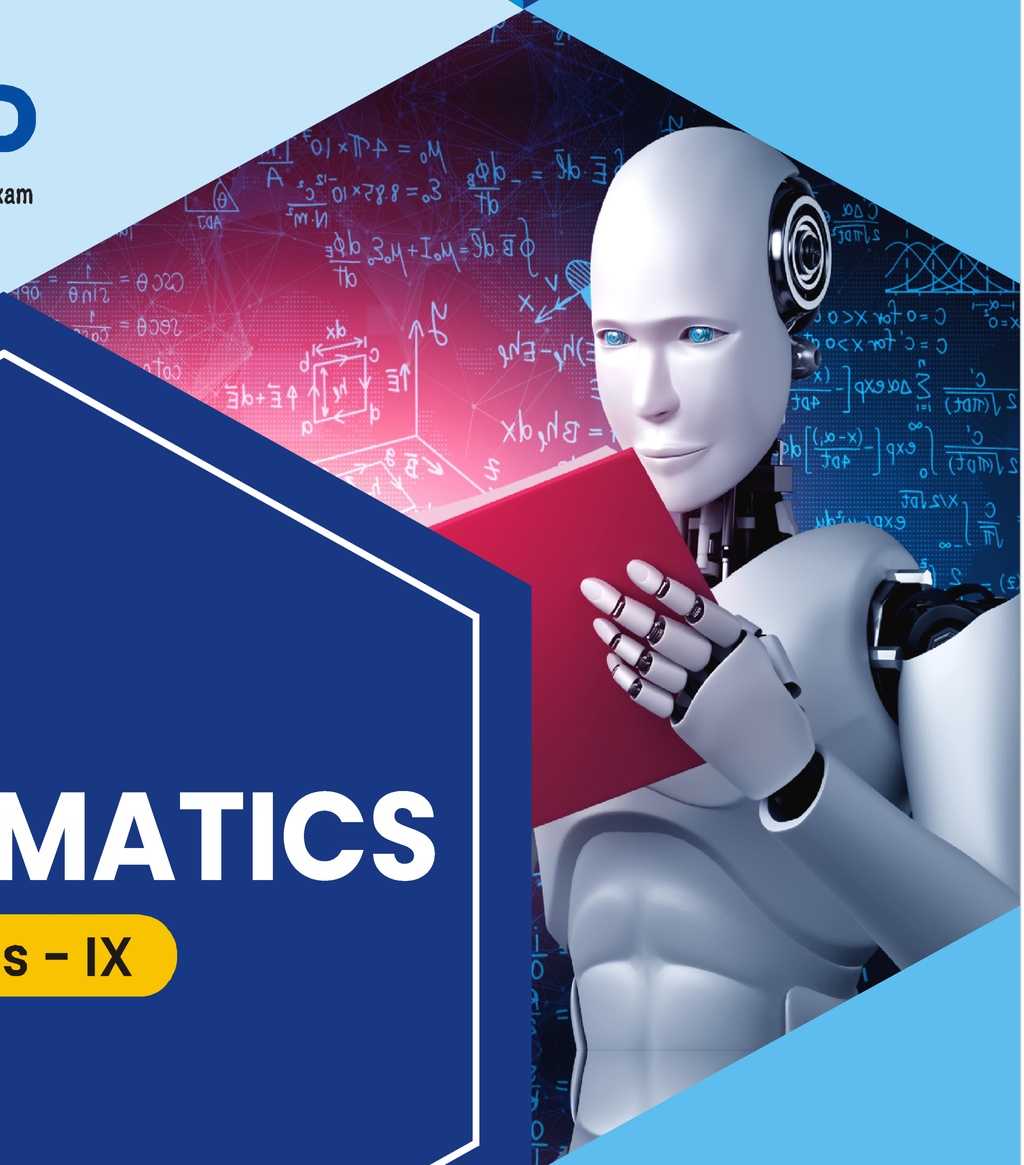
MATRIX OLYMPIAD

The Most Innovative Talent Recognition Exam



MATHEMATICS

Class - IX

MATRIX

Campus : Piprali Road, Sikar, Rajasthan 332001

Phone : 01572-241911, 01572-243911

Website: www.matrixedu.in

Few words for the Readers

Dear Reader,

"*Matrix Olympiad is established to encourage school students to go a step further than their regular studies, and get a chance and exposure to competition on a wide scale. It also helps students enhance their learning of basic cognitive skills and deeper knowledge of subjects like Science, Mathematics, English, Mental Ability, Social Studies.“Matrix Olympiad helps students nurture their minds for higher targets of tomorrow and enables them to study School for JEE, NEET, CLAT, NDA, Olympiads , NSEJS, NTSE , STSE etc.*"

The above thought has been our guiding principle while designing and collating the study material for **Matrix Olympiad** . And hence, we hope that this particular material will be helpful towards your preparation for **Matrix Olympiad**.

Our team at **MATRIX** has put in their best efforts for making this particular module interesting and relevant for you. Additional efforts have been made to ensure that the content is easy to understand and error free to the extent possible. However, there might remain some inadvertent errors in answer keys and theoretical portion and we would welcome your valuable feedback regarding the same.
If there are any suggestions for corrections, please write to us at **smd@matrixacademy.co.in** and we would be highly grateful.

Finally, we would like to end this message by a famous quote by Ernest Hemingway - "*There is no friend as loyal as a book*." So, please give your study material the time and attention it deserves, and it will surely help you reach newer heights in your fight with competition examinations.

With love and best wishes !
Team MATRIX

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NUMBER SYSTEM 1

Concepts

INTRODUCTION

In earlier classes, we have learnt about natural numbers, whole numbers and integers. As we have learnt that the counting numbers 1, 2, 3, ... etc. are called natural numbers and all the natural numbers together with zero are called whole numbers. In this chapter, we shall introduce the system of rational numbers and we shall also extend our study on real numbers, their decimal representation, representation on the number line and operations on real numbers.

1. TYPES OF NUMBERS

(i) Natural Numbers : Counting numbers are called natural numbers.

N = {1, 2, 3, 4,........} is a set of all natural numbers.

(ii) Whole Numbers : All counting numbers together with zero form a set of all whole numbers.

W = {0, 1, 2, 3, 4, ...} is a set of all whole numbers.

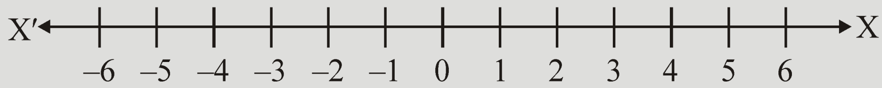
(iii) Integers : All natural numbers, 0 and negative of natural number form set of integers.

I = {..........–4, –3, –2, –1, 0, 1, 2, 3, ...} is a set of all integers.

Focus Point

All integers can be represented on the number line.

Number line :



Positive Integers : On the right hand side of 0, the points at distances of 1 unit, 2 units, 3 units etc. from 0 denote respectively the integers 1, 2, 3 etc.

Negative Integers : On the left side of 0, the points at distances of 1 unit, 2 units, 3 units etc. from 0 denote respectively the integers –1, –2, –3, ... etc.

Note : "0" is neither positive, nor negative.

2. RATIONAL NUMBERS

A number which can be expressed in the form $\frac{p}{q}$ where p, q are integers, and $q \neq 0$ is called a rational number. Each integer is a rational number, an integer m can be written as $\frac{m}{1}$ to put in the form $\frac{p}{q}$ where p, q are integers and $q \neq 0$.

Equivalent Rational Numbers : Rational numbers do not have a unique representation. For instance, $\frac{2}{3}$ can be represented by one of the following

$\frac{4}{6}, \frac{6}{9}, \frac{10}{15}, \frac{-44}{-66}, ...$

All such numbers are called equivalent rational numbers.

Focus Point

If a, b, c are three rational numbers, then

(i) Commutative property of addition : $a + b = b + a$

(ii) Associative property of addition : $(a + b) + c = a + (b + c)$

(iii) Inverse property of addition : $a + (-a) = 0$, where 0 is the identity element and $-a$ is called the inverse of a.

(iv) Commutative property of multiplication : $a \cdot b = b \cdot a$

(v) Associative property of multiplication : $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

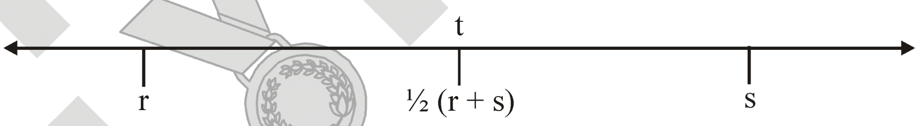
(vi) Multiplicative inverse property : $a \cdot \frac{1}{a} = 1$, where 1 is called the multiplicative identity and $\frac{1}{a}$ is called the multiplicative inverse of a or reciprocal of a.

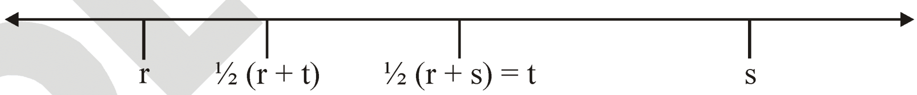
(vii) Distributive property of multiplication over addition : $a \cdot (b + c) = a \cdot b + a \cdot c$

3. INSERTING RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

Between any two distinct rational numbers x and y, there exists infinitely many rational numbers.

(i) First Method : To insert two rational numbers between r and s, we first insert the number $\frac{1}{2}(r + s) = t$(say) and then repeat the procedure with r and t or with t and s.





(ii) Second Method : If we wish to insert 'n' rational numbers between r and s. We write r and s as equivalent fractions, whose denominators are one more than n, the number of rational numbers to be inserted. That is, we write

$$r = \frac{r'}{n+1} \text{ and } s = \frac{s'}{n+1}$$

Then the desired n rational numbers are : $\frac{r'+1}{n+1}, \frac{r'+2}{n+1}, \frac{r'+3}{n+1}, \ldots\ldots, \frac{r'+n}{n+1}$

Example 1

Insert 5 rational numbers between 2 and 3.

Solution :

HWe write 2 and 3 as fractions whose denominator is 6.

$2 = \frac{2 \times 6}{6} = \frac{12}{6}$; $3 = \frac{3 \times 6}{6} = \frac{18}{6}$

Then the desired 5 rational numbers are : $\frac{13}{6}, \frac{14}{6}, \frac{15}{6}, \frac{16}{6}, \frac{17}{6}$

Example 2

Write 2 equivalent rational numbers of $\frac{2}{7}$.

Solution :

$\frac{2}{7} = \frac{2 \times 2}{7 \times 2} = \frac{4}{14}$; $\frac{2}{7} = \frac{2 \times 4}{7 \times 4} = \frac{8}{28}$

4. DECIMAL REPRESENTATION OF RATIONAL NUMBERS

4.1 FINITE TERMINATING DECIMAL

Every fraction $\frac{11}{9}$ can be expressed as a decimal, if the decimal expression of $\frac{p}{q}$ terminates, *i.e.* comes to an end, then the decimal so obtained is called a terminating decimal.

e.g., (i) $\frac{1}{4} = 0.25$ (ii) $\frac{5}{8} = 0.625$ (iii) $2\frac{3}{5} = \frac{13}{5} = 2.6$

Thus, each of the numbers $\frac{1}{4}, \frac{5}{8}$ and $2\frac{2}{3}$ can be expressed in the form of a terminating decimal.

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Focus Point

A fraction $\frac{p}{q}$ is a terminating decimal only, when prime factors of q are 2 and 5 only.

e.g. Each one of the fractions $\frac{1}{2}, \frac{3}{4}, \frac{7}{20}, \frac{13}{25}$ is a terminating decimal, since the denominator of each has no prime factor other then 2 and 5.

4.2 REPEATING (RECURRING) DECIMALS

A decimal in which a digit or a set of digits repeats periodically, is called a repeating or a recurring decimal.

In a recurring decimal, we place a bar over the first block of the repeating part and omit the other repeating blocks.

e.g. (i) $\frac{2}{3} = 0.666.................... = 0.\overline{6}$

(ii) $\frac{15}{7} = 2.142857142857............... = 2.\overline{142857}$

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Special Characteristics of Rational Numbers :

(i) Every rational number is expressible either as a terminating decimal or as a repeating decimal.

(ii) Every terminating decimal is a rational number.

(iii) Every repeating decimal is a rational number.

Example 3

Express $\frac{2}{11}$ in decimal form.

Solution :

By actual division, we have :

```
11) 2.0 (0.1818.....
   – 0
   ----
    2 0
   –1 1
   -----
      9 0
     –8 8
     ----
        2 0
       –1 1
       -----
          9 0
         –8 8
         ----
            2
```

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$\therefore \quad \frac{2}{11} = 0.1818......$

$= 0.\overline{18}$

Example 4

Express 15.75 in the form $\frac{p}{q}$

Solution :

We have, $15.75 = \frac{1575}{100} = \frac{1575 \div 25}{100 \div 25} = \frac{63}{4}$

5. CONVERSION OF DECIMAL NUMBERS INTO RATIONAL NUMBERS OF THE FORM P/Q

5.1 CONVERSION OF TERMINATING DECIMAL TO THE FORM P/Q

Step 1 : Count the number of numerals to the right of the decimal point. Let it be m.

Step 2 : Drop the decimal point and in the denominator write 1 followed by m zeros.

Step 3: Simplify the fraction.

Example 5

Convert 8.675 to the form $\frac{p}{q}$.

Solution :

Step 1 : Number of numerals to the right of decimal is 3 *i.e.* m = 3.

Step 2 : Write $8.675 = \frac{8675}{1000}$

Step 3 : Simplify (divide the numerator and denominator by 25) = $8.675 = \frac{347}{40}$.

5.2 CONVERSION OF PURE RECURRING DECIMAL TO THE FORM P/Q

Step 1 : Obtain the repeating decimal and put it equal to x.

Step 2 : Write the number in decimal form by removing bar from the top of repeating digits and listing repeating digits at least twice.

e.g. write $x = 0.\overline{8}$ as $x = 0.888.......$

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Step 3 : Determine the no. of digits having bar on their heads.

Step 4 : If the repeating decimal has 1 place repetition, multiply by 10, a two place repetition, multiply by 100, a three place repetition, multiply by 1000 and so on.

Step 5: Subtract the number in step 2 from the numbers obtained in step 4.

Step 6 : Divide both sides of the equation by the coefficient of x.

Step 7: Write the rational number in its simplest form.

Example 6

Express $0.\overline{585}$ in the form $\frac{p}{q}$.

Solution :

Let $x = 0.\overline{585}$

$\Rightarrow$ x = 0.585585585 ....(i)

Here, we have 3 repeating digits after the decimal point. So, multiply both sides of (i) by $10^3 = 1000$ to get

$\Rightarrow$ 1000 x = 585.585585 ....

Subtracting (i) from (ii), we get

$\Rightarrow$ 1000 x – x = (585.585585 ....) – (0.585585...)

$\Rightarrow 999\,x = 585 \Rightarrow x = \frac{585}{999}$

5.3 CONVERSION OF A MIXED RECURRING DECIMAL TO THE FORM P/Q

Step 1: Obtain the mixed recurring decimal and write it equal to x.

Step 2: Determine the number of digits after the decimal point which do not have bar on them. Let there be n digits without bar just after the decimal point.

Step 3: Multiply both sides of x by 10^n, so that only the repeating decimal is on the right side of the decimal point.

Step 4: Use the method of converting pure recurring decimal to the form $\frac{p}{q}$ and obtain the value of x.

Example 7

Express $0.12\overline{3}$ in the form $\frac{p}{q}$.

Solution :

Let x = $0.12\overline{3}$ .....(i)

The no of digits after the decimal point which do not have bar on them is 2.

$\therefore$ Multiply both sides of x by 10^2.

$\Rightarrow \ 100x = 12.\overline{3}$ .....(ii)

Here, we have 1 repeating digit after the decimal point. So, multiply both sides of (ii) by 10 to get.

$\Rightarrow \ 1000\ x = 123.33 \ldots\ldots\ldots$ .....(iii)

Subtracting (ii) from (iii)

$\Rightarrow \ 1000\ x - 100\ x = (123.33 \ldots\ldots) - (12.33 \ldots\ldots.)$

$\Rightarrow \ 900\ x = 111$

$\Rightarrow \ x = \dfrac{111}{900} = \dfrac{37}{300}$

6. IRRATIONAL NUMBERS

A number is a irrational number, if it has a non terminating and non-repeating decimal representation. A number that cannot be put in the form $\dfrac{p}{q}$ where, p, q are integers and $q \neq 0$ is called irrational number.

e.g. $\sqrt{2}, \sqrt{3}, \sqrt{11}, \pi$.

Focus Point

Real Numbers :

- The collection of real numbers consists of all the rational and irrational numbers and is denoted by R.
- Every real number corresponds to a point on the line and conversely, every point on the number line represents a real number.

6.1 REPRESENTING THE SQUARE ROOT OF A POSITIVE NUMBER ON THE NUMBER LINE

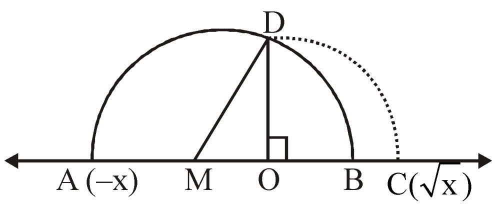
Let x be a positive real number. We will now locate $\sqrt{x}$ on the number line.

Step 1 : Mark –x on the number line. Let this point be represented by A. Mark 1 unit on the number line. Let this be represented by B.

Step 2 : Locate the midpoint M of AB.

Step 3 : With M as the centre and MA or MB as radius draw a semicircle. Since diameter AB = (x + 1) units, MA = MB = ½ (x + 1) units.

Step 4 : Draw OD perpendicular to AB meeting the semicircle in D. Join MD. Note the Δ DMO is a right triangle with MD = ½ (x + 1) units and MO = [½ (x + 1) – 1] units = [½ (x – 1)] units.



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Step 5 : Using the Pythagorean theorem, we obtain :

$OD^2 = MD^2 - MO^2$

$= \frac{1}{4}(x + 1)^2 - \frac{1}{4}(x - 1)^2$

$= \frac{1}{4}(4x)$

$= x$

$= OD = \sqrt{x}$

With O as the centre and OD as the radius, draw an arc to meet the number line at C. The point C represents $\sqrt{x}$.

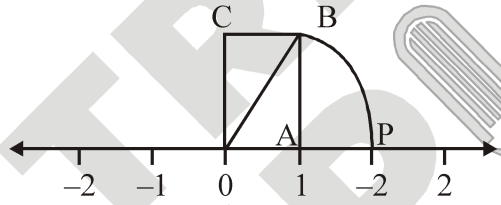
Example 8

Locate $\sqrt{2}$ on the number line.

Solution :

Step 1 : Draw the number line with O representing the number 0 and A representing the number 1.

Step 2 : Construct a square OABC with each side equal to 1 unit.



By the Pythagorean theorem :

$OB^2 = OA^2 + AB^2$

$= 1^2 + 1^2$

$= 1 + 1 = 2$

$OB = \sqrt{2}$

Step 3 : With O as centre an OB as radius, draw an arc to meet the number line at point P.

Since OP = OB = $\sqrt{2}$, the point P represents $\sqrt{2}$ on the number line.

6.2 REPRESENTING REAL NUMBERS ON THE NUMBER LINE

The decimal representation of a real number is quite useful to find its location on the number line.

Example 9

Visualise 2.437 on the number line.

Solution :

Observe that 2.437 lies between 2 and 3.

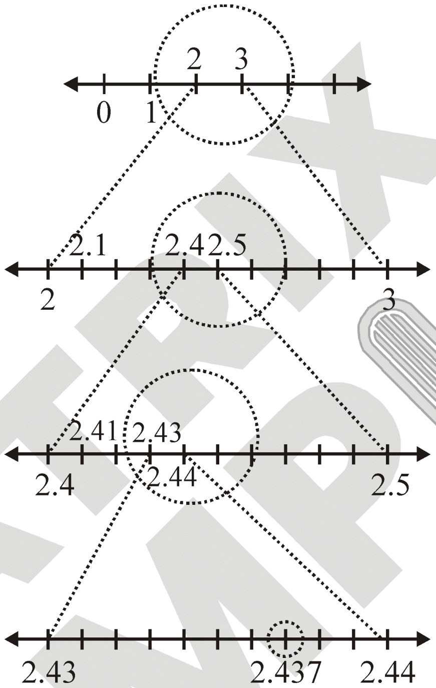
Step 1 : Locate 2 on the number line.

Step 2 : Locate 2.4 on the number line as follows.

Divide the segment between 2 and 3 into ten equal parts and mark each point of the division. The first mark is 2.1, the second 2.2, and so on. The fourth mark represents point 2.4.

Step 3 : Locate 2.43 on the number line as follows : Divide the segment 2.4 to 2.5 into equal parts. Mark the first part 2.41, the next as 2.42 and so on. The third mark repersents the number 2.43.

Step 4 : Locate 2.437 on the number line by dividing the segment 2.43 to 2.44 into 10 equal parts. Mark each part and take the 7^{th} part.



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Operations on Real Numbers

(i) The sum, difference, or product of two rational numbers is again a rational number.

(ii) For all rational numbers a and b, the quotient $\frac{a}{b}$, where $b \neq 0$, is a rational number.

(iii) The sum or difference of a rational and an irrational number is always an irrational number.

(iv) The product or quotient of a non zero rational number and an irrational number is an irrational number.

(v) The sum, difference, product and quotient of two irrational numbers may be rational or irrational.

Example 10

Classify the following numbers as rational or irrational.

(i) $2\sqrt{7}$

(ii) $(3+\sqrt{29}-\sqrt{29})$

(iii) $(3+\sqrt{5})(3-\sqrt{5})$

Solution :

(i) $2\sqrt{7}$ is irrational.

Assume on the contrary that $2\sqrt{7}$ is a rational number, say, r. Then :

$2\sqrt{7} = r$

$\Rightarrow \sqrt{7} = \frac{r}{2}$

As both r and 2 are rational numbers, $\frac{r}{2}$ is a rational number $\sqrt{7}$ is a rational number, which is a contradiction.

Thus, $2\sqrt{7}$ must be irrational.

(ii) $(3+\sqrt{29})-\sqrt{29} = 3,$ which is a rational number.

(iii) $(3+\sqrt{5})(3-\sqrt{5}) = 3^2-(\sqrt{5})^2 = 9-5=4,$ which is a rational number.

7. SURDS OR RADICALS

- An expression written under a radical sign is called a radical expression. The radicand is the number under the radical.
- A surd is the simplest type of irrational number, one whose radicand is a rational number.
- *e.g.* $\sqrt{5}, 3\sqrt{7}$ and $\frac{1}{\sqrt{3}}$ are surds whereas $3\sqrt{5-\sqrt{2}}$ and $\sqrt{\sqrt{3}}$ are not surds.
- The order of a surd is indicated by its index.
- The order of a radical is the denominator of its fractional exponent.
- Order $\rightarrow$ $\sqrt[n]{a} = a^{\frac{1}{n}}$; order = n

7.1 TYPES OF SURDS

(i) Pure surd : A surd in which the whole of the rational number is under the radical sign. & makes the radicand, is called pure surd.

For *e.g.* $\sqrt{10}, \sqrt[3]{50}, \sqrt[4]{6}$ etc.

(ii) Mixed surd : A surd which has a rational factor other than unity, the other factor being irrational, is called a mixed surd.

For *e.g.* $2\sqrt{3}, 5\sqrt[3]{12}, 2\sqrt[4]{5}$ are mixed surds.

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- Every surd is an irrational number but every irrational number is not a surd.
- A surd consisting of one term only is called a monomial surd.
- An expression consisting of the sum or difference of two monomial surds or the sum or difference of a monomial surd and a rational number is called binomial surd. *e.g.* $\sqrt{2}+\sqrt{5}, \sqrt{3}+2, \sqrt{2}-\sqrt{3}$ etc.
- The binomial surds which differ only in sign (+ or –) between the terms connecting them, are called conjugate surds *e.g.* $\sqrt{3}+\sqrt{2}$ and $\sqrt{3}-\sqrt{2}$ or $2+\sqrt{5}$ and $2-\sqrt{5}$ are conjugate surds.

7.2 LAWS OF RADICALS

If a, b are positive rational numbers and m, n, p are positive integers, then :

(i) $(\sqrt[n]{a})^n = a$

(ii) $\sqrt[n]{a^n} = a = \left(\sqrt[n]{a}\right)^n$

(iii) $\left(\sqrt[n]{a}\right)(\sqrt[n]{b}) = \sqrt[n]{ab}$

(iv) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

(v) $\sqrt[n]{a^p} = \sqrt[mn]{a^{pm}}$

Example 11

Simplify the following :

(i) $\sqrt[4]{1875}$ (ii) $\sqrt{\dfrac{125}{63}}$ (iii) $3\sqrt[5]{7}$

Solution :

(i) We have $1875 = 5^4 \times 3$

$$\sqrt[4]{1875} = \sqrt[4]{5^4 \times 3} = 5\sqrt[4]{3}$$

(ii) $\sqrt{\frac{125}{63}} = \sqrt{\frac{125}{3^2 \times 7}} = \sqrt{\frac{5^2 \times 5}{3^2 \times 7}} = \frac{5}{3}\sqrt{\frac{5 \times 7}{7 \times 7}} = \frac{5}{21}\sqrt{35}$

(iii) $3\sqrt[5]{7} = \sqrt[5]{3^5 \times 7} = \sqrt[5]{243 \times 7} = \sqrt[5]{1701}$

Example 12

Simplify the following.

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

(ii) $\sqrt{252} - \sqrt{6} + \sqrt{294} - 3\sqrt{\frac{1}{6}}$

(iii) $\sqrt[3]{3} \times \sqrt[4]{5}$

(iv) $\frac{\sqrt[6]{12}}{(\sqrt{3})(\sqrt[3]{2})}$

Solution :

(i) We first transform the radicals :

$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

We have,

$\sqrt{45} - 3\sqrt{20} + 4\sqrt{5} = 3\sqrt{5} - 3(2\sqrt{5}) + 4\sqrt{5}$

$= (3 - 6 + 4)\sqrt{5} = \sqrt{5}$

(ii) $\sqrt{252} = \sqrt{2^2 \times 3^2 \times 7} = 2 \times 3\sqrt{7} = 6\sqrt{7}$

$\sqrt{294} = \sqrt{2 \times 3 \times 7^2} = 7\sqrt{6}$

Thus, $\sqrt{252} - 5\sqrt{6} + \sqrt{294} - 3\sqrt{\frac{1}{6}} = 6\sqrt{7} - 5\sqrt{6} + 7\sqrt{6} - \frac{3}{6}\sqrt{6} = 6\sqrt{7} + \left(-5 + 7 - \frac{1}{2}\right)\sqrt{6} = 6\sqrt{7} + \frac{3}{2}\sqrt{6}$

(iii) We have LCM of 3 and 4 is 12. Now,

$$\sqrt[3]{3} = 3^{\frac{1}{3}} = \left(3^4\right)^{\frac{1}{12}} = \sqrt[12]{81}$$

$$\sqrt[4]{5} = 5^{\frac{1}{4}} = \left(5^3\right)^{\frac{1}{12}} = \sqrt[12]{125}$$

Thus, $(\sqrt[3]{5}) \times (\sqrt[4]{5}) = (\sqrt[12]{81}) \times (\sqrt[12]{125}) = \sqrt[12]{81 \times 125} = \sqrt[12]{10125}$

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(iv) We first multiply $(\sqrt{3})$ and $(\sqrt[3]{2})$

The LCM of 2 and 3 is 6 :

Thus, $\sqrt{3} = 3^{\frac{1}{2}} = (3^3)^{\frac{1}{6}} = \sqrt[6]{27}$

and, $\sqrt[3]{2} = 2^{\frac{1}{3}} = (2^2)^{\frac{1}{6}} = \sqrt[6]{4}$

$\therefore \quad (\sqrt{3}) \times (\sqrt[3]{2}) = \sqrt[6]{27} \times \sqrt[6]{4} = \sqrt[6]{27 \times 4}$

Now, $\dfrac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}} = \dfrac{\sqrt[6]{12}}{\sqrt[6]{27 \times 4}} = \sqrt[6]{\dfrac{12}{27 \times 4}}$

$\Rightarrow \sqrt[6]{\dfrac{1}{9}} = \dfrac{1}{\sqrt[3]{3}}$

7.3 RATIONALISATION OF REAL NUMBERS

(i) $\dfrac{1}{a + b\sqrt{x}}$ (ii) $\dfrac{1}{\sqrt{x} + \sqrt{y}}$

We can simplify the expression involving real numbers of the types $\dfrac{a}{\sqrt{x}}, \dfrac{1}{a + b\sqrt{x}}, \dfrac{1}{\sqrt{x} + \sqrt{y}}$ and $\dfrac{1}{a\sqrt{x} + b\sqrt{y}}$ where a, b are non-zero integers and x, y are positive real numbers.

To make the denominators in such cases free from square roots such as $\sqrt{x}$ and $\sqrt{y}$, we multiply the numerator and denominator by a suitable factor. This factor is called the **rationalising factor**. The process is known as simplification by rationalising the denominator.

For example :

(i) $\dfrac{3}{\sqrt{5}} = \dfrac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \dfrac{3\sqrt{5}}{5}$.

(ii) $\dfrac{1}{3 + \sqrt{5}} = \dfrac{1 \times (3 - \sqrt{5})}{(3 + \sqrt{5}) \times (3 - \sqrt{5})} = \dfrac{3 - \sqrt{5}}{9 - 5} = \dfrac{3 - \sqrt{5}}{4}$.

(iii) $\dfrac{2}{\sqrt{5} - \sqrt{3}} = \dfrac{2 \times (\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3}) \times (\sqrt{5} + \sqrt{3})} = \dfrac{2 \times (\sqrt{5} + \sqrt{3})}{5 - 3} = \sqrt{5} + \sqrt{3}$.

Thus, we observe that $\sqrt{5}, 3 - \sqrt{5}$ and $\sqrt{5} + \sqrt{3}$ are respectively, the rationalising factors for the numbers $\dfrac{3}{\sqrt{5}}, \dfrac{1}{3 + \sqrt{5}}$ and $\dfrac{2}{\sqrt{5} - \sqrt{3}}$.

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- For $\frac{1}{a\sqrt{x}+b\sqrt{y}}$, the rationalising factor is $a\sqrt{x}-b\sqrt{y}$.
- For $\frac{1}{a-b\sqrt{x}}$, the rationalising factor is $a+b\sqrt{x}$.
- For $\frac{1}{a\sqrt{x}-b\sqrt{y}}$, the rationalising factor is $a\sqrt{x}+b\sqrt{y}$.

Example 13

Rationalise the denominator of $\frac{5}{\sqrt{3}-\sqrt{5}}$.

Solution :

$$\frac{5}{\sqrt{3}-\sqrt{5}}=\frac{5}{\sqrt{3}-\sqrt{5}}\times\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}=\frac{5(\sqrt{3}+\sqrt{5})}{3-5}=\left(\frac{-5}{2}\right)(\sqrt{3}+\sqrt{5})$$

Example 14

Rationalise the denominator of $\frac{1}{7+3\sqrt{2}}$.

Solution :

$$\frac{1}{7+3\sqrt{2}}=\frac{1}{7+3\sqrt{2}}\times\left(\frac{7-3\sqrt{2}}{7-3\sqrt{2}}\right)=\frac{7-3\sqrt{2}}{49-18}=\frac{7-3\sqrt{2}}{31}$$

Example 15

Rationalise the denominator of $\frac{a^2}{\sqrt{a^2+b^2}+b}$.

Solution :

$$\frac{a^2}{\sqrt{a^2+b^2}+b}\times\frac{\sqrt{a^2+b^2}-b}{\sqrt{a^2+b^2}-b}=\frac{a^2(\sqrt{a^2+b^2}-b)}{(\sqrt{a^2+b^2})^2-(b)^2}=\frac{a^2(\sqrt{a^2+b^2}-b)}{a^2+b^2-b^2}=\sqrt{a^2+b^2}-b$$

8. LAWS OF EXPONENTS FOR REAL NUMBERS

Positive integral power : For any real number a and a positive integer 'n', we define a^n as $a^n = a \times a \times a \times$ ....$\times$ a(n times). a^n is called the n^{th} power of a. The real number 'a' is called the base 'n' is called the exponent of the n^{th} power of a.

8.1 RATIONAL EXPONENTS OF REAL NUMBERS

If a is a positive real number and 'n' is a positive integer, then the principal n^{th} root of a is the unique positive real number x such that $x^n = a$.

The principal n^{th} root of a positive real number 'a' is denoted by $a^{\frac{1}{n}}$ or $\sqrt[n]{a}$.

For Rational Powers : For any positive real number 'a' and a rational number $\frac{p}{q}$, where q > 0, we define $a^{\frac{p}{q}} = (a^p)^{\frac{1}{q}}$.

i.e., $a^{\frac{p}{q}}$ is the principal q^{th} root of a^p.

8.2 FOR INTEGER OR RATIONAL EXPONENTS

First law : For any non-zero real number 'a', we define $a^0 = 1$

Second law : $a^m \times a^n = a^{m+n}$.

Third law : $\frac{a^m}{a^n} = a^{m-n}$

Case I : When m > n, $a^m \div a^n = a^{-(n-m)} = \frac{1}{a^{n-m}}$

Case II : When m = n, $a^m \div a^n = a^0 = 1$

Case III : When m < n, $a^m \div a^n = a^{-(n-m)} = \frac{1}{a^{n-m}}$

Fourth law : $(a^m)^n = a^{mn} = (a^n)^m$

Fifth law : (i) $(ab)^n = a^n b^n$ (ii) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}; b \neq 0$

Sixth law : $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$ *i.e.*, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Seventh law : $a^{b^n} = a^{b \times b \times b \ldots\ldots n}$ times

Here a, b are positive real numbers and m, n are rational numbers.

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Example 16

Simplify : $\left(\frac{x^l}{x^{-m}}\right)^{l^2+m^2-lm} \times \left(\frac{x^m}{x^{-n}}\right)^{m^2+n^2-mn} \times \left(\frac{x^n}{x^{-l}}\right)^{n^2+l^2-nl}$

Solution :

We have, $\left(\frac{x^l}{x^{-m}}\right)^{l^2+m^2-lm} \times \left(\frac{x^m}{x^{-n}}\right)^{m^2+n^2-mn} \times \left(\frac{x^n}{x^{-l}}\right)^{n^2+l^2-nl} X^{(l+m)(l^2-lm+m^2)} \times X^{(m+n)(m^2-mn+n^2)} \times X^{(n+l)(n^2-nl+l^2)}$

$= (x^{l+m})^{(l^2+m^2-lm)} \times (x^{m+n})^{(m^2+n^2-mn)} \times (x^{n+l})^{(n^2+l^2-nl)}$

Then

$X^{(l+m)(l^2-lm+m^2)} \times X^{(m+n)(m^2-mn+n^2)} \times X^{(n+l)(n^2-nl+l^2)}$

$= x^{l^3+m^3} \times x^{m^3+n^3} \times x^{n^3+l^3} \quad = x^{2(l^3+m^3+n^3)}$

Example 17

Simplify : $\left(\frac{2^a}{2^b}\right)^{a+b} \cdot \left(\frac{2^b}{2^c}\right)^{b+c} \cdot \left(\frac{2^c}{2^a}\right)^{c+a}$

Solution :

We have, $\left(\frac{2^a}{2^b}\right)^{a+b} \cdot \left(\frac{2^b}{2^c}\right)^{b+c} \cdot \left(\frac{2^c}{2^a}\right)^{c+a}$

$= (2^{a-b})^{a+b} \cdot (2^{b-c})^{b+c} \cdot (2^{c-a})^{c+a}$

$= 2^{a^2-b^2} \cdot 2^{b^2-c^2} \cdot 2^{c^2-a^2}$

$= 2^{a^2-b^2+b^2-c^2+c^2-a^2}$

$= 2^0$

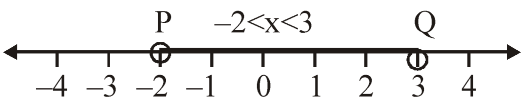
$= 1$

MATRIX OLYMPIAD

SE. 1

Represent the real numbers given by $-2 < x < 3$ on the number line.

Ans. Since all the real numbers given by $-2 < x < 3$, *i.e.* lying between -2 and 3 can be represented on the number line by the dark portion as follows:



SE. 2

Find the value of $\sqrt{4.3}$ geometrically.

Ans. (i) Draw a line segment AB = 4.3 units and extend it to C. such that BC = 1 unit.

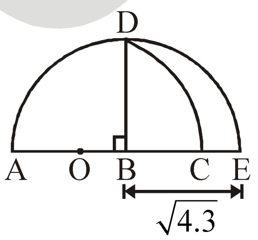
(ii) Find the midpoint O of AC.

(iii) With O as centre and OA as radius, draw a semicircle.

(iv) Now, draw BD ⊥ AC, intersecting the semi-circle at D.

Then, $BD = \sqrt{4.3}$ units.

With B as centre and BD as radius, draw an arc, meeting AC produced at E. Then $BE = BD = \sqrt{4.3}$ units.



SE. 3

Express the following in the form $\frac{p}{q}$

(i) $5.\overline{2}$

(ii) $15.7\overline{12}$

Ans. **(i)** Let $x = 5.\overline{2}$

$\Rightarrow \quad x = 5.2222 \ldots\ldots \qquad \ldots..(i)$

Multiplying both sides of (i) by 10, we get :

$\Rightarrow \quad 10x = 52.222\ldots\ldots \qquad \ldots..(ii)$

Subtracting (i) from (ii), we get :

$\Rightarrow \quad 10x - x = (52.222\ldots.) - (5.22\ \ldots..)$

$\Rightarrow \quad 9x = 47$

$\Rightarrow \quad x = \frac{47}{9}$

(ii) Let $x = 15.7\overline{12}$ then,

Multiplying both side by 10.

$\Rightarrow \quad 10x = 157.\overline{12}$

$\Rightarrow \quad 157.121212 \ldots\ldots \qquad \ldots.(i)$

Multiplying both sides of (i) by 100, we get :

$\Rightarrow \quad 1000x = 15712.1212 \ldots. \qquad \ldots.(ii)$

Subtracting (i) from (ii), we get

$\Rightarrow 1000x - 10x = (15712.1212 \ldots\ldots) - (157.121212)$

$\Rightarrow 990x = 15555$

$\Rightarrow \quad x = \frac{15555}{990}$

$\Rightarrow \quad x = \frac{5185}{330}$

SE. 4

Express $\sqrt[3]{4}$ as a surd of order 12.

Ans. Now, $\sqrt[3]{4} = \sqrt[3\times4]{4^4}$

[We are to convert order 3 into 12]

$\Rightarrow \quad \sqrt[12]{4\times4\times4\times4}$

$\Rightarrow \quad \sqrt[12]{256}$

SE. 5

Show that :

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}=1$$

Ans. We have,

$$=\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}$$

$$=\frac{x^a}{x^a+x^{b-a+a}+x^{c-a+a}}+\frac{x^b}{x^b+x^{a-b+b}+x^{c-b+b}}+\frac{x^c}{x^c+x^{b-c+c}+x^{a-c+c}}$$

[Multiplying numerator and denominator of three terms by x^a, x^b and x^c respectively]

$$=\frac{x^a}{x^a+x^b+x^c}+\frac{x^b}{x^b+x^a+x^c}+\frac{x^c}{x^c+x^b+x^a}$$

$$=\frac{x^a+x^b+x^c}{x^a+x^b+x^c}=1$$

SE. 6

Simplify by rationalising the denominator. $\dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Ans. Multiplying and dividing by the relationalising factor of the denominator, we have

$$=\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}\times\frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}$$

$$=\frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{2})}$$

$$=\frac{2\sqrt{3}\times2\sqrt{2}-2\sqrt{3}\times3\sqrt{3}-\sqrt{5}\times2\sqrt{2}+3\sqrt{5}\sqrt{3}}{\left(2\sqrt{2}\right)^2-\left(3\sqrt{3}\right)^2}$$

$$=\frac{4\sqrt{3\times2}-6\sqrt{3\times3}-2\sqrt{5\times2}+3\sqrt{5\times3}}{4\times2-9\times3}$$

$$=\frac{4\sqrt{6}-6\times3-2\sqrt{10}+3\sqrt{15}}{8-27}$$

$$=\frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19}$$

$$=\frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

SE. 7

Simplify : $\dfrac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\dfrac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}+\dfrac{2\sqrt{3}}{\sqrt{6}+2}$.

Ans. $\dfrac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\dfrac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}+\dfrac{2\sqrt{3}}{\sqrt{6}+2}=T_1-T_2+T_3$ (say)

MATRIX OLYMPIAD

$$T_1 = \frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} \times \frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}$$

$$= \frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{6-3}$$

$$= \sqrt{2}(\sqrt{6}+\sqrt{3})$$

$$= 2\sqrt{3}+\sqrt{6}$$

$$T_2 = \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

$$= \frac{4\sqrt{18}+4\sqrt{6}}{6-2}$$

$$= \frac{4(3\sqrt{2}+\sqrt{6})}{4}$$

$$= 3\sqrt{2}+\sqrt{6}$$

$$T_3 = \frac{2\sqrt{3}}{\sqrt{6}+2} \times \frac{\sqrt{6}-2}{\sqrt{6}-2}$$

$$= \frac{2\sqrt{3}(\sqrt{6}-2)}{6-(2)^2}$$

$$= 3\sqrt{2}-2\sqrt{3}$$

$\therefore$ Given expression $= T_1 - T_2 + T_3$

$$= 2\sqrt{3}+\sqrt{6}-3\sqrt{2}-\sqrt{6}+3\sqrt{2}-2\sqrt{3}$$

$$= 0$$

SE. 8

Solve the equation : $2^{2x+2} = 2^{3x-1}$.

Ans. $2^{2x+2} = 2^{3x-1}$

Equating the powers, as the bases are same,

$\therefore \quad 2x + 2 = 3x - 1$

$\Rightarrow \quad 2x - 3x = -1 - 2$

$\Rightarrow \quad -x = -3 \quad \text{or} \quad x = 3$

SE. 9

If $x = \dfrac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$, then show that $bx^2 - ax + b = 0$.

Ans. $$x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}} \times \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}+\sqrt{a-2b}}$$

$$\Rightarrow x = \frac{\left(\sqrt{a+2b}+\sqrt{a-2b}\right)^2}{(a+2b)-(a-2b)}$$

$$\Rightarrow x = \frac{a+2b+a-2b+2\sqrt{(a+2b)(a-2b)}}{4b}$$

$$\Rightarrow x = \frac{2(a+\sqrt{(a^2-4b^2)})}{2\times 2b}$$

$$\Rightarrow 2bx = a + \sqrt{(a^2-4b^2)}$$

$$\Rightarrow 2bx - a = \sqrt{(a^2-4b^2)}$$

On squaring both sides, we get :

$\Rightarrow \quad 4b^2x^2 + a^2 - 4abx = a^2 - 4b^2$

$\Rightarrow \quad 4b^2x^2 - 4abx + 4b^2 = 0$

Dividing by 4b, we get, $bx^2 - ax + b = 0$

Hence the result.

EXERCISE – I

ONLY ONE CORRECT TYPE

1. $0.12\overline{3}$ can be expressed in rational form as

(A) $\frac{900}{111}$ (B) $\frac{111}{900}$

(C) $\frac{123}{10}$ (D) $\frac{121}{900}$

2. If $x \geq 0$, then $\sqrt{x\sqrt{x\sqrt{x}}} =$

(A) $x\sqrt{x}$ (B) $x\sqrt[4]{x}$

(C) $\sqrt[8]{x}$ (D) $\sqrt[8]{x^7}$

3. $\frac{7\sqrt{3}}{(\sqrt{10}+\sqrt{3})} - \frac{2\sqrt{5}}{(\sqrt{6}+\sqrt{5})} - \frac{3\sqrt{2}}{(\sqrt{15}+3\sqrt{2})} =$

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) 3

4. The rationalising factor of $\sqrt[5]{a^2b^3c^4}$ is

(A) $\sqrt[5]{a^3b^2c}$ (B) $\sqrt[4]{a^3b^2c}$

(C) $\sqrt[3]{a^3b^2c}$ (D) $\sqrt[5]{a^3b^2c}$

5. $\frac{1}{(\sqrt{3}-\sqrt{2})}$ is not equal to

(A) $\sqrt{3}+\sqrt{2}$ (B) $\frac{\sqrt{2}}{(\sqrt{6}-2)}$

(C) $\frac{(\sqrt{3}-\sqrt{2})}{(5-2\sqrt{6})}$ (D) $\frac{\sqrt{3}}{(9-\sqrt{6})}$

6. The ascending order of the surds $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$ is

(A) $\sqrt[9]{4}, \sqrt[6]{3}, \sqrt[3]{2}$ (B) $\sqrt[9]{4}, \sqrt[3]{2}, \sqrt[6]{3}$

(C) $\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$ (D) $\sqrt[6]{3}, \sqrt[9]{4}, \sqrt[3]{2}$

7. Rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(A) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}\times\sqrt{3}}{2}$

(C) 1.5 (D) 1.8

8. The value of $\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$ is equal to

(A) $x^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$ (B) 0

(C) $x^{pq+qr+rp}$ (D) 1

9. The rational number between $\frac{1}{2}$ and $\frac{1}{3}$ is

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

10. If $\left(a+\frac{1}{a}\right)^2 = 9$, then $a^3+\frac{1}{a^3}$ equals

(A) $\frac{10\sqrt{3}}{3}$ (B) $3\sqrt{3}$

(C) 18 (D) $7\sqrt{7}$

11. If both 'a' and 'b' are rational numbers, then 'a' and 'b' from $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5}-b$, respectively are

(A) $\frac{9}{11}, \frac{19}{11}$ (B) $\frac{19}{11}, \frac{9}{11}$

(C) $\frac{2}{11}, \frac{8}{11}$ (D) $\frac{10}{11}, \frac{21}{11}$

12. $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ equals

(A) $\sqrt{2}+\sqrt{3}-\sqrt{5}$ (B) $4-\sqrt{2}-\sqrt{3}$

(C) $\sqrt{2}+\sqrt{3}+\sqrt{6}-5$ (D) $\frac{1}{2}(\sqrt{2}+\sqrt{5}-\sqrt{3})$

13. If $25^{x-1} = 5^{2x-1} - 100$, then the value of x is

(A) 3 (B) 2

(C) 4 (D) 1

14. Which of the following numbers has the terminating decimal representation ?

(A) $\frac{1}{7}$ (B) $\frac{1}{3}$

(C) $\frac{3}{5}$ (D) $\frac{17}{3}$

15. If $\frac{2^{m+n}}{2^{n-m}}=16$, $\frac{3^p}{3^n}=81$ and $a=2^{\frac{1}{10}}$, then $\frac{a^{2m+n-p}}{(a^{m-2n+2p})^{-1}}=$

(A) 2 (B) $\frac{1}{4}$

(C) 9 (D) $\frac{1}{8}$

16. Find the value of $\frac{(x^{a+b})^2(x^{b+c})^2(x^{c+a})^2}{(x^a x^b x^c)^4}$.

(A) 1 (B) 4

(C) – 1 (D) 2

17. For an integer n, a student states the following.

I. If n is odd, $(n+1)^2$ is even.

II. If n is even, $(n-1)^2$ is odd.

III. If n is even, $\sqrt{(n-1)}$ is irrational.

Which of the above statements would be true ?

(A) Both I and III (B) Both I and II

(C) All I, II and III (D) Both II and III

18. If $\sqrt{5}=2.236$ and $\sqrt{10}=3.162$, then the value of $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$ is

(A) 5.398 (B) 4.398

(C) 3.398 (D) 6.398

19. The value of $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+....+\frac{1}{99\times100}$ is

(A) Less than $\frac{99}{100}$ (B) Equal to $\frac{99}{100}$

(C) Greater than $\frac{100}{99}$ (D) Equal to $\frac{100}{99}$

20. Simplify : $\sqrt[5]{x^4\sqrt[4]{x^3\sqrt[3]{x^2\sqrt{x}}}}$

(A) $x^{\frac{23}{24}}$ (B) $x^{\frac{23}{6}}$

(C) $x^{\frac{5}{6}}$ (D) $x^{\frac{119}{120}}$

21. Which of the following statements is true ?

(A) Product of two irrational numbers is always irrational

(B) Product of rational and an irrational number is always irrational

(C) Sum of two irrational numbers can never be irrational

(D) Sum of an integer and a rational number can never be an integer

22. Find the value of $\frac{3^{-3}\times6^2\times\sqrt{98}}{5^2\times\sqrt[3]{\frac{1}{25}}\times(15)^{\frac{-4}{3}}\times3^{\frac{1}{3}}}$.

(A) $28\sqrt{2}$ (B) 28

(C) $28\sqrt{3}$ (D) None of these

23. If $a=\frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b=\frac{\sqrt{2}-1}{\sqrt{2}+1}$, then value of (a^2+ab+b^2) is

(A) 70 (B) 35

(C) 40 (D) 34

24. $2.\overline{6}-0.\overline{82}$ is equal to

(A) $\frac{181}{999}$ (B) $\frac{182}{99}$

(C) $\frac{82}{99}$ (D) None of these

25. If $x=3\sqrt{5}+2\sqrt{2}$ and $y=3\sqrt{5}-2\sqrt{2}$, then value of $(x^2-y^2)^2$ is

(A) 240 (B) 140

(C) 5760 (D) 5300

PARAGRAPH TYPE

Passage – I : $(\sqrt{x}+\sqrt{y})^2 = x+y+2\sqrt{xy}$ and $\sqrt{x}\sqrt{y} = \sqrt{xy}$, where x and y are positive real numbers.

26. If $x = 2\sqrt{5}+\sqrt{3}$ and $y = 2\sqrt{5}-\sqrt{3}$, then $x^4+y^4=$
(A) 1538 (B) 1200
(C) 1048 (D) 149

27. If $x=\sqrt{3}+3\sqrt{2}$ and $y=\sqrt{3}-3\sqrt{2}$, then $x^4+y^4-8x^2y^2=$
(A) 3914 (B) 3010
(C) – 486 (D) – 856

28. If $a = 1+\sqrt{2}+\sqrt{3}$ and $b = 1+\sqrt{2}-\sqrt{3}$, then $a^2+b^2-2a-2b=$
(A) 11 (B) 8
(C) 152 (D) 15

Passage – II : For $\dfrac{1}{a\sqrt{x}+b\sqrt{y}}$, the rationalising factor is $a\sqrt{x}-b\sqrt{y}$.

29. If $x=\dfrac{1}{3-2\sqrt{2}}$ and $y=\dfrac{1}{3+2\sqrt{2}}$, then value of xy^2+x^2y is
(A) 4 (B) 12
(C) 6 (D) 9

30. If $x=\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{80}+\sqrt{48}-\sqrt{45}-\sqrt{27}}$, then value of $4x^2+3x+5$ is
(A) 15 (B) 2
(C) 12 (D) 5

31. If $x=\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}-\dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}$, then value of x^4+x^2 is
(A) 2 (B) 1
(C) 0 (D) 12

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column-I and Column-II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the following.

Column– I	Column – II
(P) $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}} \times \left\{\left(\dfrac{25}{9}\right)^{-\frac{3}{2}} \div \left(\dfrac{5}{2}\right)^{-3}\right\}$	(i) $\dfrac{3}{80}$
(Q) $\dfrac{\sqrt[3]{0.125}\times\sqrt[5]{(0.00032)^{-2}}}{\sqrt[5]{(0.00243)^{-3}}\times(27)^{\frac{2}{3}}}$	(ii) $\dfrac{39+8\sqrt{30}}{21}$
(R) $\sqrt[4]{(81)^{-2}}$	(iii) $\dfrac{1}{9}$
(S) $\dfrac{2\sqrt{6}+\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$	(iv) 1

(A) (P) → (i), (Q) → (ii), (R) → (iv), (S) → (iii)
(B) (P) → (iii), (Q) → (ii), (R) → (i), (S) → (iv)
(C) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)
(D) (P) → (iv), (Q) → (i), (R) → (iii), (S) → (ii)

33. Match the following if $\sqrt{2}=1.414, \sqrt{3}=1.732$, $\sqrt{5}=2.236$ and π = 3.141.

Column– I	Column – II
(P) $\frac{2}{\sqrt{5}-\sqrt{3}}$	(i) 4.357
(Q) $\frac{\pi}{2}+\frac{3}{\sqrt{5}}$	(ii) 3.968
(R) $\frac{1}{2\sqrt{5}-3\sqrt{2}}$	(iii) 2.912
(S) $\pi+\frac{1}{\sqrt{2}}$	(iv) 3.848

(A) (P) → (ii), (Q) → (iii), (R) → (i), (S) → (iv)

(B) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(C) (P) → (iv), (Q) → (i), (R) → (ii), (S) → (iii)

(D) (P) → (iii), (Q) → (i), (R) → (ii), (S) → (iv)

Space for Notes :

MATRIX OLYMPIAD

EXERCISE – II

VERY SHORT ANSWER TYPE

1. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find x^2+y^2.

2. If $\sqrt{x}+\sqrt{x-\sqrt{1-x}}=1$, then show that $x = \frac{16}{25}$.

3. If $x = 3+3^{\frac{1}{3}}+3^{\frac{2}{3}}$, then show that $x^3 - 9x^2 + 18x - 12 = 0$.

4. If $x = (4+\sqrt{15})^{\frac{1}{3}}+(4-\sqrt{15})^{\frac{1}{3}}$, then show that $x^3 - 3x - 8 = 0$.

5. If $\sqrt{x}-\sqrt{12} = \sqrt{4}-\sqrt{x}$, then find x.

6. Show that $\frac{1}{\sqrt{2}+\sqrt{3}} - \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{3}{\sqrt{5}-\sqrt{2}} = 0$.

7. If $\sqrt{15-x\sqrt{14}} = \sqrt{8}-\sqrt{7}$, then find the value of x.

8. Solve : $\sqrt{9+2x}-\sqrt{2x} = \frac{5}{\sqrt{9+2x}}$.

9. Find the value of $\sqrt{\frac{1}{2}\sqrt{\frac{1}{2}\sqrt{\frac{1}{2}..........\infty}}}$.

10. Is 5.25 a rational number ? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

SHORT ANSWER TYPE

1. If $x = \frac{1}{2}\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)$, then show that $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}} = \frac{a-1}{2}$.

2. Find 10 rational numbers between $-\frac{1}{9}$ and $\frac{4}{9}$.

3. Convert $\frac{5}{11}$ into decimal form.

4. Simplify the following expressions :

   (i) $\left(\sqrt{3}+\sqrt{5}\right)^2$ (ii) $\left(\sqrt{5}-\sqrt{2}\right)^2$

5. Simplify :

   (i) $(9)^{\frac{3}{2}}$ (ii) $(9)^{-\frac{3}{2}}$

   (iii) $(25)^{\frac{3}{2}}$ (iv) $(36)^{\frac{3}{2}}$

   (v) $(49)^{-\frac{3}{2}}$ (vi) $(0.0001)^{-\frac{3}{4}}$

LONG ANSWER TYPE

1. Express the following recurring decimal expansions in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

   (i) $0.\overline{7}$ (ii) $0.2\overline{57}$

2. Show that $\sqrt[3]{ax^2+by^2+cz^2} = \sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$, if $ax^3 = by^3 = cz^3$ and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$.

3. Prove that $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+ ......+\frac{1}{\sqrt{8}+\sqrt{9}} = 2$.

4. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $3(a^2 - b^2)$.

5. If $x = \frac{7-\sqrt{45}}{2}$, find the value of $\left(x^3+\frac{1}{x^3}\right)-7\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)$.

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TRUE / FALSE TYPE

1. A terminating or non-terminating decimal expansion can be expressed as rational number.
2. There is one rational number lying between any two rational numbers.
3. An irrational number can be expressed in the form $\frac{p}{q}$.
4. Square root of a positive real number always exists.
5. The difference of a rational number and an irrational number is an irrational number.

FILL IN THE BLANKS

1. Rationalizing factor of $3\sqrt{5}$ is______.
2. R______rational mambors lie between two whole numbers .
3. Order of surd $\sqrt[7]{13}$ is _______.
4. Multiplicative inverse of $2-\sqrt{3}$ ______.
5. Every repeating deciaml is a ______ number .

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Which of the following statements is incorrect ?
   (A) There can be a real number which is both rational and irrational
   (B) The sum of any two irrational numbers is not always irrational
   (C) For any positive integers x and y, $x < y \Rightarrow x^2 < y^2$
   (D) Every integer is a rational number

2. Find the value of $\dfrac{9^{\frac{3}{2}} - 3\times 5^0 - \left[\frac{1}{81}\right]^{-\frac{1}{2}}}{\left(\frac{64}{125}\right)^{-\frac{2}{3}} + \dfrac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\dfrac{\sqrt{25}}{\sqrt[3]{64}}\right)}$.
   (A) $\frac{15}{13}$ (B) 0
   (C) $\frac{16}{5}$ (D) $\frac{48}{13}$

3. If $a = 2+\sqrt{3}+\sqrt{5}$ and $b = 3+\sqrt{3}-\sqrt{5}$, then $a^2 + b^2 - 4a - 6b - 3$ is equal to.
   (A) 2 (B) –1
   (C) 1 (D) 0

4. Find the values of the integers a and b respectively, for which the solution of the equation $x\sqrt{24} = x\sqrt{3}+\sqrt{6}$ is $\frac{a+\sqrt{b}}{7}$.
   (A) 4, 2 (B) 2, 6
   (C) 3, 2 (D) 9, 5

5. If $\sqrt{2^n} = 1024$ then $3^{2\left(\frac{n}{4}-4\right)} =$
   (A) 3 (B) 9
   (C) 27 (D) 81

NUMERICAL PROBLEMS

1. If $x = 9+4\sqrt{5}$ and $xy = 1$, then find the value of $\left(\frac{1}{x^2}+\frac{1}{y^2}\right)$.

2. The value of $\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$ is

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3. If $N = \dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}}$, then

N + 2 equals

4. Find the value of 9x, if $5^{x-3} \cdot 3^{2x-8} = 225$.

5. If $\dfrac{9^n \times 3^2 \times (3^{-\frac{n}{2}})^{-2} - (27)^n}{3^{3m} \times 2^3} = \dfrac{1}{27}$, then find m – n.

Space for Notes :

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Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	D	A	A	D	A	A	D	A	C	A	A	B	C	A
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	B	A	B	D	B	A	B	B	C	A	C	B	C	C
31	32	33												
A	D	A												

EXERCISE II

VERY SHORT ANSWER TYPE

1. 98 **2.** $\frac{16}{25}$ **3.** 0 **4.** 0 **5.** $4+2\sqrt{3}$ **6.** 0 **7.** 4 **8.** 8 **9.** $\frac{1}{2}$ **10.** Yes, $\frac{21}{4}$

SHORT ANSWER TYPE

4. (i) $8+2\sqrt{15}$ (ii) $7-2\sqrt{10}$

5. (i) 27 (ii) $\frac{1}{27}$ (iii) 125 (iv) 216 (v) $\frac{1}{343}$ (vi) 1000

LONG ANSWER TYPE

1. (i) $\frac{7}{9}$ (ii) $\frac{51}{198}$ **4.** $120\sqrt{6}$ **5.** 0

TRUE/FALSE TYPE

1. F **2.** F **3.** F **4.** T **5.** T

FILL IN THE BLANKS

1. $\sqrt{5}$ **2.** Infinate **3.** 7 **4.** $2+\sqrt{3}$ **5.** Reational

ANALYTICAL PROBLEMS & BRAIN TEASER

1. A **2.** D **3.** D **4.** A **5.** B

NUMERICAL PROBLEMS

1. 322 **2.** 1 **3.** 3 **4.** 45 **5.** 1

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SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : NUMBER SYSTEM)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

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Space for Notes :

POLYNOMIAL 2

Concepts

INTRODUCTION

In our previous classes, we have learnt about algebraic expressions and various operations on them. In this chapter, we shall review these concepts and extend them to particular types of expressions, known as polynomials.

We shall come across two types of symbols, namely, constants and variables, defined below :

Constants : A symbol having a fixed numerical value is called a constant.

e.g. $8, -6, \frac{5}{7}, \pi$ etc are all constants.

Variables : A symbol which may be assigned different numerical values is known as a variable.

e.g. circumference of a circle is given by $c = 2\pi r$

Here, 2 and π are constants, while c and r are variables.

Algebraic Expressions : A combination of constants and variables, connected by operations +, –, × and ÷ is known as an algebraic expressions.

e.g. $4 + 9x - 5x^2y + \frac{3}{5}xy$ is an algebric expression containing four terms, namely, 4, 9x, $-5x^2y$ and $\frac{3}{5}xy$.

1. POLYNOMIALS

A function p(x) of the form $p(x) = a_nx^n + a_{n-1}x^{n-1} + ...+ a_1x + a_o$

where $a_0, a_1, a_2,....,a_n$ are constants and $a_n \neq 0$ and n is a non zero negative integer (i.e. whole number) is called a polynomial.

OR

An algebric expression in which the variables involved have only non negative integral powers is called a polynomial.

Example 1

$x^2 - a^2$, $ax^2 + bx + c$, $x^3 + 3x^2 + 3x + 1$, $y^3 - 7y + 6$ etc.

Focus Point

- If the power of x or y be in either increasing or decreasing order, the polynomial in x or y is said to be in standard form.

e.g. $-2y^4 - y^3 + y^2 + 3y + 14$ or $14 + 3y + y^2 - y^3 - 2y^4$

2. DEGREE OF A POLYNOMIAL IN ONE VARIABLE

2.1 DEGREE OF A POLYNOMIAL IN ONE VARIABLE

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In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

2.2 DEGREE OF A POLYNOMIAL IN TWO OR MORE VARIABLES

In case of polynomials in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial.

Example 2

Write the degree of each of the following polynomials.

(i) $5x^3 + 4x^2 + 7x$ (ii) $7x^3 - 5x^2y^2 + 3xy + 6y + 8$

Solution :

(i) In the given polynomial $5x^3 + 4x^2 + 7x$, the highest degree of variable x is 3. Therefore, degree of polynomial $5x^3 + 4x^2 + 7x$ is 3.

(ii) The given polynomial is in 2 variables x and y. The highest sum of the powers of the x and y in all terms is 4. Therefore, degree of polynomial is 4.

3. TYPES OF POLYNOMIALS

Polynomials can be classified on the basis of number of terms and on the basis of degree.,

3.1 ON THE BASIS OF DEGREE

(i) Constant polynomial : A polynomial of degree zero is called as constant polynomial.

e.g. 2, –5, 7. Every real number is a constant polynomial.

(ii) Linear polynomial : A polynomial of degree 1 is called as linear polynomial.

e.g. $3x + 5$ is a linear polynomial in x.

and $x + y + 8$ is also a linear polynomial in x and y.

(iii) Quadratic polynomial : A polynomial of degree 2 is called a quadratic polynomial.

e.g. $3y^2 - 8y + 5$ is a quadratic polynomial in y.

and $2xy + 5x + 3y + 4$ is also a quadratic polynomial in x and y.

(iv) Cubic polynomial : A polynomial of degree 3 is called a cubic polynomial.

e.g. $4x^3 - 3x^2 + 7x + 1$ is a cubic polynomial in x

and $4x^2y + 5xy + 8$ is also a cubic polynomial in x and y.

(v) Biquadratic polynomial : A polynomial of degree 4 is called a biquadratic polynomial.

e.g. $z^4 + 6z^3 + 10z^2 + 6z + 1$ is biquadratic in z.

and $3x^2yz + 4xy^2 + 5xyz$ is also a biquadratic in, x, y and z.

3.2 ON THE BASIS OF NUMBER OF TERMS

(i) Zero polynomial : A polynomial consisting of one term namely zero only is called as zero polynomial.

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(ii) Monomial : A polynomial containing one non zero term is called as monomial. *e.g.* $5, 3x, \frac{1}{3}xy$

(iii) Binomial : A polynomial containing two non zero terms is called as binomial. *e.g.* $(5x^2y + 2yz), (x - 5y)$

(iv) Trinomial : A polynomial containing three non-zero terms is called as trinomial.

e.g. $(xy + yz + zx), (3y - 5xy + 7xy^2)$

Focus Point

- A polynomial having four or more than four terms does not have any particular name. They are simply called polynomials.
- A polynomial whose all the coefficients are zero is called a zero polynomial, degree of a zero polynomial is not defined.

Example 3

Write the degree of each of the following polynomials.

(i) $5x^3 + 4x^2 + 7x$

(ii) $4 - y^2$

(iii) $5t - \sqrt{7}$

(iv) 3

Solution :

(i) In the given polynomial $5x^3 + 4x^2 + 7x$, the highest degree of variable x is 3.

(ii) In the given polynomial $4 - y^2$, the highest degree of variable y is 2.

(iii) In the given polynomial $5t - \sqrt{7}$, the highest degree of variable t is 1.

(iv) In the given constant polynomial 3 i.e. $3x^\circ$, the highest degree of any variable like x is 0 (zero).

Example 4

Classify the following as linear, quadratic and cubic polynomials.

(i) $x^2 + x$ (ii) $x - x^3$

(iii) $y + y^2 + 4$ (iv) $1 + x$

(v) $3t$ (vi) r^2

(vii) $7x^3$

Solution :

(i) $x^2 + x$, it is quadratic polynomial as the highest degree of variable x is 2.

(ii) $x - x^3$, it is cubic polynomial as the highest degree of variable x is 3.

(iii) $y + y^2 + 4$, it is a quadratic polynomial as the highest degree of variable y is 2.

(iv) $1 + x$, it is a linear polynomial as the highest degree of x is 1.

(v) 3t, it is a linear polynomial as the highest degree of t is 1.

(vi) r^2, it is a quadratic polynomial as the highest degree of r is 2.

(vii) $7x^3$, it is a cubic polynomial as the highest degree of x is 3.

4. ZEROES AND VALUE OF A POLYNOMIAL

4.1 VALUE OF A POLYNOMIAL

The value of a polynomial f(x) at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$.

Consider the polynomial $f(x) = x^3 - 5x^2 + 2x - 4$,

If we replace x by – 1 everywhere in f(x), we get

$f(-1) = (-1)^3 - 5(-1)^2 + 2(-1) - 4$

$f(-1) = -1 - 5 - 2 - 4$

$f(-1) = -12 \neq 0.$

So, we can say that value of f(x) at x = – 1 is –12.

4.2 ZERO OF A POLYNOMIAL

The real number α is a zero of a polynomial f(x), if $f(\alpha) = 0$.

Consider the polynomial $f(x) = x^3 - 3x^2 + 3x - 1$.

If we replace x by 1 everywhere in f(x), we get

$\because$ $f(1) = (1)^3 - 3(1)^2 + 3(1) - 1$

$= 1 - 3 + 3 - 1 = 0$

Hence, x = 1 is a zero of f(x).

Example 5

Find the value of the polynomial $p(x) = 5x - 4x^2 + 3$ at

(i) x = 0

(ii) x = – 1

(iii) x = 2

Solution :

(i) $p(0) = 5(0) - 4(0)^2 + 3 = 3$

(ii) $p(-1) = 5(-1) - 4(-1)^2 + 3 = -6$

(iii) $p(2) = 5(2) - 4(2)^2 + 3 = -3$

Example 6

Verify whether x is a zero of the following polynomials.

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(i) $p(x) = 3x + 2, x = -\frac{2}{3}$ (ii) $p(x) = 3x + 1, x = \frac{1}{3}$

Solution :

(i) $p\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right) + 2 = -2 + 2 = 0$

$\therefore -\frac{2}{3}$ is a zero of $p(x) = 3x + 2$.

(ii) $p\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right) + 1 = 1 + 1 = 2 \neq 0$

$\therefore \frac{1}{3}$ is not a zero of $p(x) = 3x + 1$.

Example 7

Find the zero of the polynomial in each of the following cases

(i) $p(x) = x + 7$

(ii) $p(x) = ax, a \neq 0$

Solution :

(i) $p(x) = 0 \Rightarrow x + 7 = 0 \Rightarrow x = -7$

(ii) $p(x) = 0 \Rightarrow ax = 0 \Rightarrow x = \frac{0}{a} \Rightarrow x = 0$

5. REMAINDER THEOREM

Let f(x) be a polynomial of degree $n (n \geq 1)$ and let a be any real number. When f(x) is divided by (x–a), then the remainder is f(a).

Example 8

Find the remainder when $p(x) = 4x^3 - 3x^2 + 2x - 4$ is divided by $(x - 1)$.

Solution :

By remainder theorem, we know that when p(x) is divided by x – 1, then remainder is p(1).

Now, $p(1) = 4(1)^3 - 3(1)^2 + 2(1) - 4 = 4 - 3 + 2 - 4 = -1$

Hence, the required remainder is – 1.

Example 9

Find the remainder when the polynomial $f(x) = x^3 - 3x^2 + 4x + 50$ is divided by $(x + 3)$.

Solution :

By the remainder theorem, we know that when f(x) is divided by (x + 3), the remainder is f(– 3).

Now, $f(-3) = [(-3)^3 - 3 \times (-3)^2 + 4 \times (-3) + 50] = [-27 - 27 - 12 + 50] = -16$

Hence, the required remainder is –16.

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6. FACTOR THEOREM

Let f(x) be a polynomial of degree $n \geq 1$ and let a be any real number.

(i) If f(a) = 0, then (x – a) is a factor of f(x).

(ii) If (x – a) is a factor of f(x) then f(a) = 0.

Example 10

Show that (x – 3) is a factor of the polynomial $f(x) = x^3 + x^2 - 17x + 15$.

Solution :

By the factor theorem, (x – 3) will be a factor of f(x) if f(3) = 0

Now, $f(x) = x^3 + x^2 - 17x + 15$

$f(3) = (3^3 + 3^2 - 17 \times 3 + 15) = (27 + 9 - 51 + 15) = 0$

Hence (x – 3) is a factor of the given polynomial f(x).

Example 11

Find the value of a for which (x + a) is a factor of the polynomial $f(x) = x^3 + ax^2 - 2x + a + 6$.

Solution :

(x + a) is a factor of $f(x) = x^3 + ax^2 - 2x + a + 6$

$\Rightarrow f(-a) = 0$ $[\because x + a = 0; x = -a]$

$\Rightarrow (-a)^3 + a(-a)^2 - 2(-a) + a + 6 = 0$ $\Rightarrow 3a = -6$

$\Rightarrow a = -2$ Hence, the required value of a is –2.

7. FACTORIZATION OF POLYNOMIALS

Factors : A polynomial g(x) is called a factor of the polynomial p(x) if g(x) divides p(x) exactly.

Example 12

(i) (x – 2) is a factor of $(x^2 + 3x - 10)$

Factorization : To express a given polynomial as the product of polynomials, each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorization.

Example 13

$(x^2 - 16) = (x - 4)(x + 4)$;

$(x^2 - 3x + 2) = (x - 2)(x - 1)$

7.1 METHODS OF FACTORIZATION

(I) By taking out the common factor : When each term of an expression has a common factor, we divide each term by this factor and take it out as a multiple.

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Example 14

Factorize :

(i) $5x^2 - 20xy$

(ii) $6(2a + 3b)^2 - 8(2a+3b)$

Solution :

(i) $5x^2 - 20xy = 5x(x - 4y)$

(ii) $6(2a + 3b)^2 - 8(2a + 3b) = 2(2a + 3b)[3(2a + 3b) - 4] = 2(2a + 3b)[6a + 9b - 4]$

(II) By Grouping : Sometimes in a given expression it is not possible to take out a common factor directly. However, the terms of the given expression are grouped in such a manner that we may have a common factor.

Example 15

Factorize :

(i) $9x^2 - 16y^2$

(ii) $x^3 - x$

Solution :

(i) $(3x)^2 - (4y)^2 = (3x + 4y)(3x - 4y)$

(ii) $x(x^2 - 1) = x(x + 1)(x - 1)$

7.2 FACTORIZING OF QUADRATIC TRINOMIALS

For polynomials of the form $ax^2 + bx + c$, we find integers p and q such that $p + q = b$ and $pq = ac$.

Then, $ax^2 + bx + c = ax^2 + (p + q)x + \frac{pq}{a}$

$= a^2x^2 + apx + aqx + pq = ax(ax + p) + q(ax + p) = (ax + p)(ax + q)$

Hence, $(ax^2 + bx + c) = (ax + p)(ax + q)$.

Example 16

Factorize : $6x^2 + 7x - 3$

Solution :

The given expression is $6x^2 + 7x - 3$. Here $6 \times (-3) = -18$

So, we try to split 7 into two parts whose sum is 7 and product –18.

Clearly, $9 + (-2) = 7$ and $9 \times (-2) = -18$

$\therefore\ 6x^2 + 7x - 3 = 6x^2 + 9x - 2x - 3 = 3x(2x + 3) - (2x + 3) = (2x + 3)(3x - 1)$

Hence, $6x^2 + 7x - 3 = (2x + 3)(3x - 1)$

Example 17

Factorize : $2x^2 - \frac{5x}{6} + \frac{1}{12}$

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Solution :

We have, $\frac{(24x^2-10x+1)}{12}$

$=\frac{1}{12}(24x^2-10x+1)$

$=\frac{1}{12}(24x^2-6x-4x+1)$

$=\frac{1}{12}[6x(4x-1)-(4x-1)]$

$=\frac{1}{12}(4x-1)(6x-1)$

Hence, $2x^2-\frac{5x}{6}+\frac{1}{2}=\frac{1}{12}(4x-1)(6x-1)$.

7.3 ALGEBRAIC IDENTITIES

(i) $(x+y)^2 = x^2 + y^2 + 2xy$
(ii) $(x-y)^2 = x^2 + y^2 - 2xy$
(iii) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
(iv) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
(v) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$
(vi) $(x^2-y^2) = (x+y)(x-y)$
(vii) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
(viii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
(ix) $(x^3 + y^3 + z^3 - 3xyz) = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$
(x) Conditional Identity : $(x+y+z) = 0$, then $x^3 + y^3 + z^3 = 3xyz$

Example 18

(i) Expand : $(2a + 3b + 4c)^2$

(ii) Factorize : $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

(iii) Evaluate : $(997)^2$

Solution :

(i) $(2a + 3b + 4c)^2 = (2a)^2 + (3b)^2 + (4c)^2 + 2(2a)(3b) + 2(3b)(4c) + 2(4c)(2a)$

$= 4a^2 + 9b^2 + 16c^2 + 12ab + 24bc + 16ca$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

$=(-\sqrt{2}x)^2+y^2+(2\sqrt{2}z)^2+2(-\sqrt{2}x)(y)+2y(2\sqrt{2}z)+2(2\sqrt{2}z)(-\sqrt{2}x) = (-\sqrt{2}x+y+2\sqrt{2}z)^2$

(iii) $(997)^2 = (1000 - 3)^2$

$= (1000)^2 + 32 - 2 \times 1000 \times 3$

$= (1000000 + 9 - 2 \times 6000) = 994009$

Example 19

Factorize : $27x^3 + 125y^3$

Solution :

Using the identity $(x^3 + y^3) = (x + y)(x^2 - xy + y^2)$, we have

$(27x^3 + 125y^3) = (3x)^3 + (5y)^3 = (3x + 5y)[(3x)^2 - (3x)(5y) + (5y)^2]$

$= (3x + 5y)(9x^2 - 15xy + 25y^2)$

Example 20

Factorize : $a^3 - b^3 - a + b$

Solution :

$\Rightarrow (a^3 - b^3) - (a - b)$

$\Rightarrow (a - b)(a^2 + ab + b^2) - (a - b)$ [Since, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$\Rightarrow (a - b)(a^2 + ab + b^2 - 1)$

Hence, $(a^3 - b^3 - a + b) = (a - b)(a^2 + ab + b^2 - 1)$

Example 21

Factorize : $a^3 - 8b^3 + 64c^3 + 24abc$

Solution :

We have, $a^3 + (-2b)^3 + (4c)^3 - 3 \times a \times (-2b) \times (4c)$

$x^3 + y^3 + z^3 - 3xyz$, where $a = x$, $(-2b) = y$ and $4c = z$

$= [a + (-2b) + 4c][a^2 + (-2b)^2 + (4c)^2 - a(-2b) - (-2b)(4c) - a(4c)]$

$= (a - 2b + 4c)(a^2 + 4b^2 + 16c^2 + 2ab + 8bc - 4ac)$

Example 22

Factorize : $(p - q)^3 + (q - r)^3 + (r - p)^3$

Solution :

Putting $(p - q) = x$, $(q - r) = y$ and $(r - p) = z$, we get $(p - q)^3 + (q - r)^3 + (r - p)^3$

$= x^3 + y^3 + z^3$, where $(x + y + z) = (p - q) + (q - r) + (r - p) = 0$

$= 3xyz$ [since, $(x + y + z) = 0 \Rightarrow (x^3 + y^3 + z^3) = 3xyz$] $= 3(p - q)(q - r)(r - p)$

BUILD THE CONCEPT

Some useful relations :

(i) $a^2 + b^2 = (a + b)^2 - 2ab$, if $a + b$ and ab are given.

(ii) $a^2 + b^2 = (a - b)^2 + 2ab$, if $a - b$ and ab are given.

(iii) $a + b = \sqrt{(a - b)^2 + 4ab}$, if $a - b$ and ab are given.

(iv) $a - b = \sqrt{(a + b)^2 - 4ab}$, if $a + b$ and ab are given.

(v) $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$, if $a + \frac{1}{a}$ is given.

(vi) $a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2$, if $a - \frac{1}{a}$ is given.

(vii) $a^3 + b^3 = (a + b)^3 + 3ab(a + b)$, if $(a + b)$ and ab are given.

(viii) $a^3 - b^3 = (a - b)^3 - 3ab(a - b)$, if $(a - b)$ and ab are given.

(ix) $a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$, if $a + \frac{1}{a}$ is given.

(x) $a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right)$, $a - \frac{1}{a}$ is given.

(xi) $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = [(a + b)^2 - 2ab](a + b)(a - b)$.

Example 23

Factorize : $x^3 - 23x^2 + 142x - 120$.

Solution :

Let $p(x) = x^3 - 23x^2 + 142x - 120$

Factors for constant terms are :

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 30, \pm 40, \pm 60$

Check : $p(1) = 1 - 23 + 142 - 120 = 143 - 143 = 0 \Rightarrow (x - 1)$ is a factor

Check : $p(10) = 10^3 - 23 \times 102 + 142 \times 10 - 120 = 2420 - 2420 = 0 \Rightarrow (x - 10)$ is a factor

Check : $p(12) = 12^3 - 23 \times 122 + 142 \times 12 - 120 = 1728 - 3312 + 1704 = 0 \Rightarrow (x - 12)$ is a factor

Since the coefficient of x^3 is 1, so p(x) can be factorized into $(x - a)(x - b)(x - c)$

Then, we have $p(x) = (x - a)(x - b)(x - c) = (x - 1)(x - 10)(x - 12)$

SOLVED EXAMPLES

SE. 1

Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer.

(i) $4x^2 - 3x + 7$

(ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Ans. (i) $4x^2 - 3x + 7$ is a polynomial over one variable 'x'. Degree : 2

(ii) $y^2 + \sqrt{2}$ is a polynomial over one variable 'y'. Degree : 2

(iii) $3\sqrt{t} + t\sqrt{2}$ is not a polynomial as variable 't' has $\frac{1}{2}$ (a fraction) as exponent (degree).

(iv) $y + \frac{2}{y}$ is not a polynomial over 'y' as the second term $\frac{2}{y}$ has degree (– 1) as exponent.

(v) $x^{10} + y^3 + t^{50}$ is a polynomial over integral powers of x (10), y (3) and t(50).

Hence it is a polynomial having three variables x, y and t.

SE. 2

Find the value of a if x – a is a factor of the polynomial

$x^6 - ax^5 + x^4 - ax^3 + 3x^2 - 3ax + a - 7$.

Ans. x – a is a factor of a polynomial p(x) if and only if

$\Rightarrow p(a) = 0$

$\Rightarrow a^6 - a(a^5) + a^4 - a(a^3) + 3(a)^2 - 3a(a) + a - 7 = 0$

$\Rightarrow a^6 - a^6 + a^4 - a^4 + 3a^2 - 3a^2 + a - 7 = 0$

$\Rightarrow a - 7 = 0$

$\Rightarrow a = 7$

SE. 3

Prove that :

$(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a) = 2(a^3 + b^3 + c^3 - 3abc)$.

Ans. LHS $= (a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a)$

$= (a + b + b + c + c + a)[(a + b)^2 + (b + c)^2 + (c + a)^2 - (a + b)(b + c) - (b + c)(c + a) - (c + a)(a + b)]$

$= 2(a + b + c)(a^2 + b^2 + b^2 + c^2 + 2ab + 2bc + c^2 + a^2 + 2ac - ab - ac - b^2 - bc - ab - c^2 - ca - ca - bc - a^2 - ab)$

$= 2(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

$= 2(a^3 + b^3 + c^3 - 3abc) =$ RHS

Hence the result.

SE. 4

Factorize :

(i) $7x^2 - 2\sqrt{14}x + 2$

(ii) $\sqrt{2}x^2 + 3x + \sqrt{2}$

Ans. (i) $7x^2 - 2\sqrt{14}x + 2$

$= \left(\sqrt{7}x\right)^2 - 2\sqrt{7}\sqrt{2}x + \left(\sqrt{2}\right)^2$

Using identity : $(x - y)^2 = x^2 - 2xy + y^2$

$= \left(\sqrt{7}x\right)^2 - 2\left(\sqrt{7}\right)\left(\sqrt{2}\right)x + \left(\sqrt{2}\right)^2$

$= \left(\sqrt{7}x - \sqrt{2}\right)^2$

$= \left(\sqrt{7}x - \sqrt{2}\right)\left(\sqrt{7}x - \sqrt{2}\right)$

(ii) We take two integers l and m such that

$l + m = 3$ and $lm = \sqrt{2} \times \sqrt{2} = 2$

Lets try $l = 1$, m = 2

Then,

$\sqrt{2}x^2 + 3x + \sqrt{2} = \sqrt{2}x^2 + x + 2x + \sqrt{2}$

$= \sqrt{2}x^2 + x + \sqrt{2}\left(\sqrt{2}x + 1\right)$

$= x(\sqrt{2}\,x+1) + \sqrt{2}(\sqrt{2}\,x+1)$

$= (x+\sqrt{2})(\sqrt{2}x+1)$

SE. 5

Find the values of a and b so that the polynomial $x^3 + 10x^2 + ax + b$ is exactly divisible by $x - 1$ as well as $x - 2$.

Ans. Let $p(x) = x^3 + 10x^2 + ax + b$ be the given polynomial. If p(x) is exactly divisible by $(x - 1)$ and $(x - 2)$, then $(x - 1)$ and $(x - 2)$ are factors of p(x).

$\therefore$ p(1) and p(2) = 0

$\Rightarrow 1^3 + 10 \times 1^2 + a \times 1 + b = 0$ and $2^3 + 10 \times 2^2 + a \times 2 + b = 0$

$\Rightarrow 1 + 10 + a + b = 0$ ....(i)

and $8 + 40 + 2a + b = 0$ ....(ii)

$\Rightarrow a + b = -11$ and $2a + b = -48$

Subtracting second equation from first, we get

$(a + b) - (2a + b) = -11 - (-48)$

$\Rightarrow a + b - 2a - b = -11 + 48$

$\Rightarrow -a = 37$

$\Rightarrow a = -37$

Putting $a = -37$ in $a + b = -11$, we get

$\Rightarrow -37 + b = -11 \Rightarrow b = 26$

Hence, $a = -37$ and $b = 26$.

SE. 6

Factorize :

(i) $(x + 1)^3 - (x - 1)^3$

(ii) $x^9 - y^9$

Ans. (i) We have $(x + 1)^3 - (x - 1)^3$

$= \{(x + 1) - (x - 1)\}\,\{(x + 1)^2 + (x + 1)(x - 1) + (x - 1)^2\}$

$= \{x + 1 - x + 1\}\,\{(x^2 + 2x + 1) + (x^2 - 1) + (x^2 - 2x + 1)\}$

$= 2\,(x^2 + 2x + 1 + x^2 - 1 + x^2 - 2x + 1)$

$= 2\,(3x^2 + 1)$

(ii) We have $x^9 - y^9$

$= (x^3)^3 - (y^3)^3$

$= (x^3 - y^3)\,\{(x^3)^2 + x^3y^3 + (y^3)^2\}$

$= (x - y)(x^2 + xy + y^2)(x^6 + x^3y^3 + y^6)$

SE. 7

Simplify :

$$\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}.$$

Ans. We have

$(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$

$\therefore\ (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

$= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

$= 3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)$

Similarly, we have

$(a - b) + (b - c) + (c - a) = 0$

$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3$

$= 3(a - b)(b - c)(c - a)$

$$\therefore \frac{(a^2-b^2)+(b^2-c^2)+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$$

$$= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$$

$= (a+b)(b+c)(c+a)$

MATRIX OLYMPIAD

SE. 8

Let R_1 and R_2 are the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively. If $2R_1 + R_2 = 6$, find the value of a.

Ans. Let $p(x) = x^3 + 2x^2 - 5ax - 7$ and $q(x) = x^3 + ax^2 - 12x + 6$ be the given polynomials.

Now,

R_1 = remainder when p(x) is divided by x + 1

$R_1 = p(-1)$

$R_1 = (-1)^3 + 2(-1)^2 - 5a \times (-1) - 7$

[Since $p(x) = x^3 + 2x^2 - 5ax - 7$]

$R_1 = -1 + 2 + 5a - 7$

$R_1 = 5a - 6$

and,

R_2 = remainder when q(x) is divided by x – 2

$R_2 = q(2)$

$R_2 = (2)^3 + a \times (2)^2 - 12 \times 2 + 6$

$R_2 = 4a - 10$

Substituting the values of R_1 and R_2 in $2R_1 + R_2 = 6$, we get $2(5a - 6) + (4a - 10) = 6$

$\Rightarrow 10a - 12 + 4a - 10 = 6$

$\Rightarrow 14a - 22 = 6$

$\Rightarrow 14a = 28$

$\Rightarrow a = 2$

SE. 9

Factorize : $x^4 + 2x^3y - 2xy^3 - y^4$.

Ans. $= (x^4 - y^4) + 2xy\,(x^2 - y^2)$

$= [(x^2 + y^2)(x^2 - y^2)] + 2xy\,[x^2 - y^2]$

$= (x^2 - y^2)\,[x^2 + y^2 + 2xy]$

$= (x + y)(x - y)(x + y)^2$

$= (x + y)(x + y)(x + y)(x - y)$

SE. 10

Factorize : $(x^2 - 5x + 6)^2 - (x^2 - 6x + 8)^2$.

Ans. $(x^2 - 5x + 6)^2 - (x^2 - 6x + 8)^2$

$= [(x - 2)(x - 3)]^2] - [(x - 2)(x - 4)]^2$

$= (x - 2)^2\,[(x - 3)^2 - (x - 4)^2]$

$= (x - 2)^2\,[(x - 3 + x - 4)(x - 3 - x + 4)]$

$= (x - 2)^2\,[(2x - 7)(1)]$

$= (x - 2)^2\,(2x - 7)$

MATRIX OLYMPIAD

EXERCISE – I

ONLY ONE CORRECT TYPE

1. Which one of the following algebraic expressions is a polynomial in variable x ?

(A) $x^2 + \frac{2}{x^2}$ (B) $\sqrt{x} + \frac{1}{\sqrt{x}}$

(C) $x^2 + \frac{3x^{3/2}}{\sqrt{x}}$ (D) None of these

2. Which of the following algebraic expressions is not a polynomial ?

(A) $\frac{17}{2}x^2 + x - 3$ (B) $\sqrt{7}x^3 + 3x^{2/3} - 8$

(C) 3 (D) 0

3. Which of the following is a quadratic polynomial in one variable ?

(A) $\sqrt{2x^3} + 5$ (B) $2x^2 + 2x^{-2}$

(C) x^2 (D) $2x^2 + y^2$

4. Degree of the polynomial $p(x) = 3x^4 + 6x + 7$ is

(A) 4 (B) 5

(C) 3 (D) 1

5. The coefficient of x^2 in $(3x^2 – 5)(4 + 4x^2)$ is.

(A) 12 (B) 8

(C) –8 (D) 5

6. If a polynomial is given by $f(x) = 5x^4 – 3x^3 + 2x^2 – 1$, then the value of $\frac{f(1)+f(-1)}{f(2)}$ is

(A) $\frac{4}{63}$ (B) $\frac{4}{21}$

(C) $\frac{9}{63}$ (D) None of these

7. If $p(x) = x^3 + 3x^2 – 2x + 4$, then find the value of $[p(2) + p(– 2) – p(0)]$.

(A) 28 (B) 14

(C) 12 (D) 16

8. The zeroes of the poynomial $p(x) = x^2 + x – 6$ are.

(A) 2, 3 (B) –2, 3

(C) 2, –3 (D) –2, –3

9. If $8x^4 – 8x^2 + 7$ is divided by $2x + 1$, the remainder is

(A) $\frac{11}{2}$ (B) $\frac{13}{2}$

(C) $\frac{15}{2}$ (D) $\frac{17}{2}$

10. The value of $(x + 2y + 2z)^2 + (x – 2y –2z)^2$ is

(A) $2x^2 + 8y^2 + 8z^2$

(B) $2x^2 + 8y^2 + 8z^2 + 8xyz$

(C) $2x^2 + 8y^2 + 8z^2 – 8yz$

(D) $2x^2 + 8y^2 + 8z^2 + 16yz$

11. If $(x^2 + 3x + 5)(x^2 – 3x + 5) = m^2 – n^2$, then m is

(A) $x^2 – 3x$ (B) $3x$

(C) $x^2 + 5$ (D) $x^2 + 2x + 1$

12. When $x^{11} + 1$ is divided by $x + 1$, then the remainder is

(A) 0 (B) 2

(C) 1 (D) – 1

13. The value of m, if $2y^3 + my^2 + 11y + m + 3$ is exactly divisible by $2y – 1$ is

(A) 7 (B) – 7

(C) 6 (D) – 6

14. If the polynomials $4x^3 + ax^2 – 2x + 7$ and $2x^3 + x^2 + x – a$ leave the same remainder when divided by $x – 3$, then the value of a is

(A) $x – 3$ (B) $4x^3 + ax^2 – 2x + 7$

(C) $\frac{-43}{10}$ (D) $\frac{-3}{13}$

15. What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $x^2 + 2x - 3$?
(A) $-x + 2$ (B) $x - 2$
(C) $2x - 1$ (D) $2x + 1$

16. If both x - 2 and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, then p =
(A) $\frac{3}{4}r$ (B) 2r
(C) $\frac{r}{2}$ (D) r

17. Which of the following is true if (x + 1) and (x + 2) are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$?
(A) $2\alpha + 3\beta = 2$ (B) $2\alpha - 3\beta = -2$
(C) $\alpha - 7\beta = 5$ (D) $7\alpha - \beta = 2$

18. The value of $\frac{7.83\times7.83-1.17\times1.17}{6.66}$ is
(A) 9 (B) 6.66
(C) 1.176 (D) 18

19. If $x^3 + y^3 + z^3 = 3xyz$, then which one of the following is true ?
(A) x + y + z = 1
(B) x – y + z = 0
(C) Either x + y + z = 0 or x = y = z
(D) Neither x + y + z = 0 nor x = y = z

20. If $a^{1/2} + b^{1/2} - c^{1/2} = 0$, then the value of $(a + b - c)^2$ is
(A) 2ab (B) 2bc
(C) 4ab (D) 4ac

21. If a + b + c = 10 and $a^2 + b^2 + c^2 = 80$, find the value of $a^3 + b^3 + c^3 - 3abc$.
(A) 700 (B) 710
(C) 1280 (D) 950

22. If $x + \frac{1}{x} = 5$, then find the value of $x^2 + \frac{1}{x^2}$.
(A) 26 (B) 23
(C) 30 (D) 22

23. If $3x + \frac{2}{x} = 7$, then $\left(9x^2 - \frac{4}{x^2}\right) =$
(A) 25 (B) 35
(C) 49 (D) 30

24. If $x^2 - 5x + 1 = 0$ $(x \neq 0)$, then the value of $x^3 + \frac{1}{x^3}$ is :
(A) 125 (B) 110
(C) 150 (D) 140

25. If p = 2 – a, then what is the value of $a^3 + 6ap + p^3 - 8$?
(A) 0 (B) – 1
(C) 1 (D) 2

PARAGRAPH TYPE

PASSAGE – I :

For a polynomial p(x) of degree ≥ 1, p(a) = 0, where a is a real number, then (x – a) is a factor of the polynomial p(x).

26. $p(x) = x^3 - 3x^2 + 4x - 12$, then p(3) is
(A) 0 (B) 1
(C) – 1 (D) 2

27. For what value of k, the polynomial $2x^4 + 3x^3 + 2kx^2 + 3x + 6$ is exactly divisible by (x + 2) ?
(A) 0 (B) – 1
(C) 1 (D) 2

28. Find the value of k if x – 1 is a factor of $4x^3 + 3x^2 - 4x + k$.
(A) 0 (B) 1
(C) – 3 (D) 2

PASSAGE – II :

It is given that when polynomials $p(x) = x^3 + ax^2 + 3x + 2$ and $q(x) = 2x^3 + 3x^2 - 4x - 7$ are divided by $x - 1$, the remainders obtained in each case are equal

29. What is the value of 'a' for polynomial p(x)
(A) 12 (B) – 12
(C) 0 (D) 6

30. What should be subtracted from polynomial p(x) so as to make remainder zero
(A) 8 (B) – 8
(C) 6 (D) – 6

31. If polynomial p(x) + q(x) is divided by x – 1, the remainder obtained is
(A) 6 (B) 7
(C) – 12 (D) 4

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column – I and Column – II are given as options (A), (B), (C) and (D) out of which one is correct.

32.

Column – I	Column – II
(P) Degree of $x^4 + 3x - 2$ is	(i) 2
(Q) The coefficient of x^3 in the expansion of $(3x - 5)^3$ is	(ii) 4
(R) Remainder when $x^{100} + 1$ is divided by $x + 1$ is	(iii) – 3
(S) Zero of polynomial $f(x) = 2x + 6$ is	(iv) 27

(A) (P) → (ii), (Q) → (iv), (R) → (i), (S) → (iii)
(B) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)
(C) (P) → (iv), (Q) → (i), (R) → (ii), (S) → (iii)
(D) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)

33.

Column – I	Column – II
(P) If x – 2 is factor of $5x + ax + 6$ then a =	(i) 16
(Q) If a + b = 5 and ab = 6 then $a^2 + b^2$ =	(ii) 1
(R) If a + b + c = 7 and $a^2 + b^2 + c^2 = 17$, then ab + bc + ca =	(iii) – 8
(S) $(0.36)^2 + (0.64)^2 + 0.72 \times 0.64$ =	(iv) 13

(A) (P) → (iii), (Q) → (iv), (R) → (i), (S) → (ii)
(B) (P) → (i), (Q) → (iii), (R) → (iv), (S) → (ii)
(C) (P) → (iv), (Q) → (i), (R) → (ii), (S) → (iii)
(D) (P) → (iv), (Q) → (iii), (R) → (i), (S) → (ii)

MATRIX OLYMPIAD

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Which of the following expressions are polynomials or not ?

   (i) $\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+\frac{1}{2}$ (ii) $\frac{1}{x}(x-1)(x-2)$

   (iii) $\frac{(x^2+x+1)(x+1)}{(1+x)}$ (iv) $x^2+\frac{1}{x^2}$

2. Write the coefficient of x^2 in each of the following :

   (i) $6 - 2x^2 + 3x^3 + x^4$ (ii) $\pi x^2 - x + 2$

   (iii) $\sqrt{3}x - 4$

3. Rewrite the following polynomials in the standard form :

   (i) $x - 7 + 8x^2 + 9x^3$ (ii) $- 5x^2 + 6 - 3x^3 + 4x$

4. Without actual division, find the remainder when :

   (i) $x^6 - 3x^5 + 2x^2 + 8$ is divided by $x - 3$.

   (ii) $x^2 + 5x + 4$ is divided by $x + 2$.

5. Simplify :

   $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

6. Simplify :

   (i) $(x + y - 2z)^2 - x^2 - y^2 - 3z^2 + 4xy$

   (ii) $(x^2 - x + 1)^2 - (x^2 + x + 1)^2$

7. Find zeroes of the polynomial given below :

   (i) $3x + \pi$ (ii) $ly + m; l \neq 0$

8. Given possible expressions for the length and breadth of the rectangle whose area is given as

   $16a^2 - 32a + 15$ square units; $a > \frac{5}{4}$.

9. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leaves the ramainder R_1 and R_2 respectively. Find the value of a if $R_1 + R_2 = 0$.

10. Show that $(3a + 2b - c + d)^2 - 12a (2b - c + d)$ is a perfect square.

SHORT ANSWER TYPE

1. Factorize :

   (i) $x^4 + x^2 + 1$

   (ii) $x^4 - x^4y^4$

   (iii) $x^2+\frac{12}{35}x+\frac{1}{35}$

   (iv) $2\sqrt{2}x^3+16\sqrt{2}y^3+z^3-12xyz$

2. Factorize :

   (i) $(x^3 + y^3 + 2x^2 - 2y^2)$ (ii) $(x^4 + 4)$

   (iii) $(x + y)^3 - (x - y)^3$ (iv) $\left(x^2+\frac{1}{x^2}-3\right)$

   (v) $a^3 - 5\sqrt{5}\ b^3$

3. If the perimeter of a rectangle is 24 units and the length exceeds the breadth by 4 units, then find the area of a rectangle.

4. Without actually calculating the cubes, evaluate the expression $(30)^3 + (- 18)^3 + (- 12)^3$.

5. If $x=\sqrt{7}-\sqrt{5}, y=\sqrt{5}-\sqrt{3}, z=\sqrt{3}-\sqrt{7}$, then find the value of $x^3 + y^3 + z^3 - 2xyz$.

LONG ANSWER TYPE

1. In each of the following questions divide the polynomial p(x) by g(x) and find the remainder. Find in which cases g(x) is a factor of p(x).

   (i) $p(x) = x^3 - 14x^2 + 37x - 60$; $g(x) = x - 2$

   (ii) $p(t) = t^3 + 6t^2 + 11t - 6$; $g(t) = t^2 - 5t + 6$

2. Find the values of m and n in the polynomial $2x^3 + mx^2 + nx - 14$ such that $(x - 1)$ and $(x + 2)$ are its factors.

3. Given $p(x) = 2x^5 + 3x^2 - 3x - 2$ and $q(x) = x - 1$. Find by actual division, whether q(x) is a factor of p(x). Verify your answer by factor theorem.

4. Find the product of : $(a^{1/8} + a^{-1/8}) (a^{1/8} - a^{-1/8}) (a^{1/4} + a^{-1/4}) (a^{1/2} + a^{-1/2})$

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5. Given that $px^2 + qx + 6$ leaves the remainder as 1 on division by $2x + 1$ and $2qx^2 + 6x + p$ leaves the remainder as 2 on division by $3x – 1$. Find p and q.

TRUE / FALSE TYPE

1. Zero of a polynomial is always 0.
2. The degree of the sum of two polynomials each of degree 5 is always 5.
3. $x^2 + x^{½} + 1$ is not a polynomial.
4. $(x+ 1)$ is a factor of $x^n + 1$, if n is odd natural number.
5. A binomial may have degree 5.

FILL IN THE BLANKS

1. For a polynomial p(x) of degree $\geq$ 1, x-k is a factor of the polynomial p(x) if and only if p(k) is __________ .
2. The number of zeroes of a polynomial is the __________ of that polynomial .
3. Zero of the polynomial $2y+\pi$ is __________.
4. If $p(x) = 2x^3-22x^2+141x-120$, then p(1) is __________ .
5. If $p(x) = (x+2)^3 + 3x-7$, then degree of p(x) is__________.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. Consider the following polynomials, $f(x) = x^3 – 2\sqrt{2}x^2 - 2x + 6\sqrt{2}$ and $g(x) = 2x^3 – 3x^2 + 9x – 25$ when f(x) is divided by $\left(x+\sqrt{2}\right)$, then we get remainder R_1. Also, when g(x) is divided by $(2x – 5)$, we get remainder R_2. Find the value of $(R_1 + R_2 – R_1R_2)$.

(A) $10 - 18\sqrt{2}$ (B) $10 + 18\sqrt{2}$

(C) $10 + 8\sqrt{2}$ (D) $10 - 8\sqrt{2}$

2. If $x^2 – 3x + 1 = 0$ $(x \neq 0)$, then the value of $x^3 - \dfrac{1}{x^3}$ is

(A) $2\sqrt{3}$ (B) $4\sqrt{5}$

(C) $4\sqrt{3}$ (D) $8\sqrt{5}$

3. If $k^3 = \dfrac{1\times2\times4+2\times4\times8+.....+100\times200\times400}{1\times3\times9+2\times6\times18+.....+100\times300\times900}$ and $(x – 3k)$ is a factor of the polynomial $p(x) = x^2 + ax – 4$, then the value of a is :

(A) 0 (B) $\dfrac{2}{3}$

(C) $\dfrac{8}{27}$ (D) 1

4. Let a, b, c and k be real numbers and p(x) be the polynomial $(x – a) (x – b) (x – c) + x$. If $p(k) = k$, then the sum of all possible values of k is :

(A) 0 (B) $a + b + c$

(C) $– (a + b + c)$ (D) $\dfrac{a+b+c}{2}$

5. Let $a = \sqrt[3]{6+\sqrt{2+\sqrt[3]{6+\sqrt{2+\sqrt[3]{6+\sqrt{2+.....}}}}}}$. If p(x) is a polynomial of degree 6 such that p(a) = 0, then p(2) equals

(A) 0 (B) 2

(C) 3 (D) 4

NUMERICAL PROBLEMS

1. $p(x) = \sqrt{2}$ is a polynomial of degree.
2. If $p(x) = x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2}) =$
3. Find the constant term in the expansion of $(x + 3)^3$.
4. Find the remainder when the polynomial $p(x) = x^{100} – x^{97} + x^3$ is divided by $x + 1$.

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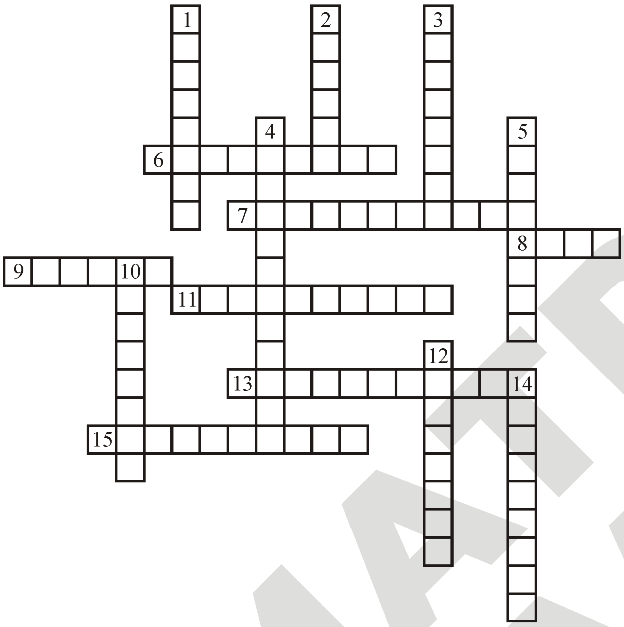
5. Value of $\frac{a^3+b^3+c^3-3abc}{ab+bc+ca-a^2-b^2-c^2}$, when a = – 5, b = – 6, c = 10 is

CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

6. 5x, 6x are _______.
7. 3x, 5y, 9h are ________.
8. Can be a number or a variable or even the product of numbers.
9. The _________ of 6xy is 2.
11. A phrase that contains numbers and variables, and is connected by operators.
13. When looking at 7x, the number 7 is a ______.
15. A ______ is an algebraic expression that is made up with adding or subtracting terms.

Down

1. 5x is one term, this is also referred to as a ____.
2. Numbers you multiply together, to get another number.
3. When looking at 5x, x is a _______.
4. When in a polynomial, the terms are re-put into order from the highest degree to the lowest degree.
5. A _______ has no variables in its term.
10. 5t – 2 = 3t + 4 is a _______.
12. 3x + 4 has two terms, this is also referred to as a _________.
14. 5x – 2f + 4y has three terms, this is also referred to as a _________.

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	B	C	A	C	B	A	C	A	D	C	A	B	C	B
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
D	B	A	C	C	A	B	B	B	A	A	B	C	B	D
31	32	33												
C	A	A												

EXERCISE II

VERY SHORT ANSWER TYPE

1. (i) Yes, (ii) No, (iii) Yes, (iv) No **2.** (i) – 2, (ii) π, (iii) 0

3. (i) $9x^3 + 8x^2 + x - 7$, (ii) $-3x^3 - 5x^2 + 4x + 6$ **4.** (i) R = 26, (ii) R = – 2 **5.** 1

6. (i) $z^2 + 6xy - 4yz - 4zx$, (ii) $-4x(x^2 + 1)$ **7.** (i) $x = -\frac{\pi}{3}$, (ii) $y = -\frac{m}{l}$

8. Length = 4a – 3, breadth = 4a – 5 **9.** $\frac{-153}{65}$

SHORT ANSWER TYPE

1. (i) $(x^2 + x + 1)(x^2 - x + 1)$ (ii) $x^4(1 - y)(1 + y)(1 + y^2)$, (iii) $\frac{1}{35}(7x + 1)(5x + 1)$

(iv) $\left(\sqrt{2}x + 2\sqrt{2}y + z\right)\left(2x^2 + 8y^2 + z^2 - 4xy - 2\sqrt{2}yz - \sqrt{2}xz\right)$

2. (i) $(x + y)(x^2 + y^2 - xy + 2x - 2y)$, (ii) $(x^2 + 2x + 2)(x^2 - 2x + 2)$, (iii) $2y(3x^2 + y^2)$

(iv) $\left(1 - \frac{1}{x} + 1\right)\left(x - \frac{1}{x} - 1\right)$, (v) $\left(a - \sqrt{5}b\right)\left(a^2 + \sqrt{5}ab + 5b^2\right)$

3. 32 sq. units **4.** 19440 **5.** $-4\sqrt{5} + 2\sqrt{3} + 2\sqrt{7}$

LONG ANSWER TYPE

1. (i) R = – 34, R ≠ 0 g is not factor of P, (ii) R = 60t – 72

2. m = 9, n = 3 **3.** $Q = 2x^4 + 2x^3 + 2x^2 + 5x + 2$, R = 0 **4.** $(a - a^{-1})$ **5.** p = – 2, q = 9

TRUE/FALSE TYPE

1. F **2.** T **3.** T **4.** T **5.** T

FILL IN THE BLANKS

1. 0 **2.** DEGRE **3.** $-\Pi / 2$ **4.** 1 **5.** 3

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ANALYTICAL PROBLEMS AND BRAIN TEASER

1.	A	**2.**	D	**3.**	A	**4.**	B	**5.**	A

NUMERICAL PROBLEMS

1.	0	**2.**	1	**3.**	27	**4.**	1	**5.**	1

CROSS WORD PUZZLE

1. Monomial	**2.** Factor	**3.** Variable	**4.** Standard form
5. Constant	**6.** Like terms	**7.** Unlike terms	**8.** Term
9. Degree	**10.** Equation	**11.** Expression	**12.** Binomial
13. Coefficient	**14.** Trinomial	**15.** Polynomial	

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SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : POLYNOMIAL)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

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NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

COORDINATE GEOMETRY 3

Concepts

INTRODUCTION

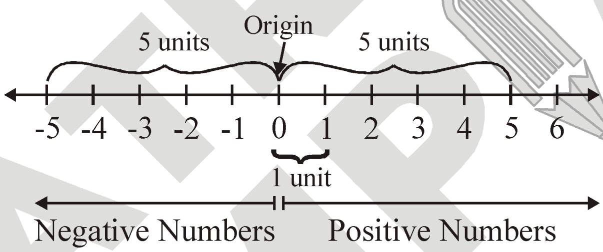
Coordinate Geometry is the branch of Mathematics which deals with the unification of algebra and geometry in which algebra is used in the study of geometrical relation and geometrical figures are represented by means of equations. The most popular coordinate system is the rectangular cartesian system.

1. CARTESIAN SYSTEM

1.1 NUMBER LINE

Any number can be represented geometrically on the number line. Positive numbers are represented on right side of zero and negative numbers on the left side of zero.

For example : To represent 5 on number line. Distance of each point from the previous is 1 unit. In the above example, the point 5 is located at the distance of 5 units on the right side from origin. Similarly, – 5 is located at the distance of 5 units from origin but on the left side of origin. Thus, sign decides the position of a point with reference to origin.



1.2 CARTESIAN PLANE

To locate the position of an object in a plane (Two-dimensions), two number lines are used. These two number lines are perpendicular lines, one of them is horizontal and the other is vertical.

(i) Rectangular coordinate axes : Let X'OX and Y'OY be two mutually perpendicular lines through any point O in the plane of the paper. This point O, is called origin. Now choose a convenient unit of length and starting from the origin as zero, mark off a number scale on the horizontal line X'OX, positive to the right of the origin O and negative to the left of origin O.Also, mark off the same scale on the vertical line Y'OY, positive upwards and negative downwards of the origin O.

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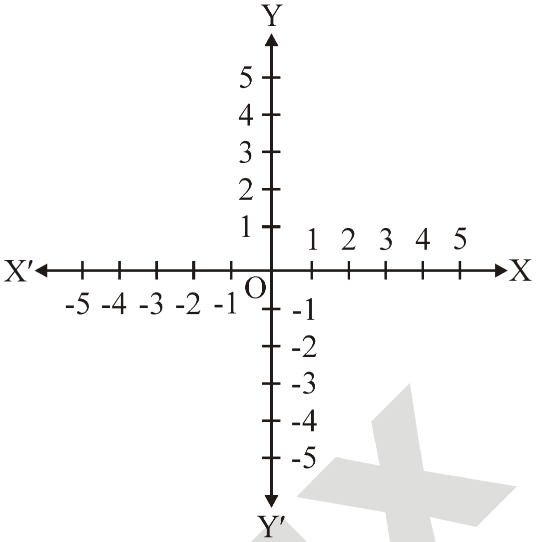


Note : (i) The line X'OX is called the **x-axis** or **axis of x**.

(ii) The line Y'OY is known as the **y-axis** or **axis of y**.

(iii) The x-axis and y-axis taken together are called the **co-ordinate axis** or the **axes of coordinates**.

Example 1

Fill in the blanks :

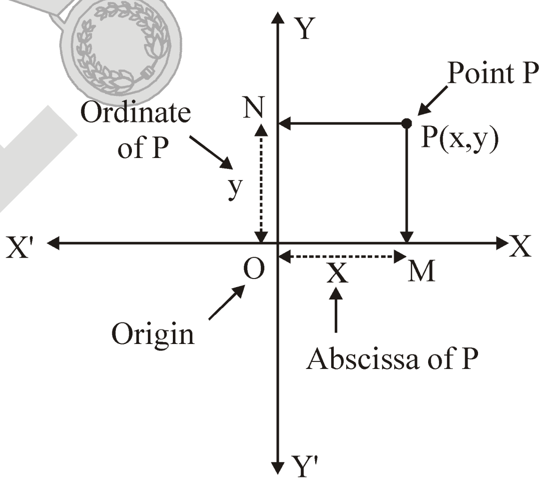
(i) The point in the positive direction at a distance of '4' units from origin lies on __________.

(ii) The point at a distance of 'x' units above the origin lies on _____________.

Solution :

(i) positive x-axis, (ii) positive y-axis

(ii) Cartesian coordinates of a point : Let X'OX and Y'OY be the coordinate axes, and let P be any point in the plane. Draw perpendiculars PM and PN from P on x and y axis respectively.



Abscissa : The coordinate representing the position of a point along a line perpendicular to the y-axis in plane Cartesian coordinate system or we can say that x-coordinate of any point is called abscissa. In the given figure PN is the abscissa of the point P.

Ordinate : The coordinate representing the position of a point along a line perpendicular to the x-axis in plane Cartesian coordinate system or we can say that y-coordinate of any point is called ordinate. In the given figure, PM is the ordinate of the point P.

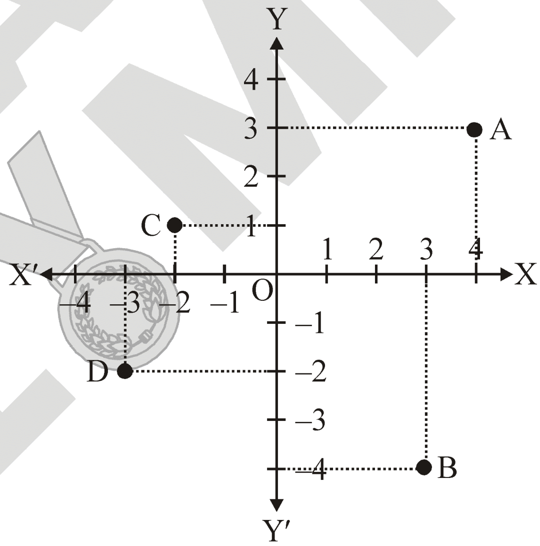
Focus Point

(i) The position of the point P in the plane with respect to the coordinate axes is represented by the ordered pair (x, y). The ordered pair (x, y) is called the coordinates of point P.

(ii) A pair of numbers a and b listed in a specific order with a at the first place and b at the second place is called an ordered pair (a, b). Note that (a, b) ≠ (b, a).

Example 2

See figure & complete the following statements :



(i) The abscissa and the ordinate of the point A are ___ and ___, respectively. Hence, the coordinate of A are (___, ___).

(ii) The abscissa and the ordinate of the point B are ___ and ___, respectively. Hence the coordinates of B are (___, ___).

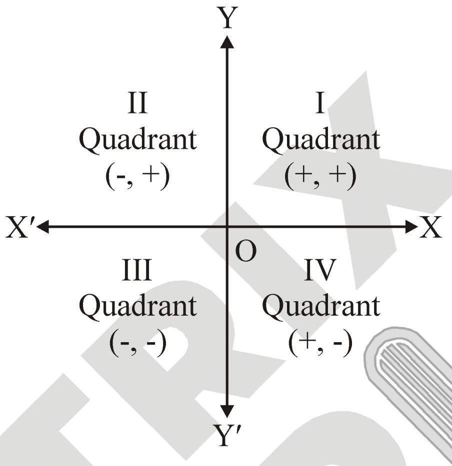
(iii) The distance of point C form x axis is (___).

(iv) Point D is at a perpendicular distance of ___ from y axis.

Solution :

(i) 4, 3, (4, 3) (ii) 3, – 4, (3, – 4) (iii) 1 (iv) 3

(iii) Quadrants : The x-axis and y-axis divide the cartesian plane into four regions, called the quadrants. Let X'OX and Y'OY be the coordinate axes, which divide the plane in four quadrants. Thus, quadrant is 1/4 part of plane divided by coordinate axes.



(i) XOY plane is Ist quadrant.

(ii) X′OY plane is IInd quadrant.

(iii) X′OY′ plane is IIIrd quadrant.

(iv) XOY′ plane is IVth quadrant.

Sign convention of Coordinates :

Quadrant	x coordinate	y coordinate	Po int
Frist Quadrant	+	+	(+,+)
Second Quadrant	–	+	(–,+)
Third Quadrant	–	–	(–,–)
Fourth Quadrant	+	–	(+,–)

Focus Point

(i) The coordinates of the origin are (0, 0).

(ii) The coordinates of any point on x axis are of the form (x, 0).

(iii) The coordinates of any point on y axis are of the form (0, y).

Example 3

In which quadrant or on which axis are the points (–2, 4), (3, –1), (–1, 0) and (–3, –5) lie ?

Solution :

(i) $\because$ x coordinate < 0, y coordiante > 0, point (–2, 4) lies in the II quadrant.

(ii) $\because$ x coordinate > 0, y coordinate < 0, point (3, –1) lies in the IV quadrant.

(iii) $\because$ x coordinate < 0, y coordinate = 0, point (–1, 0) lies on x axis.

(iv) $\because$ x coordinate < 0, y coordinate < 0, point (–3, –5) lies in the III quadrant.

Example 4

Write the coordinates of a point lying on x-axis to the left of origin at a distance of 2 units.

Solution :

(–2, 0)

Example 5

Write the coordinate of a point lying on y-axis at a distance of 5 units above origin.

Solution :

(0, 5)

1.3 PLOTTING OF POINTS IN THE CARTESIAN PLANE

In order to plot the points in a plane, we use the following algorithm :

(i) Draw two mutually perpendicular lines on the graph paper, one horizontal and other vertical.

(ii) Mark their intersection point as O (origin). The horizontal line as X'OX and the vertical line as Y'OY. The X'OX is the x-axis and the line Y'OY is the y-axis.

(iii) Choose a suitable scale on x-axis and y-axis and mark the points on both the axes.

(iv) Obtain the coordinates of the point which is to be plotted. Let the point be P(a, b). To plot this point, start from the origin and move |a| units along OX or OX' according as 'a' is positive or negative. Suppose we arrive at point M. From point M move vertically upward or downward through |b| units according as 'b' is positive or negative. The point where we arrive finally is the required point P(a, b).

Example 6

Mark the points P(4, 5), Q(2, – 1), R(– 3, 2) and S(– 3, – 4) on the Cartesian plane.

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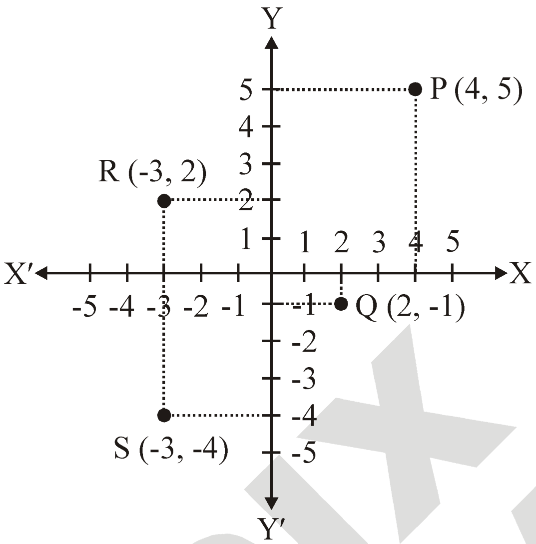
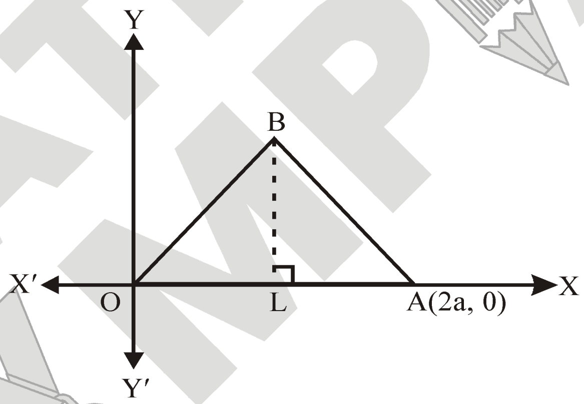
Solution :



Example 7

In the given figure, find the co–ordinates of the vertices of an equilateral triangle of side 2a.



Solution :

Since OAB is an equilateral triangle of side 2a. Therefore OA = AB = OB = 2a

Let BL be the perpendicular from B on OA. Then

OL = LA = a

In ΔOLB,

$OB^2 + OL^2 + LB^2$

$\Rightarrow (2a)^2 = a^2 + LB^2$

$\Rightarrow LB = \sqrt{3}\,a$

$\therefore$ coordinates of O are (0, 0), A(2a, 0) and B (a, $\sqrt{3}$ a).

Focus Point

We may take x-axis or y-axis as the mirror. Then, the image of different points are given below.

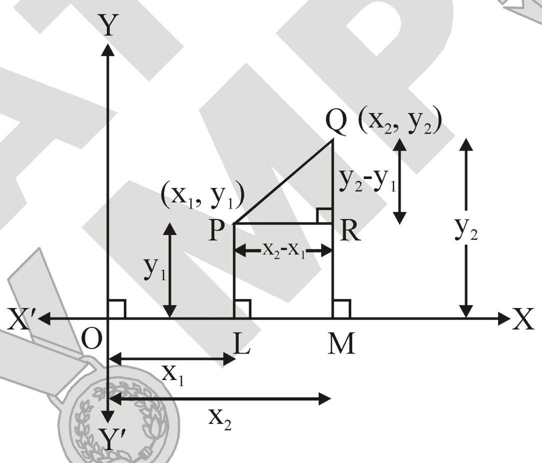
Point	Mirror-image in x-axis	Mirror-image in y-axis
(x, y)	(x, -y)	(-x, y)
(-x, y)	(-x, -y)	(x, y)
(-x, -y)	(-x, y)	(x, -y)
(x, -y)	(x, y)	(-x, -y)

1.4 DISTANCE BETWEEN TWO POINTS

Theorem : Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in a rectangular coordinate system is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Proof : Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points in the plane. Let us assume that both the points P and Q are in the 1st quadrant.



From P and Q, draw PL and QM perpendiculars to x-axis. From P, draw PR perpendicular to QM and join PQ.

Then, $OL = x_1$, $OM = x_2$, $PL = y_1$, $QM = y_2$

$\therefore$ $PR = LM = OM - OL = x_2 - x_1$ and $QR = QM - RM = QM - PL = y_2 - y_1$

Since, ΔPRQ is a right triangle, therefore by Pythagoras theorem, $(PQ)^2 = (PR)^2 + (QR)^2$

$\therefore PQ = \sqrt{(PR)^2 + (QR)^2}$ [$\because$ PQ is always positive]

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ The distance PQ between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

or $\sqrt{(\text{difference of x} - \text{coordinates})^2 + (\text{difference of y} - \text{coordinates})^2}$

Focus Point

Corollary 1 : The above formula is true for all positions of the points (i.e., points can lie in any quadrant) keeping in mind, the proper signs of their coordinates.

Corollary 2 : The distance of the point P(x, y) from the origin O(0, 0) is given by

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{(x^2 + y^2)}$$

Corollary 3 : The distance formula can also be used as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Corollary 4 : (i) If PQ is parallel to x-axis, then $y_1 = y_2$ and so $PQ = \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$

(ii) If PQ is parallel to y-axis, then $x_1 = x_2$ and so $PQ\sqrt{(y_2 - y_1)^2} = |y_2 - y_1|$

Example 8

Find the distance between the two points P(–5, 7) and Q(–1, 3).

Solution :

$$PQ = \sqrt{(-1-(-5))^2 + (3-7)^2} = \sqrt{(-1+5)^2 + (-4)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$$

$\therefore$ The distance between two points P and Q is $\sqrt{32}$ units.

Example 9

Find the area of the triangle formed by O(0, 0), A(4, 1) and B(– 2, – 5) such that the length of the perpendicular from origin to AB is $\frac{3}{\sqrt{2}}$ units.

Solution :

$$AB = \sqrt{(-2-4)^2 + (-5-1)^2} = \sqrt{36+36} = 6\sqrt{2} \text{ units}$$

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AB \times OP = \frac{1}{2} \times \frac{3}{\sqrt{2}} \times 6\sqrt{2} = 9 \text{ square units.}$$

1.5 APPLICATION OF DISTANCE FORMULA

Parallelogram : A quadrilateral with each pair of opposite sides parallel.

Properties : (i) Opposite sides are equal.
(ii) Opposite angles are equal.
(iii) Diagonals bisect each other.

Rhombus : A parallelogram with all sides of equal length.

Properties : (i) All the properties of parallelogram.
(ii) Diagonals are perpendicular to each other.

Rectangle : A parallelogram having one angle 90°.

Properties : (i) All the properties of parallelogram.
(ii) Each of the angles is 90°.
(iii) Diagonals are equal.

Square : A rectangle with all sides of equal length.

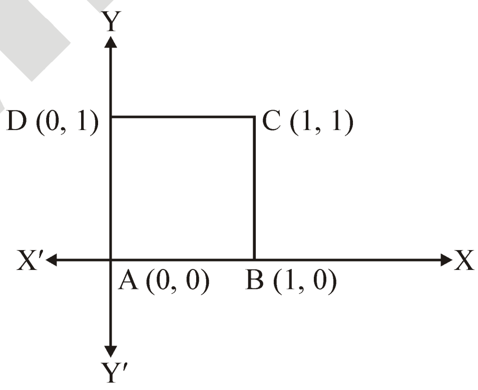
Properties : All the properties of parallelogram, rhombus and rectangle.

Example 10

If the points A(0, 0), B(1, 0), C(1, 1) and D(0, 1) are the vertices of a quadrilateral, then find out which special type of quadrilateral is it ?

Solution :

Using distance formula,



$AB = \sqrt{(1-0)^2 + 0^2} = 1 \text{ unit}$

$BC = \sqrt{(1-1)^2 + (0-1)^2} = 1 \text{ unit}$

$CD = \sqrt{(1-0)^2 + (1-1)^2} = 1 \text{ unit}$

$DA = \sqrt{0^2 + (1-0)^2} = 1 \text{ unit}$

Since, AB = BC = CD = DA = 1 unit.

Also, $\angle DAB = 90°$. [Since, D lies on y-axis, B lies on x-axis and A lies on the origin and both the axes are perpendicular]

∴ ABCD is a square.

Example 11

Prove that the points A(0, 0), B(1, 3), C(4, 2) and D(3, – 1) are the vertices of a square. Also, find its area.

Solution :

Here,

$AB = \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{1+9} = \sqrt{10}$ units

$BC = \sqrt{(4-1)^2 + (2-3)^2} = \sqrt{1+9} = \sqrt{10}$ units

$CD = \sqrt{(4-3)^2 + (2+1)^2} = \sqrt{1+9} = \sqrt{10}$ units

$AD = \sqrt{(3-0)^2 + (-1-0)^2} = \sqrt{10}$ units

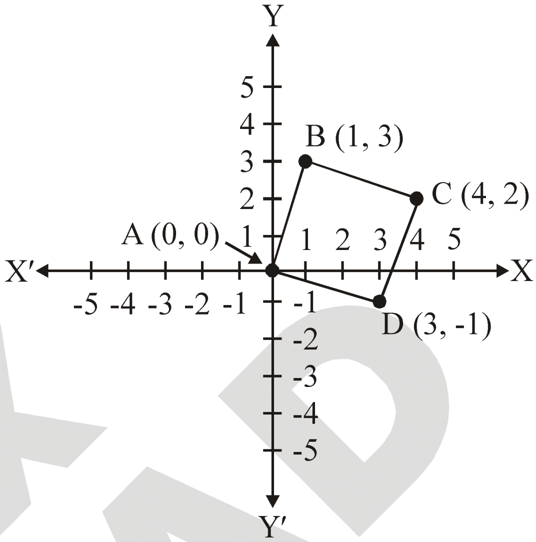
$\therefore$ AB = BC = CD = AD = 10 units

Also, AC = $\sqrt{16+4} = \sqrt{20}$ units

BD = $\sqrt{4+16} = \sqrt{20}$ units

Since, diagonals are also equal. $\therefore$ ABCD is a square.

Area of square = (side)^2 = (AB)^2 = $\left(\sqrt{10}\right)^2 = 10$ square units.



BUILD THE CONCEPT

Condition for Collinearity : Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are given points, then by using distance formula, if sum of lengths of any two line segment is equal to the length of third line segment, then we can say points are collinear.

Note : (i) If AB + BC = AC, then the points A, B and C are collinear.

(ii) If AC + CB = AB, then the points A, C and B are collinear.

(iii) If BA + AC = BC, then the points B, A and C are collinear.

(iv) If the points are collinear, then area of triangle formed by using these three points is 0, since triangle cannot be formed with three collinear points.

Example 12

Show that the points A(2, 3), B(3, 4) and C(4, 5) are collinear.

Solution :

Given, A = (2, 3), B = (3, 4) and C = (4, 5)

$AB = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{2}$ units

$BC = \sqrt{(4-3)^2 + (5-4)^2} = \sqrt{2}$ units

$AC = \sqrt{(4-2)^2 + (5-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

Now, $AB + BC = \sqrt{2} + \sqrt{2} = 2\sqrt{2} = AC$

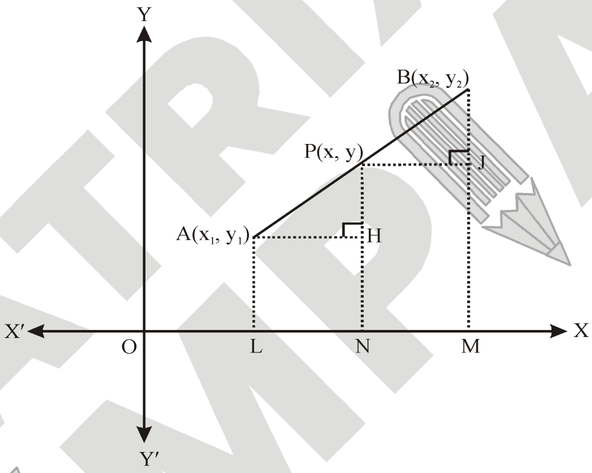
That is, AB + BC = AC.

Hence, the points A, B and C are collinear.

1.6 DIVISION OF A LINE–SEGMENT IN A GIVEN RATIO

Internal Division

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points and point P(x, y) divides the line segment AB in the ratio $m_1 : m_2$ internally, then $(x, y) \equiv \left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$



Proof : The given points are $A(x_1, y_1)$ and $B(x_2, y_2)$. Let us assume that points A and B are both in I quadrant. Since P(x, y) divides AB internally in ratio $m_1 : m_2$ i.e. $AP : PB = m_1 : m_2$.

Now, Draw AL, BM and PN perpendiculars to x–axis.

Draw, $AH \perp PN$ and $PJ \perp BM$, then

$OL = x_1$, $ON = x$, $OM = x_2$

$AL = y_1$, $PN = y$, $BM = y_2$

$\therefore\ AH = LN = ON - OL = x - x_1$

$PJ = NM = OM - ON = x_2 - x$

$PH = PN - HN = PN - AL = y - y_1$

$BJ = BM - JM = BM - PN = y_2 - y$

clearly, ΔAHP and ΔPJB are similar.

$$\therefore \frac{AH}{PJ} = \frac{PH}{BJ} = \frac{AP}{PB}$$

or $\dfrac{x - x_1}{x_2 - x} = \dfrac{y - y_1}{y_2 - y} = \dfrac{m_1}{m_2}$

Now, $\dfrac{x - x_1}{x_2 - x} = \dfrac{m_1}{m_2} \Rightarrow m_2x - m_2x_1 = m_1x_2 - m_1x$

$\Rightarrow (m_1 + m_2)x = m_2x_1 + m_1x_2 \qquad \Rightarrow x = \dfrac{m_2x_1 + m1x_2}{m_1 + m_2}$

and $\dfrac{y - y_1}{y_2 - y} = \dfrac{m_1}{m_2} \Rightarrow (m_1 + m_2)\, y = m_2y_1 + m_1\, y_2 \qquad \Rightarrow y = \dfrac{m_2y_1 + m_1y_2}{m_1 + m_2}$

Example 13

Find the coordinates of the point which divides the line segment joining the points (4, –3) and (8, 5) in the ratio 3 : 1 internally.

Solution :

Let P(x, y) be the required point. Using the section formula, we get

$m_1 = 3 \qquad x_1 = 4 \qquad x_2 = 8$

$m_2 = 1 \qquad y_1 = -3 \qquad y_2 = 5$

$x = \dfrac{3(8) + 1(4)}{3 + 1} = 7\,;\; y = \dfrac{3(5) + 1(-3)}{3 + 1} = 3$

Therefore, (7, 3) is the required point.

Focus Point

Note (i) : The ratio $m_1 : m_2$ can also be written as $\dfrac{m_1}{m_2} : 1$ or k : 1 where $k = m_1 : m_2$. So, coordinates of point P(x, y) dividing line segment joining points $A(x_1, y_1)$ $B(x_2, y_2)$

is given $(x, y) = \left(\dfrac{kx_2 + x_1}{k + 1}, \dfrac{ky_2 + y_1}{k + 1}\right)$

(ii) Mid – point of a line segment : Let A (x_1, y_1) and $B(x_2, y_2)$ be the end–points of AB, if P divides the line segment AB in the ratio 1 : 1 $\therefore P(x, y) \equiv \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

(iii) The coordinate of the **Centroid of a triangle** whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

(iv) The coordinate of the **Incentre of a triangle** ABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is $\left(\dfrac{ax_1 + bx_2 + cx_3}{a + b + c}, \dfrac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

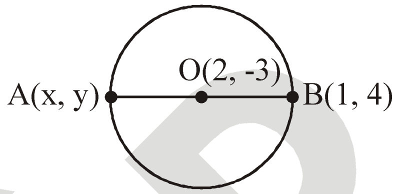
Example 14

Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

Solution :

Here, centre of the circle is O(2, – 3).

Let the end points of the diameter be A(x, y) and B(1, 4).



The centre of a circle bisects the diameter.

$\therefore 2 = \frac{x+1}{2} \Rightarrow x+1 = 4$ or $x = 3$

And $-3 = \frac{y+4}{2} \Rightarrow y+4 = -6$ or $y = -10$

Hence the coordinates of A are (3, – 10).

Example 15

In what ratio does the point (–4, 6) divide the line segment joining the points A(–6, 10) and B(3, –8) ?

Solution :

Let (–4, 6) divide AB internally in the ratio m : n. Using the section formula, we get

$(-4, 6) = \left(\frac{3m-6n}{m+n}, \frac{-8m+10n}{m+n}\right)$ We know if (x, y) = (a, b), then x = a and y = b

So, $-4 = \frac{3m-6n}{m+n}$ and $6 = \frac{-8m+10n}{m+n}$

Now,

$-4(m + n) = 3m - 6n$

$\Rightarrow -4m - 4n = 3m - 6n$

$\Rightarrow -4m - 3m = -6n + 4n$

$\Rightarrow -7m = -2n$

$\Rightarrow 7m = 2n$

$\Rightarrow \frac{m}{n} = \frac{2}{7}$

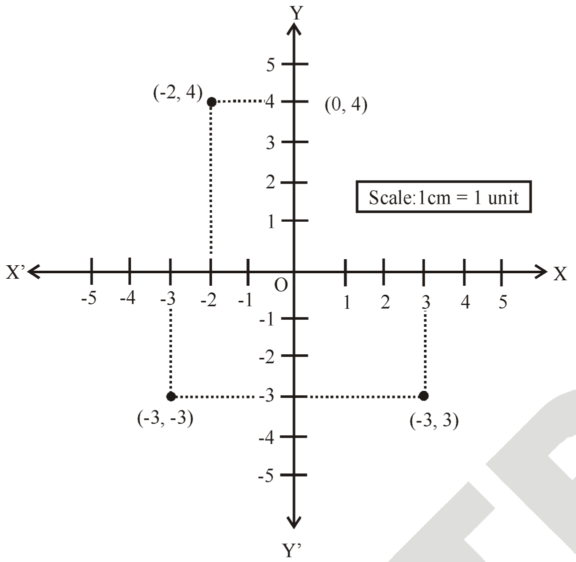
$\Rightarrow m : n = 2 : 7$

Therefore, the point (–4, 6) divides the line segment joining points A(–6, 10) and B(3, –8) in the ratio 2 : 7.

SOLVED EXAMPLES

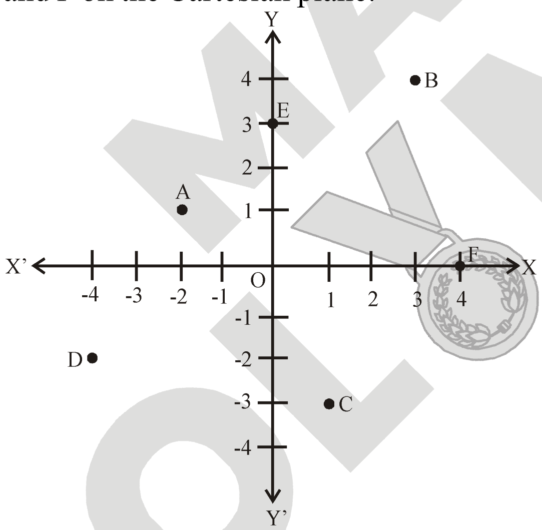
SE. 1

Plot the points (3, –3), (0, 4), (–2, 4) and (–3, –3) in the cartesian plane. Use the scale 1 cm = 1 unit on the co ordinate axes .



SE. 2

Write the coordinates of points A, B, C, D, E and F on the Cartesian plane.



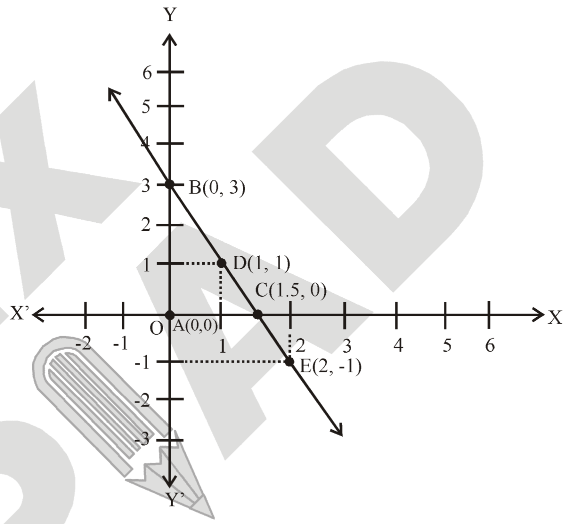
Ans. A = (–2, 1), B = (3, 4), C = (1, –3), D = (–4, –2), E = (0, 3), F = (4, 0)

SE. 3

Plot the given points on a graph, A(0, 0), B (0, 3), C (1.5, 0), D(1, 1), E(2, –1).

Is there a line passing through more than two points? If so which are those points?

Ans.



By inspection, we can see that a line passes through more than two points i.e., point B, D, C and E. But A is not lying on that line.

SE. 4

On which axes and on which side of origin do the given point lie?

(i) (9, 0) (ii) (0, –3)

(iii) (0, 6) (iv) (–5, 0)

Ans. (i) In (9, 0), we have the ordinate = 0.

Since , the abscissa is positive.

$\therefore$ (9, 0) lies on the x–axis and right of origin.

$\therefore$ (9, 0) lies on the positive x – axis.

(ii) In (0, –3) we have the abscissa = 0.

Since, the ordinate is negative.

$\therefore$ (0, –3) lies on the y – axis, below to the origin.

$\therefore$ (0, –3) lies on the negative y – axis.

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(iii) In (0, 6) have the abscissa = 0.
Since, the ordiante is positive.
∴ (0, 6) lies on the y – axis and above the origin.
∴ (0, 6) lies on the positive y – axis.
(iv) In (–5, 0) we have the ordinate = 0.
Since, the abscissa is negative.
∴ (–5, 0) lies on the x–axis, left of origin.
∴ (–5, 0) lies on negative x–axis.

SE. 5

Find the distance between A(2, –5) and B(–4, 2).

Ans. $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(2+4)^2 + (-5-2)^2} = \sqrt{6^2 + (-7)^2} = \sqrt{85}$.

SE. 6

Find the mid–point of line segment AB, if A is (2, –7) and B is (4, –3).

Ans. Mid points of AB $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{2+4}{2}, \dfrac{-7-3}{2}\right)$ = (3, –5)

SE. 7

Prove that the points (2, –2), (8, 4), (5, 7) and (–1, 1) are the angular points of a rectangle.

Ans. Here A(2, –2), B(8, 4), C(5, 7) and D(–1, 1) are angular points.

$AB = \sqrt{(8-2)^2 + (-4-2)^2} = \sqrt{36+36} = 6\sqrt{2}$ units.

$BC = \sqrt{(5-8)^2 + (7-4)^2} = \sqrt{9+9} = 3\sqrt{2}$ units.

$CD = \sqrt{(1+5)^2 + (1-7)^2} = 6\sqrt{2}$ units.

$AD = \sqrt{(1+2)^2 + (-1-2)^2} = 3\sqrt{2}$ units.

AB = CD and BC = AD.

So □ ABCD may be parallelogram or rectangle

$AC = \sqrt{(2-5)^2 + (-2-7)^2} = \sqrt{9+81} = \sqrt{90}$ units.

$BD = \sqrt{(-1-8)^2 + (1-4)^2} = \sqrt{81+9} = \sqrt{90}$ units.

AC = BD Diagonals are equal.

So ABCD must be rectangle.

SE. 8

Determine if the points (1, 5), (2, 3) and (–2, –11) are collinear.

Ans. Let the points be A(1, 5), B(2, 3) and C(–2, –11),
A, B and C are collinear, if
AB + BC = AC, AC + CB = AB, BA + AC = BC

$\because\ AB = \sqrt{(2-1)^2 + (3-5)^2}$

$= \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$

$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$

$= \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

$AC = \sqrt{(-2-1)^2 + (-11-5)^2}$

$= \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$

But AB + BC ≠ AC, AC + CB ≠ AB, BA + AC ≠ BC
∴ A, B and C are not collinear.

ONLY ONE CORRECT TYPE

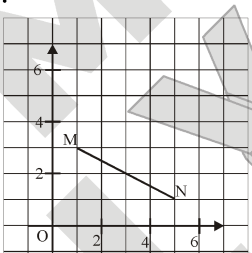
1. If P (a, b) lies in II quadrant then which of the following is true about a and b ?
(A) $a > 0, b > 0$ (B) $a > 0, b < 0$
(C) $a < 0, b > 0$ (D) $a < 0, b < 0$

2. A point lies on negative side of x–axis. Its distance from origin is 10 units. The coordinates of the point are
(A) (10, 0) (B) (–10, 0)
(C) (0, 10) (D) (0, –10)

3. Point (0, 3) lies
(A) on x–axis (B) on y–axis
(C) in I quadrant (D) at origin

4. The point for which the abscissa and ordinate have same signs will lie in
(A) I and II quadrants
(B) I and III quadrants
(C) I and IV quadrants
(D) III and IV quadrants

5. The diagram shows two points, M and N on a Cartesian plane.



The abscissa of M and ordinate of N are
(A) 3, 2 (B) 5, 1
(C) 1, 1 (D) 3, 5

6. In which quadrant abscissa is negative and ordinate is positive ?
(A) II (B) III
(C) I (D) IV

7. Signs of the coordinates of a point in the III quadrant are
(A) (+, +) (B) (–, +)
(C) (+, –) (D) (–, –)

8. If perpendicular distance of a point P from the x–axis be 3 units along the negative direction of the y–axis, then the point P has
(A) x–coordinate = – 3
(B) y–coordinate = –3
(C) y–coordinate = 3
(D) None of these

9. If the coordinates of the point P are (3, –5), then the perpendicular distance of P from the y–axis
(A) 4 (B) 5
(C) 3 (D) 2

10. If $a > 0$ and $b < 0$, then the point P(a, b) lies in
(A) IV quadrant (B) II quadrant
(C) III quadrant (D) I quadrant

11. Reflections of D(–2, –3) in x–axis and y–axis respectively are
(A) (–2, 3) and (–2, –3)
(B) (–2, –3) and (–2, 3)
(C) (2, 3) and (–2, 3)
(D) None of these

12. If the coordinates of two points A and B are (10, 5) and (–7, –4) respectively. Then the value of (x–coordinate of A) – (y – coordinate of B) is
(A) –14 (B) 14
(C) –10 (D) –12

13. If (x + 3, 5) = (2, 2 – y) then the value of the x and y are
(A) x = 5, y = 3 (B) x = –1, y = –3
(C) x = 0, y = –3 (D) x = 1, y = 3

14. The point is at a distance of 5 units from x–axis and 7 units from y–axis. Then, the coordinates of point could be
(A) (5, 7) (B) (7, 5)
(C) (0, 7) (D) (7, 0)

15. P is the point (–5, 3) and Q is the point (–5, m). If sum of abscissas and ordinates of both points is equal then the possible value of m is
(A) –5 (B) –13
(C) –10 (D) 3

16. Which of the following statements is true ?
(A) The point P(6, 0)lies in the quadrant I.
(B) The perpendicular distance of the point A(5, 5) from x–axis is 5 units.
(C) The mirror image of the point A(4, 5) in the x–axis is A'(–4, 5).
(D) The mirror image of the point A(4, 5) in the y–axis is A' (4, – 5)

17. Find the perimeter of the figure obtained by plotting points M(4, 3), N(4, 0), O(0, 0), P(0, 3).
(A) 14 units (B) 12 units
(C) 7 units (D) 24 units

18. Plot the point P(–6, 3) on a graph paper. Draw PL ⊥ x–axis and PM ⊥ y–axis. Then coordinates of points L and M respectively are
(A) (0, 6), (3, 0) (B) (–6, 0), (0, 3)
(C) (0, –6), (0, 3) (D) (–6, 0), (3, 0)

19. The distance of the point P(4, –3) from the origin is
(A) 1 unit (B) 3 units
(C) 5 units (D) 7 units

20. The distance between the points A (2, –3) and B(2, 2) is
(A) 4 units (B) 5 units
(C) 3 units (D) 2 units

21. If the distance between the points A(2, –2) and B(–1, x) is 5, then x =
(A) –3, 4 (B) 3, –4
(C) –6, 2 (D) 6, –2

22. A point P divides the line segment joining points A(5, –2) and B(9, 6) in the ratio 3 : 1. The coordinates of P are
(A) (4, 7) (B) (8, 4)
(C) (12, 8) (D) $\left(\frac{11}{2}, 5\right)$

23. The point which divides the line segment joining the point A(7, –6) and B(3,4) in the ratio 1 : 2 lies in
(A) I quadrant (B) II quadrant
(C) III quadrant (D) IV quadrant

24. If P(–1, 1) is the mid point of the line segment joining A(–3, b) and B(1, b + 4), then b = ?
(A) 1 (B) –1
(C) 2 (D) 0

25. If A (–1, 0), B(5, –2) and C(8, 2) are the vertices of ΔABC, then its centroid is
(A) (12, 0) (B) (4, 15)
(C) (–4, –15) (D) (4, 0)

PARAGRAPH TYPE

PASSAGE – I : The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Three points P, Q and R are said to be collinear if they lie on the same line. We can find condition of collinearity of three points by using distance formula. Based on the above passage, answer the following questions.

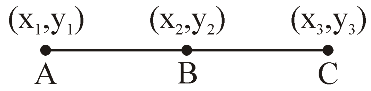
26. If two points $A(a_1, a_2)$ and $B(b_1, b_2)$ are given, then the distance between them is :
(A) $\sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$
(B) $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$
(C) $\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$
(D) $\sqrt{a_1 a_2 + b_1 b_2}$

27. If two points A(5, 3) and B(– 6, – 3) are given then the distance between them is :

(A) 1 unit (B) $\sqrt{79}$ units

(C) $\sqrt{139}$ units (D) $\sqrt{157}$ units

28. In the below figure, distance AC is given by



(A) $\sqrt{(x_1-x_3)^2+(y_1-y_3)^2}$

(B) $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(x_2-x_3)^2+(y_2-y_3)^2}$

(C) $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}+\sqrt{(x_2-x_3)^2+(y_2-y_3)^2}$

(D) Both (A) and (C)

PASSAGE–II : If one end point of a line segment is the origin and other end point is (x, y). Then distance between end points of the line segment = $\sqrt{x^2+y^2}$.

29. Which of the following points is not 10 units from the origin ?

(A) (–6, 8) (B) (8, –6)

(C) (–6, –8) (D) (6, 4)

30. Which of the following points is the nearest to the origin ?

(A) (0, –6) (B) (–8, 0)

(C) (–3, –4) (D) (7, 0)

31. What is the distance between (0, 0) and (–5, –2) ?

(A) 29 units (B) $\sqrt{52}$ units

(C) $\sqrt{29}$ units (D) 52 units

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column–I and Column–II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the following :

Column – I	Column – II
(P) coordinates of any point on x–axis	(1) (0, 0)
(Q) coordinates of any point on y–axis	(2) (x, y)
(R) ordered pair of x and y	(3) (x, 0)
(S) coordiantes of origin	(4) (0, y)

(A) P–3, Q–4, R–2, S–1

(B) P–4, Q–3, R–1, S–2

(C) P–4, Q–1, R–3, S–2

(D) P–3, Q–4, R–1, S–2

33. Match the following :

Column – I	Column – II
(P) The distance between (3, –9) and (– 2, 3) is	(1) 10 square units
(Q) Points (2, 0), (5, 0) and (7, 0) are	(2) 13 units
(R) In a rectangle ABCD, the area of ΔABC is $\frac{31}{2}$ sq. units. The area of rectangle ABCD is equal to	(3) Collinear
(S) The area of the triangle enclosed between the coordinate axes with the vertices (4, 0) and (0, 5) is	(4) 31 square units

(A) P–1, Q–2, R–3, S–4

(B) P–1, Q–3, R–2, S–4

(C) P–2, Q–3, R–4, S–1

(D) P–3, Q–2, R–1, S–4

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EXERCISE – II

VERY SHORT ANSWER TYPE

1. Find out the quadrants in which the following points lie.
   (i) Point A(3, –4) (ii) Point B(–3, 4)
   (iii) Point C(–3, –4) (iv) Point D(3, 4)
2. On which axis, do the following points lie ?
   (i) P(5, 0) (ii) Q(0, – 2)
   (iii) R(– 4, 0) (iv) S(0, 5)
3. Draw a rectangle PQRS in which vertices P, Q, R and S are (1, 4), (–5, 4), (–5, –3) and (1, –3) respectively.
4. Determine whether the given points lie on a same straight line or not : (0, 5), $\left(\frac{5}{2},0\right)$ and (5, –5).
5. Plot the points A(2, 0), B(2, 2), C(0, 2) and draw the line segments OA, AB, BC and CO. What figure do you obtain ?
6. Find the distance between the points A(– 1, 2) and B(3, 4).
7. Find the area of the triangle formed by joining the poitns (0, 6), (8, 0) and (0, 0).
8. Prove that A(2, 0), B(5, 0) and C(3, 0) are collinear points.
9. Coordinates of the point P(x, y), where x, y > 0. If line segment OP is making angle of 45° with positive x-axis, then by what angle should OP be rotated so as to make x and y both negative ?
10. Find the area of the figure formed by joining the points (1, 0), (0, 1), (– 1, 0) and (0, – 1).

SHORT ANSWER TYPE

1. Plot the points A(4, 4) and B(–4, 4) and join the lines OA, OB and BA. What figure, do you obtain ?
2. Draw a quadrilateral whose vertices are the points having coordinates as : (–3, 3), (3, 3), (3, –3) and (–3, –3). What is the special name of the quadrilateral so obtained ?
3. Find the value of x and y, if (x + 4, 3y –2) = (9, 6).
4. Find the value of y for which the distance between the points P(2, 3) and O(10, y) is 10 units.
5. Find the points where the graph of the equation $\frac{x}{2}+\frac{y}{3}=1$ cuts the Y-axis and the X-axis.

LONG ANSWER TYPE

1. Find the area of the triangle whose sides are represented by the graphs of the equations x = 0, y = 0 and 4x + 5y = 20.
2. Prove that the points (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order form a rhombus. Also, find its area.
3. Prove that the points (2a, 4a), (2a, 6a) and $\left(2a+\sqrt{3}a, 5a\right)$ are the vertices of an equilateral triangle.
4. If the point C(– 1, 2) divide internally the line segment joining A(2, 5) and B in ratio 3 : 4, find the coordinates of B.
5. Find the point of intersection of the diagonals of the rectangle whose vertices are (0, 8), (0, 0), (6, 0), (6, 8).

MATRIX OLYMPIAD

TRUE / FALSE TYPE

1. If $x \neq y$, $(x, y) \neq (y, x)$ and $(x, y) = (y, x)$ if $x = y$.
2. The y co-ordinate is also called as the abscissa.
3. The coordinate axes divides the plane into five parts called quadrants.
4. Point (1, – 1) and (– 1, 1) lies in the same quadrant.
5. The mirror image of (3, 9) on x-axis is (3, – 9).

FILL IN THE BLANKS

1. Every point on the X-axis has zero distance from the _______.
2. The point (-31,0) lies on _______.
3. The point (3,-7) lies in _______ quadrant
4. Distance of point (5,-12) from the origin_______.
5. Distance of the point (-5,-7) from X-axis _______.

ANALYTICAL PROBLEMS & BRAIN TEASER

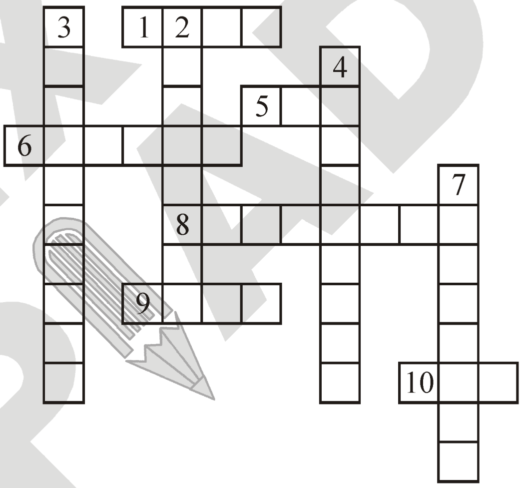
1. Show that the points (a, a), (– a, – a) and $(-\sqrt{3}a, \sqrt{3}a)$ are the vertices of an equilateral triangle. Also, find its area.
2. If the point P(x, y) is equidistant from the points A(5, 1) and B(1, 5), then prove that x – y = 0.
3. Find the distance between the points.
   (i) R(a + b, a – b) and S(a – b, – a – b)
   (ii) $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$, if $t_1t_2 = -1$
4. If opposite vertices of a square are (– 1, 1) and (1, – 1), then find the coordinates of the other vertices.
5. If P(6, – 1), Q(1, 3) and R(x, 5) are such that PR = QR, then the values(s) of x is/are
   (A) 3 (B) – 5
   (C) 6.7 (D) All of these

NUMERICAL PROBLEMS

1. What is the perpendicular distance of the point A(15, 19) from the y–axis ?
2. If the coordinates of two points are A(3, 4) and B(–2, –1), then find 4(abscissa of A) –3 (ordinate of B).
3. What is the distance of point A(16, 20) from x–axis
4. What is the ordinate of any point on x–axis ?
5. What is the abscissa of any point on y–axis ?

CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

1. The coordinate axes divide the Cartesian plane into _______ quadrants. [4]
5. The Cartesian plane consists of _____ axes. [3]
6. Point of intersection of x-axis and y-axis is called _______. [6]
8. The x-coordinate is also called ______. [8]
9. The y-coordinate of every point on x-axis is _______. [4]
10. Distance formula is valid for _____ quadrants. [3]

Down

2. The y-coordinate is also called ______. [8]
3. The _____ axes divide the plane into four quadrants. [10]
4. Three points A, B and C are such that AB + BC = AC, then these points are ________. [9]
7. Parallelogram is a quadrilateral in which each pair of opposite sides are _______. [8]

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	B	B	B	C	A	D	B	C	A	D	B	B	B	B
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
B	A	B	C	B	C	B	D	B	D	C	D	D	D	C
31	32	33												
C	A	C												

EXERCISE II

VERY SHORT ANSWER TYPE

1. (i) Fourth quadrant (ii) Second quadrant (iii) Third quadrant (iv) First quadrant

2. (i) x-axis (ii) y-axis (iii) x-axis (iv) y-axis

4. Yes 5. Square 6. $2\sqrt{5}$ units 7. 24 sq. units

9. 135° **CW/ACW** 10. 2 sq. units

SHORT ANSWER TYPE

1. Isosceles right triangle 2. Square 3. x = 5 and y = $\frac{8}{3}$

4. 9, – 3 5. (0, 3), (2, 0)

LONG ANSWER TYPE

1. 10 sq. units 2. 24 sq. units 3. 24 sq. units 4. (–5, –2) 5. (3, 4)

TRUE / FALSE

1. T 2. F 3. F 4. F 5. T

FILL IN THE BLANKS

1. x-axis 2. x-axis 3. iv 4. 13 5. 7

ANALYTICAL PROBLEMS & BRAIN TEASER

1. $2\sqrt{3}a^2$ square units 2. x – y = 0 3. (i) $2\sqrt{a^2+b^2}$ units, (ii) $a(t_1 - t_2)^2$ units

4. (1, 1) and (– 1, – 1) 5. C

NUMERICAL PROBLEMS

1. 15 2. 15 3. 20 4. 0 5. 0

CROSSWORD PUZZLE

1. Four 2. Ordinate 3. Coordinate 4. Collinear 5. Two
6. Origin 7. Parallel 8. Abscissa 9. Zero 10. All

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : COORDINATE GEOMETRY)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

MATRIX OLYMPIAD

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

LINEAR EQUATIONS IN TWO VARIABLES

4

Concepts

INTRODUCTION

In earlier classes, you have studied linear equations in one variable. Can you write down a linear equation in one variable ? You may say that $x + 1 = 0$, $x + \sqrt{2} = 0$ and $\sqrt{2}\ y + \sqrt{3} = 0$ are examples of linear equations in one variable. You also know that such equations have a unique (i.e., one and only one) solution. You may also remember how to represent the solution on a number line. In this chapter, the knowledge of linear equations in one variable shall be recalled and extended to that of two variables.

1. LINEAR EQUATION IN TWO VARIABLES

Let us now consider the following situation :

In a One-day International Cricket match between India and Sri Lanka played in Nagpur, two Indian batsmen together scored 176 runs. Express this information in the form of an equation.

Here, you can see that the score of neither of them is known, i.e., there are two unknown quantities. Let us use x and y to denote them. So, the number of runs scored by one of the batsmen is x, and the number of runs scored by the other is y. We know that

$$x + y = 176,$$

which is the required equation.

This is an example of a linear equation in two variables. It is customary to denote the variables in such equations by x and y, but other letters may also be used. Some examples of linear equations in two variables are :

$1.2s + 3t = 5$, $p + 4q = 7$, $\pi u + 5v = 9$ and $3 = \sqrt{2}\ x - 7y$.

So, any equation which can be put in the form ax + by + c = 0, where a, b and c are real numbers, and a and b are not both zero, is called a linear equation in two variables. This means that you can think of many many such equations.

Example 1

Write each of the following equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case :

(i) $2x + 3y = 4.37$ (ii) $x - 4 = \sqrt{3}\ y$ (iii) $2x = y$

Solution :

(i) $2x + 3y = 4.37$ can be written as $2x + 3y - 4.37 = 0$. Here $a = 2$, $b = 3$ and $c = -4.37$.

(ii) The equation $x - 4 = \sqrt{3}\ y$ can be written as $x - \sqrt{3}\ y - 4 = 0$. Here $a = 1$, $b = -\sqrt{3}$ and $c = -4$.

(iii) The equation $2x = y$ can be written as $2x - y + 0 = 0$. Here $a = 2$, $b = -1$ and $c = 0$.

Example 2

Write each of the following as an equation in two variables :

(i) $x = -5$ (ii) $y = 2$ (iii) $2x = 3$

Solution :

(i) $x = -5$ can be written as $1.x + 0.y = -5$, or $1.x + 0.y + 5 = 0$.

(ii) $y = 2$ can be written as $0.x + 1.y = 2$, or $0.x + 1.y - 2 = 0$.

(iii) $2x = 3$ can be written as $2x + 0.y - 3 = 0$.

2. SOLUTION OF A LINEAR EQUATION

A solution means a pair of values, one for x and one for y which satisfy the given equation.

Let us consider the equation $2x + 3y = 12$. Here, $x = 3$ and $y = 2$ is a solution because when you substitute $x = 3$ and $y = 2$ in the equation above, you find that

$$2x + 3y = (2 \times 3) + (3 \times 2) = 12$$

This solution is written as an ordered pair (3, 2), first writing the value for x and then the value for y. Similarly, (0, 4) is also a solution for the equation above.

On the other hand, (1, 4) is not a solution of $2x + 3y = 12$, because on putting $x = 1$ and $y = 4$ we get $2x + 3y = 14$, which is not 12. Note that (0, 4) is a solution but not (4, 0).

Focus Point

(i) Solution of linear equation in two variables can be represented as an ordered pair in a cartesian plane.

(ii) Conversely, if any ordered pair (α, β) is a solution of any linear equation $ax + by + c = 0$, then $a\alpha + b\beta + c = 0$.

(iii) A linear equation in two variables has infinitely many solutions. We usually find some of them.

Example 3

Check whether $x = 4$ and $y = 0$ is a solution of $4x + 3y = 16$ or not.

Solution :

$4x + 3y = 16$

At $x = 4, y = 0$ L.H.S. $= 4 \times 4 + 3 \times 0 = 16 =$ R.H.S.

$\therefore$ L.H.S. = R.H.S.

Hence, $x = 4, y = 0$ is the solution of the equation.

Example 4

Find the value of k if $x = 2$, $y = 3$ is a solution of $(k + 1)x - (2k + 3)y - 1 = 0$.

Solution :

If $x = 2, y = 3$ is a solution of given equation is true.

L.H.S. (at $x = 2, y = 3$)

$= (k + 1) \times 2 - (2k + 3) \times 3 - 1$

$= 2k + 2 - 6k - 9 - 1 = -4k - 8$

and R.H.S. = 0

Since L.H.S. = R.H.S.

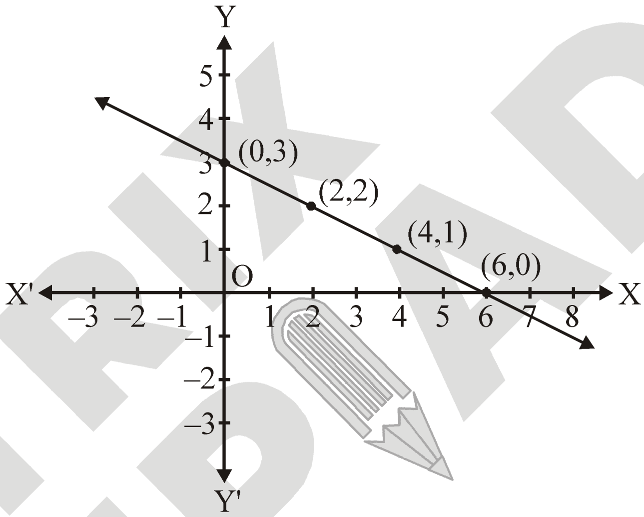
$\Rightarrow$ $-4k - 8 = 0$ $\Rightarrow$ $4k = -8$

$\therefore$ $k = -2$

3. GRAPH OF A LINEAR EQUATION IN TWO VARIABLES

A degree one, polynomial equation $ax + by + c = 0$ in its geometrical representation or graphical presentation is a straight line. Let us take a linear equation $x + 2y = 6$. The solution of given equation can be expressed in the form of table as follows :

x	0	2	4	6	...
y	3	2	1	0	...



$\Rightarrow$ By plotting the points (0, 3) (2, 2), (4, 1) and (6, 0) on a graph paper we obtained the given graph which is called the geometrical representation of linear equation in two variables.

Example 5

Write three solutions of $x - 7y - 2 = 0$.

Solution :

We have, $x - 7y - 2 = 0$

For $x = 0$, $0 - 7y - 2 = 0$ $\Rightarrow$ $y = -\frac{2}{7}$

For $x = 1$, $1 - 7y - 2 = 0$ $\Rightarrow$ $-7y = 1$

$\Rightarrow$ $y = -\frac{1}{7}$

For $x = 9$, $9 - 7y - 2 = 0$ $\Rightarrow$ $7 - 7y = 0$ $\Rightarrow$ $y = 1$

$\therefore$ Solution table is

x	0	1	9
y	$-\frac{2}{7}$	$-\frac{1}{7}$	1

Example 6

Find solution table of 2x + y = 4.

Solution :

We have, 2x + y = 4

For x = 0, y = 4

x = –1, y = 4 + 2 = 6

x = 1, y = 4 – 2 = 2

∴ The solution table is.

x	0	–1	1
y	4	6	2

Example 7

If (3, 5) lies on the graph of ax + 5y = 15. Find the value of a.

Solution :

If (3, 5) lies on the graph, so (3, 5) is the solution of ax + 5y = 15

⇒ $a \times 3 + 5 \times 5 = 15$

⇒ $3a = 15 - 25 = -10$

∴ $a = -\frac{10}{3}$

4. EQUATION OF LINES PARALLEL TO X-AXIS AND Y-AXIS

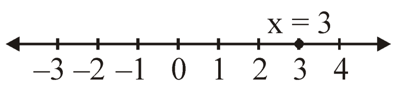
- A linear equation x = a in one variable can be represented geometrically by plotting x = a on number line. The linear equation x = a *i.e.*, $1 \cdot x + y \cdot 0 = a$ can also be represented geometrically in two variables by a line through (a, 0) and parallel to y-axis in cartesian plane.
- The linear equation y = b in one variable can be represented geometrically by plotting y = b on number line. The linear equation y = b *i.e.*, $x \cdot 0 + y \cdot 1 = b$ can also be represented geometrically in two variables by a line through (0, b) and parallel to x-axis in cartesian plane.

Example 8

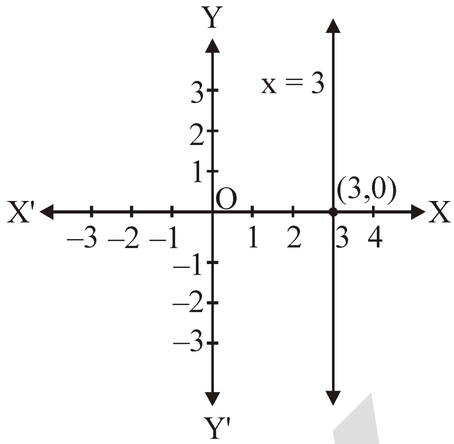
Represent x = 3 in one variable and two variables.

Solution :

In one variable x = 3 is



In two variables x = 3 is



Given equation x = 3 can be represented geometrically by a line through (3, 0) which is parallel to y-axis.

Example 9

Draw the graph of the following linear equation in cartesian plane.

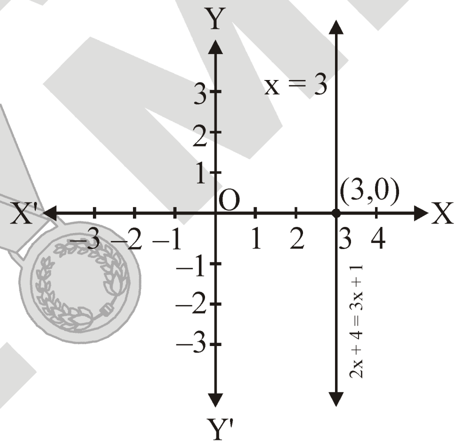
$2x + 4 = 3x + 1$

Solution :

We have, $2x + 4 = 3x + 1$

$\Rightarrow \quad 3x - 2x = 4 - 1 \quad \Rightarrow \quad x = 3$

Clearly, it does not contain y. So, its graph is a line parallel to y-axis passing through the point (3, 0) as shown in figure.

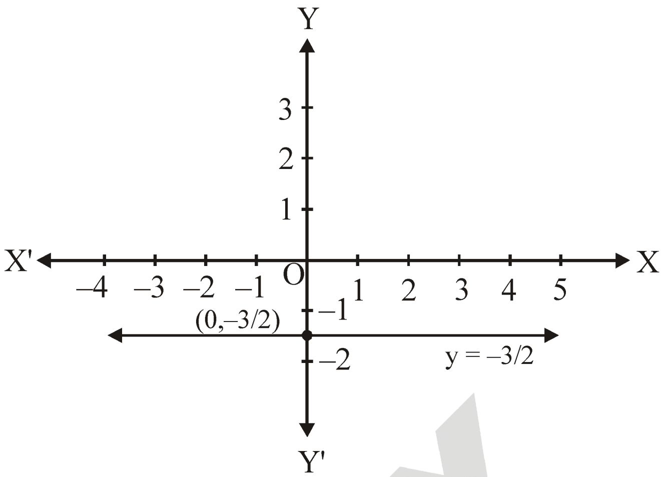


Example 10

Represent $2y + 3 = 0$ graphically in two variables.

Solution :

$2y + 3 = 0 \quad \Rightarrow \quad 2y = -3 \quad \Rightarrow \quad = -\frac{3}{2}$



$2y + 3 = 0$ can be represented geometrically by a line through $(0, -3/2)$ which is parallel to x-axis.

Focus Point

- An equation of the type $y = mx$ represents a line passing through the origin.
- A linear equation in two variables is represented geometrically by a line whose points make the collection of solutions of the equation.
- If a linear equation is of type $ax + by = 0$ (*i.e.* constant = 0) then its graph will be a straight line passing through origin because $x = 0, y = 0$ will always satisfy the equation.

BUILD THE CONCEPT

Two linear equatins $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(i) intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) coincident, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

In fact, the converse is also true for any pair of lines.

Example 11

By comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident :

(i) $5x - y + 7 = 0; 10x - 2y + 15 = 0$

(ii) $3x + y - 14 = 0; 2x + 5y - 5 = 0$

Solution :

(i) We have $5x - y + 7 = 0, 10x - 2y + 15 = 0$

Here $a_1 = 5, b_1 = -1, c_1 = 7,$

$a_2 = 10, b_2 = -2, c_2 = 15$

Now, $\frac{a_1}{a_2} = \frac{5}{10} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-1}{-1} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{7}{15}$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines are parallel.

(ii) We have $3x + y - 14 = 0, 2x + 5y - 5 = 0$

Here $a_1 = 3, b_1 = 1, c_1 = -14,$

$a_2 = 2, b_2 = 5, c_2 = -5$

Now, $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{1}{5}, \frac{c_1}{c_2} = \frac{-14}{-5} = \frac{15}{5}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the lines intersect at a point.

Example 12

Find the value of p for which the pair of linear equations $2px + 3y = 7, 2x + y = 6$ has exactly one solution.

Solution :

The given equations are $2px + 3y = 7$ and $2x + y = 6$ has exactly one solution i.e., they are intersecting lines.

We write these equations in standard form :

$2px + 3y - 7 = 0$

$2x + y - 6 = 0$

$a_1 = 2p, b_1 = 3, c_1 = -7$

$a_2 = 2, b_2 = 1, c_2 = -6$

$\frac{a_1}{a_2} = \frac{2p}{2} = p, \frac{b_1}{b_2} = \frac{3}{1}.$

Since the lines are intersecting therefore

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow p \neq 3.$

Hence, there will be a solution for all real values of p except 3.

MATRIX OLYMPIAD

SOLVED EXAMPLES

SE. 1

Write each of the following as an equation in two variables *i.e.*, in the form ax + by + c = 0.

(i) x = – 5 (ii) y = 2

(iii) 2x = 3 (iv) 5y = 2

Ans. **(i)** x = – 5 can be written as

$1\cdot x + 0\cdot y = -5$ or $1\cdot x + 0\cdot y + 5 = 0$.

(ii) y = 2 can be written as

$0\cdot x + 1\cdot y = 2$ or $0\cdot x + 1\cdot y - 2 = 0$.

(iii) 2x = 3 can be written as $2x + 0\cdot y - 3 = 0$.

(iv) 5y = 2 can be written as $0\cdot x + 5y - 2 = 0$.

SE. 2

Which of the following equations are linear equations ?

(i) 3x + 6 = 4x – 5 + 5y

(ii) 5y + 3 = 25 – 4y – 2x

(iii) $u + 4 = u^2 - 4$

(iv) $x^2 + 2 = x + 1$

Ans. **(i)** 3x + 6 = 4x – 5 + 5y

$\Rightarrow$ 11 = 4x – 3x + 5y $\Rightarrow$ x + 5y = 11

It is a linear equation in x and y.

(ii) 5y + 3 = 25 – 4y – 2x $\Rightarrow$ 2x + 9y = 22

It is a linear equation in x and y.

(iii) $u + 4 = u^2 - 4$. Since degree of the equation is 2. Hence, it is not a linear equation.

(iv) $x^2 + 2 = x + 1$. Degree of this equation is 2, so it is not a linear equation.

SE. 3

Given the point (5, 6), find the equation of a line on which it lies. How many such equations are there ?

Ans. Here (5, 6) is a solution of a required linear equation we are looking for. So, we have to find any line passing through the point (5, 6). In fact, there are infinitely many linear equations which are satisfied by the coordinates of the point (5, 6) because through a given point infinite lines can be drawn.

SE. 4

Solve the equation 3x + 2 = x + 8 and represent the solution(s) on

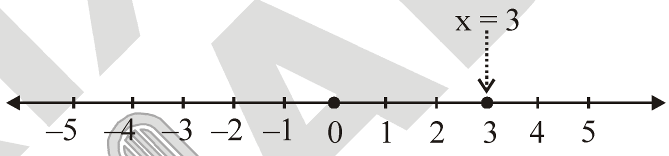
(i) the number line (ii) the cartesian plane

Ans. We have, 3x + 2 = x + 8

$\Rightarrow$ 3x – x = 8 – 2 = 6

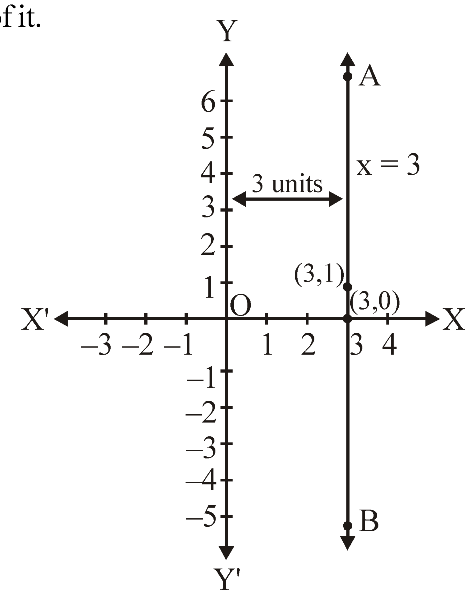
i.e., 2x = 6 $\Rightarrow$ x = 3

(i) The representation of the solution on the number line is shown in figure, where x = 3 is treated as an equation in one variable.



(ii) We know that x = 3 can be written as $x + 0\cdot y = 3$ which is a linear equation in two variables x and y. This is represented by a line. Now all the values of y are permissible because $0\cdot y$ is always 0. However, x must satisy the equation x = 3. Hence two solution of the given equation are x = 3, y = 0 and x = 3, y = 1.

AB represents graph of x = 3 as it is a line parallel to the y-axis and at a distance of 3 units to the right of it.



MATRIX OLYMPIAD

SE. 5

Draw the graph of the equation y = 3x. From your graph, find the value of x when y = – 3.

Ans. Given equation is y = 3x

When x = 1, then y = 3 × 1 = 3

When x = 2, then y = 3 × 2 = 6

When x = 0, then y = 3 × 0 = 0

Thus, we have the following table :

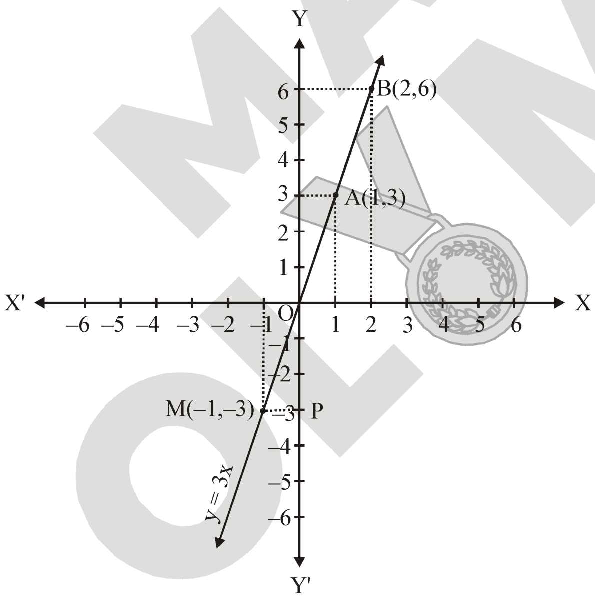
x	1	2	0
y	3	6	0

Now, plotting the points A(1, 3), B(2, 6), O(0, 0) on a graph paper and joining them, we get line AB. Line AB is the required graph of y = 3x.

Given : y = – 3. Take a point P on the y-axis such that OP = – 3.

Draw PM, parallel to the x-axis meeting the graph line at M, giving PM = –1.

∴ When y = – 3, then x = – 1.



SE. 6

How will you express the following situation as a linear equation in two variables ? "Age of father is 10 years more than 3 times the age of the son."

Ans. Let the age of father = x years and age of son = y years

According to the equation, x = 3y + 10.

SE. 7

For what value of k, x = 1, y = 0 is a solution of $\left(\frac{k+2}{k-1}\right)x - \left(\frac{3x-2}{k+2}\right)y - 3 = 0$?

Ans. Since (x = 1, y = 0) is the solution of given equation, So, x = 1, y = 0 must satisfy the equation.

$\Rightarrow \left(\frac{k+2}{k-1}\right)\times 1 - \left(\frac{3x-2}{k+2}\right)\times 0 - 3 = 0$

$\Rightarrow \frac{k+2}{k-1} - 3 = 0 \quad \Rightarrow k + 2 = 3(k-1)$

$\Rightarrow k + 2 = 3k - 3 \quad \Rightarrow 2 + 3 = 3k - k$

$\Rightarrow 5 = 2k \quad \Rightarrow k = \frac{5}{2}$

∴ For $k = \frac{5}{2}$, x = 1 and y = 0 is the solution of given equation.

SE. 8

Write the equation of following lines.

(i) passing through (7, 0) and parallel to y-axis.

(ii) passing through (0, 3) and parallel to x-axis.

Ans. **(i)** Since a line parallel to y-axis has its ordinate zero and when it passes through (7, 0) its equation is x = 7.

(ii) Since line parallel to x-axis has its abscissa zero and when it passes through (0, 3) its equation is y = 3.

SE. 9

Check which one of the following are solutions of the equation $2x - y = 4$ and which are not :
(i) (0, 2) (ii) (2, 0) (iii) (4, 4)

Ans. In the equation , we have
L.H.S. = $2x - y$ and R.H.S. = 4
(i) Putting $x = 0$ and $y = 2$ in the given equation, we get
L.H.S. = $2x - y = 2 \times 0 - 2 = -2 \neq$ R.H.S.
So, (0, 2) is not a solution of $2x - y = 4$
(ii) Putting $x = 2$ and $y = 0$ in the given equation we get
L.H.S. = $2x - y = 2 \times 2 - 0 = 4$ = R.H.S.
So, (2, 0) is a solution of $2x - y = 4$
(iii) Putting $x = 4$ and $y = 4$ in the given equation we get
L.H.S. = $2x - y = 2 \times 4 - 4 = 4$ = R.H.S.
So, (4, 4) is a solution of $2x - y = 4$.

SE. 10

Find the value of t, if $x = 2, y = 3$ is a solution of the equation $3x - 2y = t$.

Ans. $x = 2$ and $y = 3$ is a solution of $3x - 2y = t$.
So, $x = 2$ and $y = 3$ satisfy the equation $3x - 2y = t$
$\Rightarrow \quad 3 \times 2 - 2 \times 3 = t \quad \Rightarrow \quad 6 - 6 = t \quad \Rightarrow \quad t = 0$

SE. 11

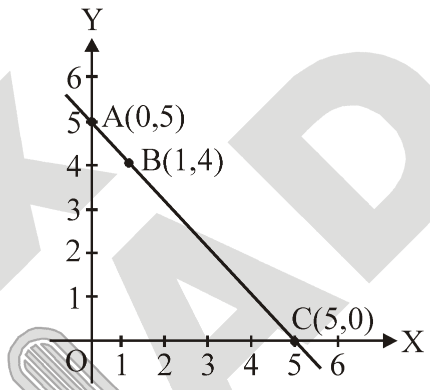
If $x = 5, y = 1$ is a solution of the equation $ax + a^2y = 6$, then find the value of a.

Ans. $x = 5$ and $y = 1$ is the solution of equation $ax + a^2y = 6$
$\therefore \quad a \times 5 + a^2 = 6$
$\Rightarrow \quad a^2 + 5a - 6 = 0$
$\Rightarrow \quad a^2 + 6a - a - 6 = 0$
$\Rightarrow \quad a(a + 6) - 1(a + 6) = 0$
$\Rightarrow \quad (a + 6)(a - 1) = 0$
$\Rightarrow \quad a + 6 = 0$ or $a - 1 = 0$
$\Rightarrow \quad a = -6$ or $a = 1$

SE. 12

Draw the graph of the equation $y + x = 5$.

Ans. Given equation is $y + x = 5$
When $x = 0$, then $y = 5$
When $x = 1$, then $y = 4$
When $x = 5$, then $y = 0$



Thus, we have following table

x	0	1	5
y	5	4	0

Plotting the points A(0, 5), B(1, 4) and C(5, 0) on the graph paper, we get the graph of the line $y + x = 5$.

SE. 13

Draw graphs of the equations :
$2x - 3y = 6$ and $x + 2y - 2 = 0$.

Ans. We have, $2x - 3y = 6$
When $x = 0$, then $y = -2$
When $x = 3$ then $y = 0$
Hence, we get the following table

x	0	3
y	–2	0

By plotting the points (0, –2) and (3, 0) on the graph paper, we get the graph of $2x - 3y = 6$
Now, for $x + 2y - 2 = 0$

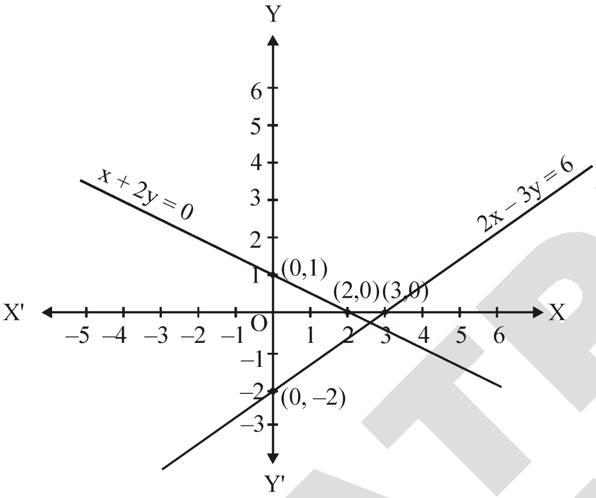
When x = 0, then y = 1

When x = 2, then y = 0

So, we have the following table.

x	0	2
y	1	0

By plotting the points (0, 1) and (2, 0) on the graph paper, we get the graph of $x + 2y - 2 = 0$.



SE. 14

If the points A(3, 5) and B(1, 4) lie on the graph of the line ax + by = 7, find the values of a and b.

Ans. It is given that the points A(3, 5) and B(1, 4) lie on the graph of the line ax + by = 7. Therefore, x = 3, y = 5 and x = 1, y = 4 are solutions of the equation ax + by = 7.

$\therefore \quad 3a + 5n = 7$ and $a + 4b = 7$

Multiplying (ii) by 3, we get

$3a + 12b = 21$

Subtracting (iii) from (i), we get

$-7b = -14 \Rightarrow b = 2$

Putting b = 2 in (i), we get

$3a + 5 \times 2 = 7 \Rightarrow 3a = -3 \Rightarrow a = -1$

SE. 15

The taxi fare in a city is as follows. For the first kilometre, the fare is Rs. 20, for the subsequent distance it is Rs. 10 per km. Taking the distance covered as x km and total fare as Rs. y, write a linear equation for this information .

Ans. Total fare = Rs. y

Total distance = x km

According to question,

$y = 20 + 10(x - 1)$

$\Rightarrow \quad y = 20 + 10x - 10 \Rightarrow y = 10x + 10$

Which is the required linear equation.

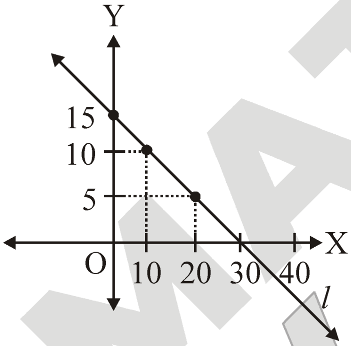
ONLY ONE CORRECT TYPE

1. $2 = -y$ can be expressed in the form $ax + by + c = 0$ as
(A) $y + 2 = 0$ (B) $y + 0 \cdot x + 3 = 0$
(C) $0 \cdot x + 1 \cdot y - 2 = 0$ (D) $0 \cdot x + 1 \cdot y + 2 = 0$

2. Age of a father is 7 years more than 3 times the present age of his son. The above statement can be expressed in a linear equation as
(A) $x - 3y - 7 = 0$ (B) $x + 3y + 7 = 0$
(C) $x + 3y - 7 = 0$ (D) $x - 3y + 7 = 0$

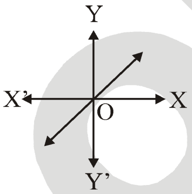
3. The number of solutions, the equation $3x + 5y + 15 = 0$ can have
(A) One only (B) Exactly two
(C) Zero (D) Infinite

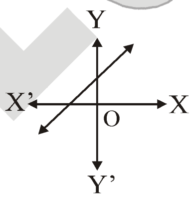
4. If $(20, -a)$ lies on l whose graph is given, then the value of a is

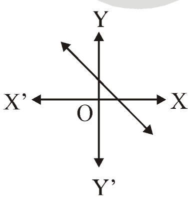


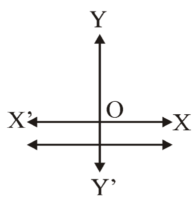
(A) – 5 (B) 5
(C) – 10 (D) 10

5. Which could be the graph of $y = x$?

(A) 

(B) 

(C) 

(D) 

6. Which of the following equation has graph parallel to y-axis ?
(A) $y = -2$ (B) $x = 1$
(C) $x - y = 2$ (D) $x + y = 2$

7. The distance between $M(-1, 5)$ and $N(x, 5)$ is 8 units. The value of x is
(A) – 9 or 9 (B) – 7 or 9
(C) – 9 or 7 (D) – 7 or – 9

8. Graph of $x = 2$ and $y = 1$ intersect at
(A) $(-1, 2)$ (B) $(2, -1)$
(C) $(1, 2)$ (D) $(2, 1)$

9. The linear equation $3x = 2y$ when expressed in the form $ax + by + c = 0$, then a, b, c are respectively
(A) 3, 2, 0 (B) 3, 2, 1
(C) 3, – 2, 0 (D) 3, – 2, 1

10. An ordered pair that satisfy an equation in two variables is called its
(A) Zero (B) Root
(C) Solution (D) Both (B) and (C)

11. Richa had 10 chocolates, let her brother borrowed y chocolates from her and then Richa had 4 chocolates. Which equation models this solution ?
(A) $10 - y = 4$ (B) $10 + y = 4$
(C) $10y = 4$ (D) $4y = 4$

12. $ax + by + c = 0$ does not represent equation of line, if:
(A) $a = c = 0, b \neq 0$ (B) $c = 0, a \neq 0, b \neq 0$
(C) $b = c = 0, a \neq 0$ (D) $a = b = 0$

13. Sonia distributed notebooks in an orphanage. On her birthday, she gave 5 notebooks to each child and 20 notebooks to adults. Taking number of children as x and total notebooks distributed as y then the linear equation representing above situation is :
(A) $y = 5x + 20$ (B) $y = 5x$
(C) $y = 5x - 20$ (D) $x + 5y = 20$

14. Find the values of p and q for which $x = 1, y = 1$ are solutions of the equations, $9px + 12py = 63$ and $5x + 2qy = 3q$.
(A) 3, 5 (B) 5, 3
(C) 9, 2 (D) None of these

15. Points A and B are 90 km apart from each other on a highway. A car starts from A with speed x km/h and another from B with speed y km/h at same time. If they go in the same direction they meet in 9 hours then linear equation representing the above situation is

(A) x + y = 10 (B) 2x + y = 9
(C) x – y = 10 (D) 3x + 4y = 20

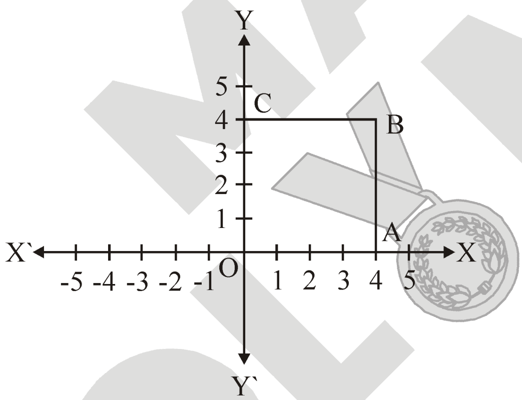
16. If (2, 1) and (1, 0) lie on the graph of $\frac{x}{a}+\frac{y}{b}=1$, then the vlaues of a and b are

(A) a = 1, b = – 1 (B) a = – 1, b = 1
(C) a = 2, b = 1 (D) a = 1, b = 2

17. The graph of y = – 9 is a line

(A) Parallel to the x-axis at a distance 9 units above the origin
(B) Parallel to the x-axis at a distance 9 units below the origin
(C) Parallel to the y-axis at a distance 9 units to left of the origin
(D) Parallel to the y-axis at a distance 9 units to the right of the origin

18. OABC is a square whose equations of sides are :



(A) x = 4, y = 4, x = – 4, y = – 4
(B) x = 4, y = 4, x = 0, y = 0
(C) x = – 4, y = 4, x = 0, y = 0
(D) x = 4, y = 4, x = – 4, y = 0

19. If the graph of the equation 4x + 3y = 12 cuts the coordinate axes at A and B, then hypotenuse of right triangle AOB is of length

(A) 4 units (B) 3 units
(C) 5 units (D) None of these

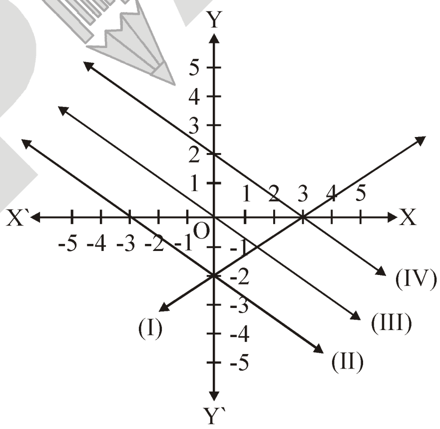
20. The point (2, 3) lies on the graph of the linear equation 3x – (a – 1) y = 2a – 1. If the same point also lies on the graph of the linear equation 5x + (1 – 2a) y = 3b, then find the value of b.

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$
(C) $\frac{1}{7}$ (D) $\frac{2}{3}$

21. The number of ways of writing the number $\frac{1}{15}$ in the form $\frac{a}{3}-\frac{b}{5}$ (a, b are real numbers and a, b $\neq$ 0) is :

(A) 0 (B) 1
(C) Finitely many (D) Infinitely many

22. The graph of the equation 2x + 3y = 6 is :



(A) I (B) II
(C) III (D) IV

23. The point of intersection of lines x + y – 1 = 0 and x – y + 1 = 0 is :

(A) (0, 1) (B) (1, 0)
(C) (1, 1) (D) (– 1, 0)

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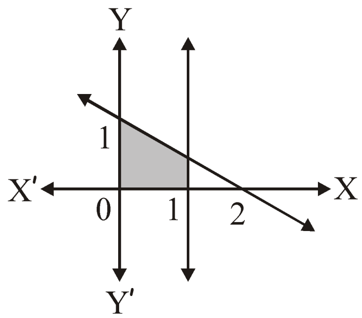
24. In the rectangular coordinate system given below, the shaded region is bounded by straight lines. Which of the following is not an equation of one of the boundary lines ?



(A) $x = 0$ (B) $x = 1$
(C) $x - y = 0$ (D) $x + 2y = 2$

25. Rakesh has x dollars more than the amount Mohan has and together they have a total of y dollars. Which of the following represents the number of dollars that Mohan has ?

(A) $\frac{y-x}{2}$ (B) $y - \frac{x}{2}$

(C) $\frac{y}{2} - x$ (D) $2y - x$

PARAGRAPH TYPE

PASSAGE – I :

The graph of the linear equation $ax + by + c = 0$ is a straight line. If the equation does not contain the constant term, then the graph of equation will pass through origin. If (x_1, y_1) is a solution of the equation $ax + by + c = 0$, then it will also lie on the graph of the equation and vice-versa.

Based on the above passage, answer the following questions.

26. Graph of the equation $3x - 7y = 0$
(A) Passes through the origin
(B) Does not pass through the origin
(C) Is a straight line parallel to x-axis
(D) Is a straight line parallel to y-axis

27. If the point $P(2a - 1, a - 2)$ lies on the graph of the equation $x - y = -\frac{1}{2}$, then the value of a is

(A) $-\frac{1}{2}$ (B) $-\frac{3}{2}$

(C) $-\frac{5}{2}$ (D) $-\frac{7}{2}$

28. The definite solution of the equation $ax - by = 0$ is
(A) (0, 0) (B) (a, b)
(C) (0, – 1) (D) (– 1, 0)

PASSAGE–II :

The system of linear equations is given as

$4x - (3k + 2)y = 20$

$(11k - 3)x - 10y = 40$, where $k \neq 0$.

Based on the above equations, answer the following questions.

29. If the given system of linear equations has infinitely many solutions, then the value of k is
(A) 0 (B) 1
(C) –1 (D) 2

30. If k = 4, then the set of linear equations has
(A) No solution (B) Unique solution
(B) Infinite solution (D) Data insufficient

31. If k = 0, then the given system of linear equations has
(A) No solution (B) Unique solution
(B) Infinite solution (D) Data insufficient

MATCH THE COLUMN TYPE

32. Match the following :

Column – I	Column – II
(P) The value of k for which (2, 1) is a solution of the equation 3x + 2y = k is	(i) 4
(Q) Equation of a straight line at an angle of 45° with the positive x-axis is	(ii) 8
(R) If (a, – 2) is a solution of x + y = 2, then the value of a is	(iii) y-axis
(S) The graph of the equation x = 4 is parallel to	(iv) y = x

(A) P – (ii), Q – (iv), R – (i), S – (iii)
(B) P – (iv), Q – (ii), R – (i), S – (iii)
(C) P – (ii), Q – (iii), R – (iv), S – (i)
(D) P – (i), Q – (iv), R – (ii), S – (iii)

33. In European countries temperature is measured in Fahrenheit, whereas in Asian countries, it is measured in Celsius. The linear equation that converts Fahrenheit to Celsius is

$$C = \frac{1}{9}(F - 32) \times 5.$$

Match the temperatures given in List – I with temperatures given in List – II.

Column – I	Column – II
(P) 26° C in F =	(i) 104°
(Q) 64° F in C =	(ii) 8.9°
(R) 48° F in C =	(iii) 78.8°
(S) 40° C in F =	(iv) 17.8°

(A) P – (iv), Q – (i), R – (iii), S – (ii)
(B) P – (iii), Q – (iv), R – (ii), S – (i)
(C) P – (iv), Q – (iii), R – (i), S – (ii)
(D) P – (ii), Q – (i), R – (iii), S – (iv)

Space for Notes :

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EXERCISE – II

VERY SHORT ANSWER TYPE

1. Find the value of k if (3, 4) is a solution of the equation $5x - 2y = k$. Find one more solution of the equation.
2. Represent the following situation by a linear equation : Age of father is 8 less than 5 times age of his son.
3. Find the value of k for which the system of equations, $x + y - 4 = 0$ and $2x + ky - 3 = 0$ has no solution.
4. Write the value of k for which the system of equations, $2x - y = 5$ and $6x + ky - 15$ has infinitely many solutions.
5. The runs scored by two batsmen in a cricket match is 105. Write a linear equation in two variables x and y for the same.
6. If $x = 2s$ and $y = s$ is a solution of the equation $3x - 5y - 7 = 0$, then find the value of s.
7. Write the value of k for which the system of equations $x + ky = 0$, $2x - y = 0$ has unique solution.
8. The cost of a plate is four times the cost of a cup. Write a linear equation in two variables to represent the statement.
9. In an ordered pair, which coordinate is always listed first ?
10. What is the standard form of a linear equation ?

SHORT ANSWER TYPE

1. Plot the following pairs of number (x, y) as points in the cartesian plane. Use the scale 1 cm = 1 unit on the axes.

x	–3	0	–1	4	5
y	2	–3	–1	4	0

2. Write the linear equations in two variables in the form $ax + by + c = 0$ by using the given values of a, b, c.
(i) $a = 0$, $b = -3$ and $c = 0$
(ii) $a = \frac{1}{2}, b = \frac{-1}{3}$ and $c = \frac{1}{6}$
3. Express y in terms of x, given that $2y - 4x = 7$. Check whether (– 1, – 1) is a solution of the line.
4. Draw the graph of the linear equation $4x - 3y + 12 = 0$ in two variables.
5. The cost of milk in a city is Rs. 40 per litre. Write an equation with y representing the number of litres and x representing the total cost (in rupees). Also, draw its graph.

LONG ANSWER TYPE

1. Solve the following equations graphically :
$2x + 3y = 12$, $x - y = 1$
Shade the region between the two lines and x-axis.
2. Draw the graph of each of the following equations in a single cartesian plane.
(i) $x = 0$ (ii) $y = 0$
(iii) $x = 4$ (iv) $y = 3$
(v) $y + 4 = 0$ (vi) $x + 2 = 0$
3. Sum of two numbers is 100. Write a linear equation to represent this statement and draw its graph. If one number is 50, using graph, find the other.
4. Present age of a father is four times the age of his son. Write a linear equation in two variables to represent this situation and draw the graph.
5. A library take charge of Rs. 5 for issuing a book for one day and Rs. 1 per day thereafter. If Ritu had taken a book for x days and y be the total amount needs to be paid, write the linear equation in two variables for this situation. Plot its graph and find the amount to be paid for 5 days.

TRUE / FALSE TYPE

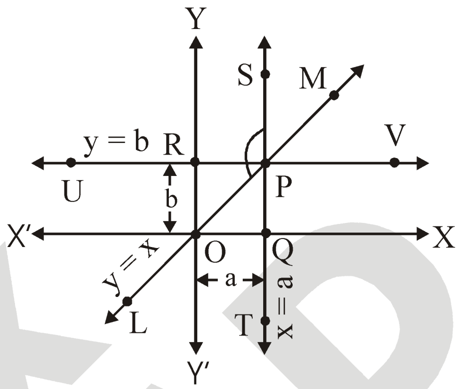
1. ax + by + c = 0, where a, b and c are real numbers and a, b ≠ 0 is a linear equation in two variables.
2. A linear equation 2x + 3y = 5 has a unique solution.
3. All the points (2, 0), (–3, 0), (4, 2) and (0, 5) don't lie on the x-axis.
4. The graph of the equation y = mx + c passes through the origin.
5. The point (0, 3) lies on the graph of the linear equation 3x + 4y = 12.

FILL IN THE BLANKS

1. The standard form of a linear equation with two variables is __________ .
2. The linear equation $3x - 11y = 10$ has ________ solutions.
3. $3x + 10 = 0$ will has _________ solution.
4. If $x = 1, y = 2$ is a solution of the equation $2x + 3y = k$, the value of $k =$ ___________ .
5. Two linear equations $a_1 + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are intersecting if _________ .

ANALYTICAL PROBLEMS

1. Draw a quadrilateral whose sides are represented by graphs of the equation x = 0, y = 0, 2y – 3x – 1 = 0 and 5x – y – 10 = 0. Determine the coordinates of the vertices of the quadrilateral.
2. Draw the graphs of y = – 9 and x – 2 = y and find the point of intersection, if any.
3. The coordinates of the point A are (x, y), where x < 0 and y > 0. If the line segment OA (O is the origin) makes an angle of 150° with the positive x-axis, then by what angle (anticlockwise) should OA be retated so as to make x positive and y negative ?
4. The graph of the equation y = x, x = a and y = b intersect each other at point P as shown in the figure. Find the value of ∠SPO.



5. If p and q are whole numbers, then find the number of ordered pairs (p, q) which satisfy the equation 2p + 3q = 25.

NUMERICAL PROBLEMS

1. If the distance between the graphs of the equations y = – 1 and y = 3 is k units, find the value of 36k.
2. If the graph of the equation 4x + 3y = 12 cut the x-axis and y-axis at A and B respectively, then find the sum of abscissa of A and ordinate of B.
3. If the system of equations 2x + 3y = 5, 4x + ky = 10 has infinitely many solutions, then find k.
4. If the system of equations 4x + 6y = 7, 4ax + 2 (a + b) y = 28 has infinitely many solutions, then b = ka. Find value of k.
5. If (3a – 4, 0) and (5, 0) are same points, then what is the value of a ?

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	A	D	A	A	B	C	D	C	C	A	D	A	A	C
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	B	B	C	A	D	D	A	C	A	A	B	A	B	B
31	32	33												
B	A	B												

EXERCISE II

VERY SHORT ANSWER TYPE

1. k = 7, (1, – 1) 2. x = 5y – 8 3. k = 2 4. k = – 3 5. x + y = 105

6. 7 7. $k \neq \frac{-1}{2}$ 8. x – 4y = 0 9. x-coordinate 10. ax + by + c = 0

SHORT ANSWER TYPE

6. (i) 0·x – 3y + 0 = 0 (ii) 3x – 2y + 1 = 0 7. $y = \frac{7+4x}{2}$, point (– 1, – 1) is not a solution

10. x – 40y = 0

LONG ANSWER TYPE

3. x + y = 100 4. x = 4y 5. y = x + 4, Rs. 9

TRUE / FALSE

1. True 2. False 3. True 4. False 5. True

FILL IN THE BLANKS

1. $Ax + By = C$ 2. Infinite many 3. Unique

4. 8 5. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

ANALYTICAL PROBLEMS

1. $\left(0, \frac{1}{2}\right), (0,0), (2,0), (3,5)$ 2. (– 7, – 9) 3. Between 120° and 210° 4. 135° 5. 4

NUMERICAL PROBLEMS

1. 144 2. 7 3. 6 4. 2 5. 3

MATRIX OLYMPIAD

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : LINEAR EQUATIONS IN TWO VARIABLES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

MATRIX OLYMPIAD

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :

LINES AND ANGLES 5

Concepts

INTRODUCTION

In previous class we have studied that minimum two points are required to draw a line. A line having one end point is called a ray. Now if two rays originate from a point, an angle is formed. If two lines intersect each other, different angles are formed. Here we will study different properties related to lines and angles.

1. BASIC TERMS AND DEFINITIONS

1.1 LINE SEGMENT

A part of line with two end points is called line segment and is denoted as $\overline{AB}$.

1.2 RAY

A part of a line with one end point is called a ray and is denoted as $\overrightarrow{AB}$.
It can be extended further from point B.

1.3 LINE

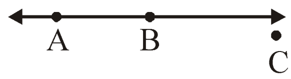
It can be extended from both sides (left and right) and is denoted as $\overleftrightarrow{AB}$.

1.4 COLLINEAR POINTS

Three or more points are said to be collinear if a single straight line passes through them. Here A, B, C are collinear.

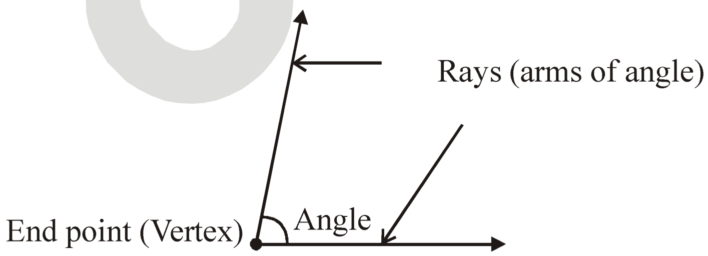
1.5 NON-COLLINEAR POINTS

Three or more points not lying on a single straight line are called non-collinear points. A , B, C are not collinear.



1.6 ANGLE

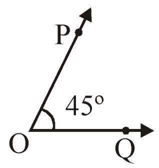
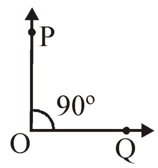
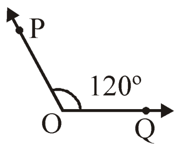
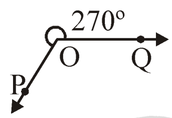
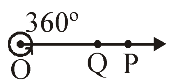
When two rays originates from the same end point they form an angle.



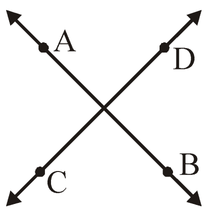
The ray are called arms of an angle and end point is called vertex.

MATRIX OLYMPIAD

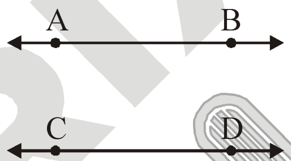
2. TYPES OF ANGLES

Measure	0° < x < 90°	x = 90°	90° < x < 180°	180°	180° < x < 360°	360°
Name	Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle	Complete angle
Illustration						

3. INTERSECTING LINES AND NON-INTERSECTING LINES



(i) Intersecting lines



(ii) Non-intersecting lines (Parallel Lines)

In figure (i) AB and CD are intersecting lines.

In figure (ii) AB and CD are non-intersecting lines (parallel lines).

In parallel lines the lengths of the common perpendiculars at different points are equal. This equal length is called the distance between two parallel lines.

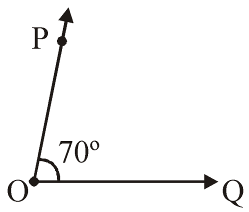
4. PAIRS OF ANGLES

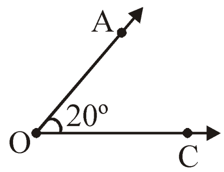
4.1 COMPLEMENTARY ANGLES

If the sum of measure of two angles is 90°, they are known as complementary angles.

For example : $\angle POQ + \angle ABC = 70^\circ + 20^\circ = 90^\circ$

$\therefore$ $\angle POQ$ and $\angle ABC$ are called complementary angles and $\angle POQ$ is called complement of $\angle ABC$ and vice-versa

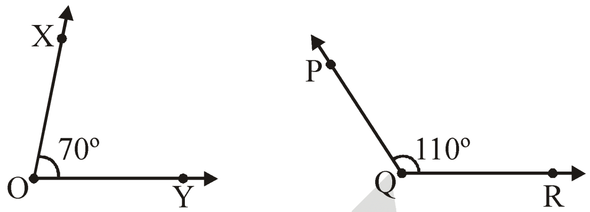




4.2 SUPPLEMENTARY ANGLES

If the sum of measure of two angles is 180º, they are called supplementary angles.

For example : ∠XOY and ∠PQR are supplementary as ∠XOY + ∠PQR = 70º + 110º = 180º and ∠XOY is called supplement of ∠PQR and vice-versa.



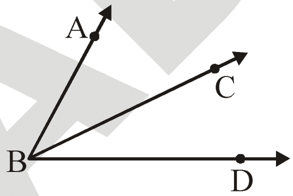
4.3 ADJACENT ANGLES

Two angles are said to be adjacent angles or adjacent to each other if

(i) They have common arm.

(ii) They have common vertex.

(iii) Non-common arms lying on the different sides of the common arm.



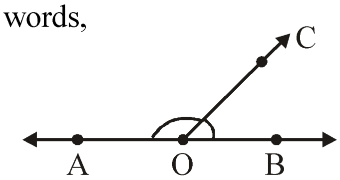
In the figure, ∠ABC and ∠CBD are adjacent angles.

They have common vertex B, common arm BC and non-common arms AB and BD lying on the different sides of BC.

4.4 LINEAR PAIR

Two adjacent angles whose sum is 180º are said to form linear pair or in other words, supplementary adjacent angles are called linear pair.

Here, ∠BOC + ∠COA = 180º, so they form linear pair.

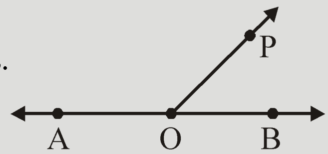


BUILD THE CONCEPT

• **Axiom-1 :-** If a ray stands on a line, then the sum of two adjacent angles so formed is 180º.

Ray OP stands on line AB, then ∠AOP and ∠POB are adjacent angles.

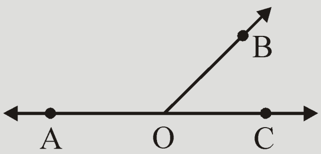
⇒ ∠AOP + ∠POB = 180º



• **Axiom-2 :-**

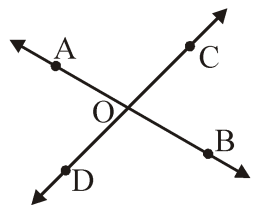
If the sum of two adjacent angles is 180º, then the non-common arms of the angles form a straight line. Suppose ∠AOB and ∠BOC are two adjacent angles, with common arm OB, non common arms OA and OC.

If ∠AOB + ∠BOC = 180º, then AOC would be a straight line.



4.5 VERTICALLY OPPOSITE ANGLES

If two lines AB and CD intersects each other at point O, then four angles are formed.



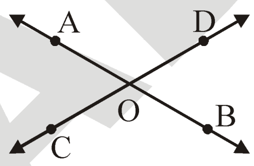
$\angle AOD$ is vertically opposite to $\angle BOC$. Similarly $\angle AOC$ is vertically opposite to $\angle DOB$.

These are called pairs of vertically opposite angles.

Theorem - 1

Statement : If two lines intersect each other, then the vertically opposite angles are equal.

Given : AB and CD are two lines intersecting each other at point O. $\angle AOC$ is vertically opposite to $\angle DOB$ and $\angle AOD$ is vertically opposite to $\angle COB$.



To Prove : $\angle AOC = \angle DOB$

$\angle AOD = \angle COB$

Proof : $\angle AOC + \angle AOD = 180^\circ$ ......(1) [Linear pair]

$\angle AOD + \angle DOB = 180^\circ$ .....(2) [Linear pair]

From (1) and (2), we get $\angle AOC + \angle AOD = \angle AOD + \angle DOB$

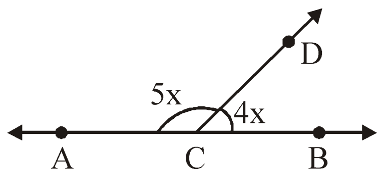
$\Rightarrow$ $\angle AOC = \angle DOB$

Similarly, $\angle AOD = \angle COB$

Example 1

ACB is a line such that $\angle DCA = 5x$ and $\angle DCB = 4x$.

Find the value of x and hence find $\angle DCA$ and $\angle DCB$.



Solution :

Since ACB is a line,

$\therefore$ $\angle ACD$ and $\angle DCB$ form a linear pair

$\Rightarrow$ $\angle ACD + \angle DCB = 180^\circ$

$\Rightarrow$ $5x + 4x = 180^\circ$

$\Rightarrow$ $9x = 180^\circ$

$\Rightarrow$ $x = 20^\circ$

$\therefore$ $\angle ACD = 5x = 5 \times 20^\circ = 100^\circ$

$\angle DCB = 4x = 4 \times 20^\circ = 80^\circ$

Example 2

If the supplement of an angle is two-third of itself. Determine the angle and its supplement.

Solution :

Let the angle be x

$\therefore$ Its supplement $= \frac{2}{3}$ of $x = \frac{2}{3}x$

$\Rightarrow \quad x + \frac{2}{3}x = 180^\circ$

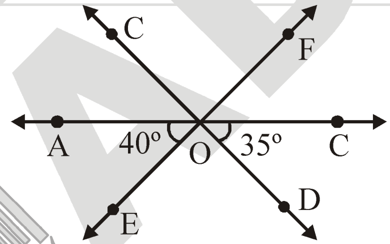
$\Rightarrow \quad x = \frac{180^\circ \times 3}{5} = 108^\circ$

$\therefore$ The angle is 108° and its supplement

$\Rightarrow \quad \frac{2}{3} \times 108^\circ = 2 \times 36^\circ = 72^\circ$

Example 3

Lines AB, CD and EF intersect at O. Find the measures of $\angle AOC$, $\angle COF$ and $\angle BOF$



Solution :

AB and EF intersect at point O.

$\therefore \quad \angle AOE = \angle FOB$ [Vertically opposite angles]

$\therefore \quad \angle FOB = 40^\circ$

Similarly, $\angle AOC = \angle DOB$ [Vertically opposite angles]

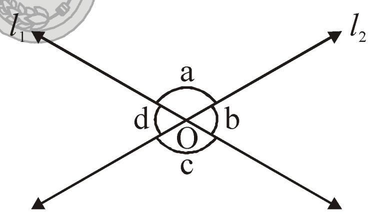
$\Rightarrow \quad \angle AOC = 35^\circ$

Also, $\angle AOE + \angle AOC + \angle COF = 180^\circ$ [Straight angle]

$\therefore \quad \angle COF = 180^\circ - 75^\circ = 105^\circ$

Example 4

Lines l_1 and l_2 intersects at point O forming angles a, b, c, d. If a = 45°, find b, c, d, a + b, b + c and verify that a + d = b + c.



Solution :

$a = 45^\circ \Rightarrow c = 45^\circ$ [Vertically opposite angles]

$a + d = 180^\circ$ [Linear pair]

$\Rightarrow d = 180^\circ - a = 180^\circ - 45^\circ = 135^\circ$

$d = b$ [Vertically opposite angles]

$\Rightarrow b = 135^\circ$

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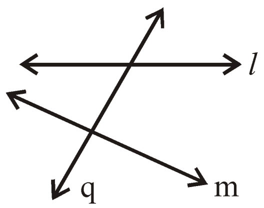
Now, a + d = 45° + 135° = 180°

b + c = 135° + 45° = 180°

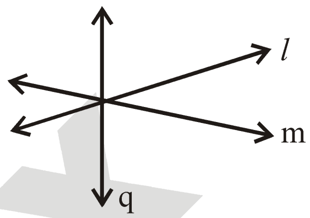
⇒ a + d = b + c

5. TRANSVERSAL

A line which intersects two or more lines at distinct points is called a transversal.



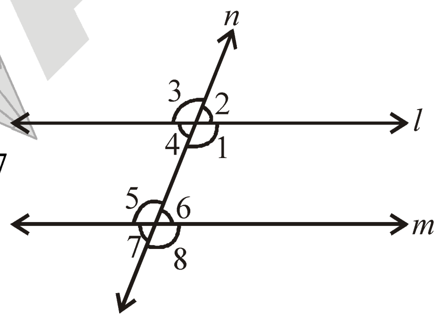
(q is a transversal of *l* and m)



(q is a transversal of *l* and m)

5.1 ANGLES FORMED BY A TRANSVERSAL

Suppose *l* and m are two parallel lines intersected by a transversal n.



- **Interior Angles :-** ∠1, ∠4, ∠5 and ∠6
- **Exterior Angles :-** ∠3, ∠2, ∠7 and ∠8
- **Corresponding Angles :-** ∠1 and ∠8, ∠2 and ∠6, ∠3 and ∠5, ∠4 and ∠7

[Note that pairs of angles lie either above or below the lines.]

- **Alternate Interior Angles :-** ∠4 and ∠6, ∠1 and ∠5
- **Alternate Exterior Angles :-** ∠3 and ∠8, ∠2 and ∠7
- **Co-Interior Angles :-** (Also called consecutive interior angles or allied angles) ∠4 and ∠5, ∠1 and ∠6.
- **Co-Exterior Angles :-** ∠2 and ∠8, ∠3 and ∠7.

6. RESULTS WHEN A TRANSVERSAL INTERSECTS TWO PARALLEL LINES

6.1 CORRESPONDING ANGLES AXIOM AND ITS CONVERSE

(i) If a transversal intersects two parallel lines, then each pair of corresponding angles are equal.

(ii) If a transversal intersects two lines such that a pair of corresponding angles are equal, then the two lines are parallel to each other.

6.2 ALTERNATE INTERIOR ANGLES THEOREM AND ITS CONVERSE

(i) If a transversal intersects two parallel lines, then each pair of alternate interior angles are equal.

(ii) If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.

6.3 CO–INTERIOR ANGLES THEOREM AND ITS CONVERSE

(i) If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary. In other words, co–interior angles are supplementary.

(ii) If a transversal intersects two lines such that a pair of co–interior angles are supplementary, then the two lines are parallel.

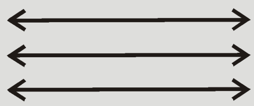
6.4 CO–EXTERIOR ANGLES THEOREM AND ITS CONVERSE

(i) If a transversal intersects two parallel lines then each pair of co–exterior angles are supplementary.

(ii) If a transversal intersects two lines such that a pair of co–exterior angles are supplementary, then two lines are parallel.

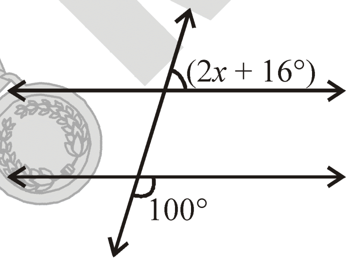
Focus Point

- Lines which are parallel to the same line are parallel to each other.
  This means if $l \,||\, n$ and $m \,||\, n \Rightarrow l \,||\, m$.



Example 5

In the given figure if $l \,||\, m$ and t is a transversal, determine x.



Solution :

$l \,||\, m$

$\Rightarrow 2x + 16° + 100° = 180°$ [Co–exterior angles]

$\Rightarrow 2x = 180° - 116° \Rightarrow 2x = 64° \Rightarrow x = 32°$

Example 6

AB || CD and EF || DQ.

Determine ∠PDQ, ∠AED and ∠DEF.

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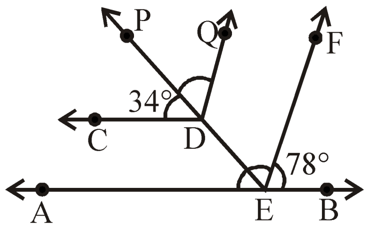


Solution :

AB || CD and transversal DE intersects them at E and D respectively.

$\therefore$ ∠AED = ∠CDP [Corresponding angles]

$\Rightarrow$ ∠AED = 34° ....(i)

Now ray EF stands on AB at E

$\therefore$ ∠AEF + ∠BEF = 180° [Linear pair]

$\Rightarrow$ ∠AEP + ∠PEF + ∠BEF = 180°

[∠AEF = ∠AEP + ∠PEF]

$\Rightarrow$ 34° + PEF + 78° = 180° [From (i)]

$\Rightarrow$ ∠PEF = 180° – 112°

$\Rightarrow$ ∠PEF = 68° ...(ii)

Now EF || DQ and transversal DE intersects them at E and D respectively .

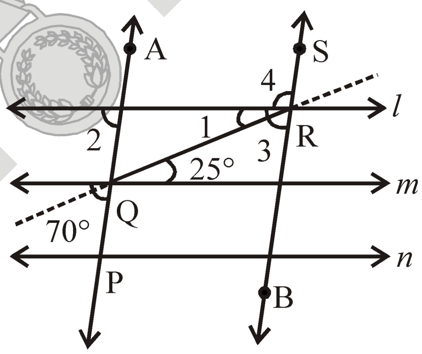
$\therefore$ ∠FED = ∠PDQ [Corresponding angles]

$\Rightarrow$ ∠PDQ = 68° [From (ii)]

$\Rightarrow$ ∠PDQ = ∠DEF = 68° and ∠AED = 34°

Example 7

If *l* || m || n and ∠PQ || ∠RS, find ∠QRS.



Solution :

Extend QP to point A and SR to point B.

l || m $\Rightarrow$ ∠1 = 25° [Alternate interior angles]

and ∠2 = 70° [Corresponding angles]

Also, $\angle 3 = \angle 2$ [AP || SB, corresponding angles]

$\Rightarrow \angle 3 = 70^\circ$

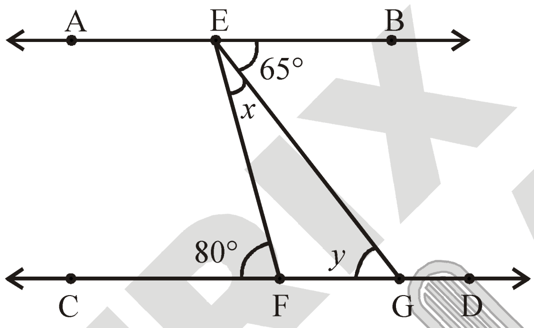
Now, $\angle 3 + \angle 4 = 180^\circ$ [Linear pair]

$\Rightarrow 70^\circ + \angle 4 = 180^\circ \Rightarrow \angle 4 = 110^\circ$

$\therefore \angle QRS = \angle 1 + \angle 4 = 25^\circ + 110^\circ = 135^\circ$

Example 8

In figure, if AB || CD, $\angle BEG = 65^\circ$ and $\angle EFC = 80^\circ$, then find x and y.



Solution :

$\angle BEF = \angle EFC$ [Alternate interior angles]

$\Rightarrow 65^\circ + x = 80^\circ \Rightarrow x = 80^\circ - 65^\circ \Rightarrow x = 15^\circ$

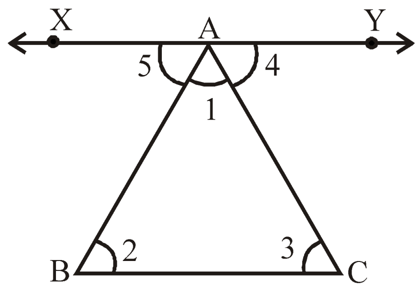
Now, $\angle FGE = \angle BEG$ [Alternate interior angles]

$\Rightarrow y = 65^\circ$.

7. PROPERTIES OF TRIANGLES

7.1 ANGLE SUM PROPERTY OF A TRIANGLE

Theorem - 2



Statement : The sum of the three angles of a triangle is 180°.

i.e., $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

Proof : Draw a line XY through point A parallel to BC.

$\angle 3 = \angle 4$ [Alternate interior angles]

$\angle 5 = \angle 2$ [Alternate interior angles]

Also $\angle 5 + \angle 1 + \angle 4 = 180^\circ$ [Straight angle]

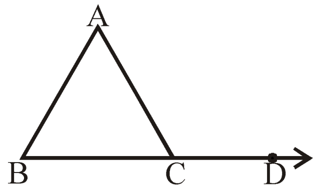
Replacing $\angle 5$ and $\angle 4$ by $\angle 2$ and $\angle 3$ respectively, we get

$\angle 2 + \angle 1 + \angle 3 = 180^\circ \Rightarrow \angle 1 + \angle 2 + \angle 3 = 180^\circ$

$\therefore$ Sum of all angles of a triangle is 180°

7.2 EXTERIOR ANGLE PROPERTY OF A TRIANGLE

Theorem - 3



Statement : If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior oposite angles.

Given : In ΔABC, side BC is extended to point D. ∠ACD is an exterior angle.

To Prove : ∠ABC + ∠CAB + ∠BCA = 180°

Proof : ∠ABC + ∠CAB + ∠BCA = 180° ...(1) [Angle sum property of triangle]

Also, ∠BCA + ∠ACD = 180° ...(2) [Linear pair]

From (1) and (2), we get ∠ABC + ∠CAB + ∠BCA = ∠BCA+ ∠ACD ⇒ ∠ACD = ∠ABC + ∠CAB

Example 9

If one angle of a triangle is 72° and the difference of the other two angles is 12°, find the other two angles.

Solution :

One angle of the triangle = 72°

Let other two angles be x and 12° + x

[∵ The difference between the two angles is 12°]

So, x + 12° + x + 72° = 180°

[Angle sum property of triangle]

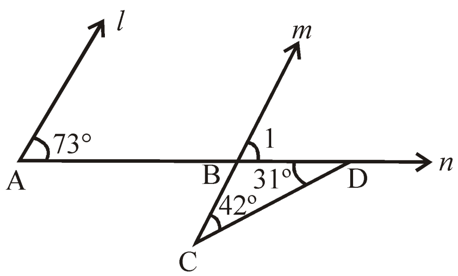
⇒ 2x + 84° = 180° ⇒ 2x = 180° – 84° = 96°

⇒ x = 96°/2 = 48°

∴ The other angles is 48° + 12° = 60°

Example 10

In the given figure show that *l* || m.



Solution :

$\angle CBD + \angle BDC + \angle BCD = 180^\circ$

[Angle sum property of triangle]

$\Rightarrow \angle CBD + 31^\circ + 42^\circ = 180^\circ$

$\Rightarrow \angle CBD = 180^\circ - 73^\circ$

$\Rightarrow \angle CBD = 107^\circ$

$\angle 1 + \angle CBD = 180^\circ$ [Linear pair]

$\angle 1 = 180^\circ - 107^\circ = 73^\circ$

Since, $\angle 1 = 73^\circ$ and $\angle A = 73^\circ$. So, $\angle 1 = \angle A$

$\Rightarrow l \parallel m$ [As corresponding angles are equal]

Example 11

The sides BC, CA and AB of ΔABC, are produced in order, forming exterior angles $\angle ACD$, $\angle BAE$ and CBF. Prove that $\angle ACD + \angle BAE + \angle CBF = 360^\circ$.

Solution :

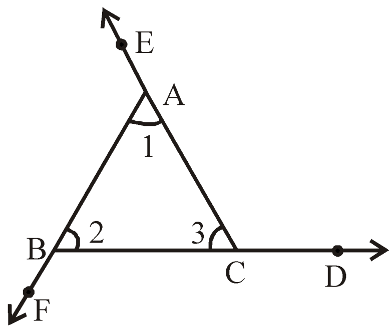
From exterior angle property of triangle,

$\angle FBC = \angle 1 + \angle 3$ ...(i)

$\angle BAE = \angle 2 + \angle 3$ ...(ii)

$\angle ACD = \angle 1 + \angle 2$ ...(iii)

Adding (i), (ii) and (iii), we get



$\angle FBC + \angle BAE + \angle ACD$

$= \angle 1 + \angle 3 + \angle 2 + \angle 3 + \angle 1 + \angle 2$

$= 2(\angle 1 + \angle 2 + \angle 3) = 2 \times 180^\circ$

[By angle sum property of triangle]

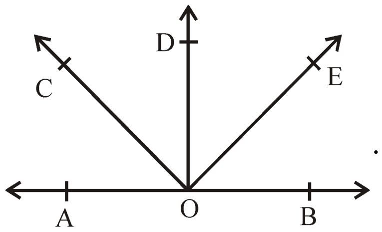
$\angle FBC + \angle BAE + \angle CD = 360^\circ$

SOLVED EXAMPLES

SE. 1

Prove that the sum of all angles formed on the same side of a line at a given point on the line is 180°.

Ans. Given : AOB is a straight line and rays OC, OD and OE stands on it, forming $\angle AOC$, $\angle COD$, $\angle DOE$ and $\angle EOB$



To Prove :

$\angle AOC + \angle COD + \angle DOE + \angle EOB = 180°$.

Proof : Ray OC stands on line AB.

$\therefore \angle AOC + \angle COB = 180°$ [Linear pair]

$\Rightarrow \angle AOC + (\angle COD + \angle DOE + \angle EOB) = 180°$

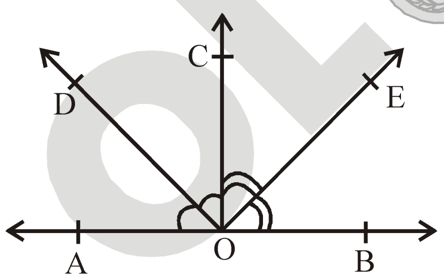
$\Rightarrow \angle AOC + \angle COD + \angle DOE + \angle EOB = 180°$

Hence, the sum of all the angles formed on the same side of line AB at a point O on it is 180°

SE. 2

Prove that the bisectors of the angles of a linear pair are at right angle.

Ans. **Given :** $\angle AOC$ and $\angle BOC$ form a linear pair of angles. OD and OE are the bisectors of $\angle AOC$ and $\angle BOC$ respectively.



To prove : $\angle DOE = 90°$

Proof : $\angle AOC + \angle BOC = 180°$ [Linear pair]

$\Rightarrow \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = 90°$

$\Rightarrow \angle DOC + \angle COE = 90°$

[$\because$ OD and OE are the bisectors of $\angle AOC$ and $\angle BOC$]

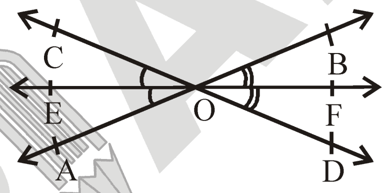
$\Rightarrow \angle DOE = 90°$

Hence, the bisectors of the angles of a linear pair are at right angle.

SE. 3

Prove that the bisectors of a pair of vertically opposite angles are in the same straight line.

Ans. Given : Two lines AB and CD intersecting each other at a point O. Also, OE and OF are the bisectors of $\angle AOC$ and $\angle BOD$ respectively.



To Prove : EOF is a straight line.

Proof : Since, the sum of all angles around a point is 360°,

$\therefore \angle AOC + \angle BOC + \angle BOD + \angle AOD = 360°$

$\Rightarrow 2\angle EOC + 2\angle BOC + 2\angle BOF = 360°$

[$\because \angle BOC = \angle AOD$ (Vertically opposite angles).

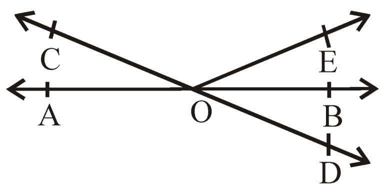
OE is bisector of $\angle AOC$, OF is bisector of $\angle BOD$]

$\Rightarrow \angle EOC + \angle BOC + \angle BOF = 180° \Rightarrow \angle EOF = 180°$

Hence, EOF is a straight line.

SE. 4

In the figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 80°$ and $\angle BOD = 30°$, find $\angle BOE$ and reflex $\angle AOD$.



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Ans. $\angle AOC + \angle BOE = 80°$ ...(i)[Given]

$\angle BOD = 30°$ ...(ii)[Given]

Lines AB and CD intersect at O.

$\therefore \angle AOC = \angle BOD$ [Vertically opposite angles]

$\Rightarrow \angle AOC = 30°$ [From (ii)]

Now, putting the value of $\angle AOC$ in (i), we have

$30° + \angle BOE = 80°$

$\Rightarrow \angle BOE = 80° - 30° = 50°$.

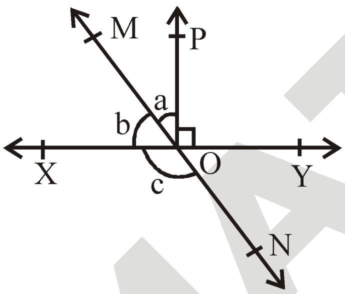
Also, $\angle BOD + \angle AOD = 180°$ [Linear pair]

$\angle AOD = 180° - 30° = 150°$

and reflex $\angle AOD = 360° - 150° = 210°$.

SE. 5

In the figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a = b$, find c.



Ans. Given a = b

Let a = x then b = x $\angle XOM + \angle POM + \angle POY = 180°$ [Angles on a straight line]

$\Rightarrow x + x + 90° = 180°$

$\Rightarrow 2x + 90° = 180° \Rightarrow 2x = 90° \Rightarrow x = 90°/2 = 45°$

$\therefore \angle XOM = b = 45°$ and $\angle POM = a = 45°$

Now, $\angle XON = c = \angle MOY$

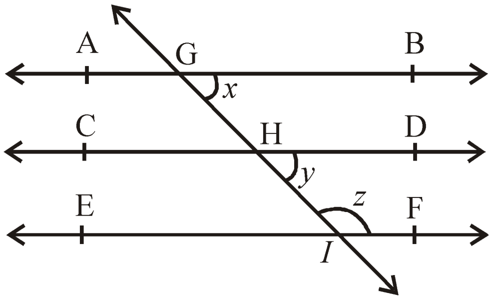
[Vertically opposite angles]

$= \angle POM + \angle POY = 45° + 90°$

Hence, c = 135°

SE. 6

In the figure, if AB || CD, CD || EF and y : z = 2 : 3, find x.



Ans. Let y = 2a and z = 3a

$\angle DHI + \angle FIH = 180°$ [CD || EF, Co-interior angles]

$\Rightarrow y + z = 180° \Rightarrow 2a + 3a = 180°$

$\Rightarrow 5a = 180° \Rightarrow a = 180°/5 = 36°$

$\therefore y = 2a = 2 \times 36° = 72°$ and $z = 3a = 3 \times 36° = 108°$

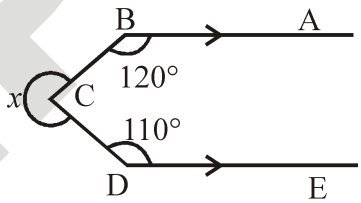
Also, AB || CD and GI is a transversal

$\therefore \angle BGI = \angle DHI$ [Corresponding angles]

$\Rightarrow x = y \Rightarrow x = 72°$

SE. 7

If AB || DE, then find the value of x.

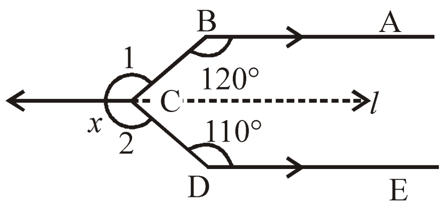


Ans. Construct a line l through C parallel to AB.

$\Rightarrow l$ || AB || DE and $x = \angle 1 + \angle 2$ Since, AB || l

$\Rightarrow \angle 1 = 120°$ [Alternate interior angles]

Also, l || DE



$\Rightarrow \angle 2 = 110°$ [Alternate interior angles]

Now, $x = \angle 1 + \angle 2 \Rightarrow x = 120° + 110° = 230°$

SE. 8

The side BC of a ΔABC is produced such that D is on ray BC. The bisector of $\angle A$ meets BC in L. Prove that $\angle ABC + \angle ACD = 2\angle ALC$.

Ans. In ΔABC, we have $\angle ACD = \angle B + \angle A$

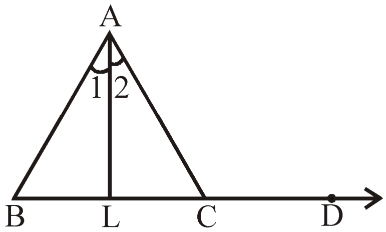
[Exterior angle property of a triangle]

$\Rightarrow \angle ACD = \angle B + 2\angle 1$ ...(i)

[$\because$ AL is the bisector of $\angle A$, $\therefore \angle A = 2\angle 1$]

In ΔABL, we have $\angle ALC = \angle B + \angle BAL$

[Exterior angle property of a triangle]



$\Rightarrow \angle ALC = \angle B + \angle 1$

$\Rightarrow 2\angle ALC = 2\angle B + 2\angle 1$

Subtracting (i) from (ii), we get

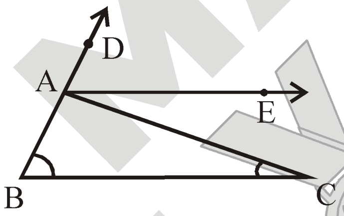
$2\angle ALC - \angle ACD = \angle B$

$\Rightarrow \angle ACD + \angle B = 2\angle ALC$

$\Rightarrow \angle ACD + \angle ABC = 2\angle ALC.$

SE. 9

In the figure, AE bisects $\angle CAD$ and $\angle B = \angle C$, prove that AE || BC.



Ans. In ΔABC, we have $\angle CAD = \angle B + \angle C$

[Exterior angle property of a triangle]

$\Rightarrow \angle CAD = 2\angle C$ [Given $\angle B = \angle C$]

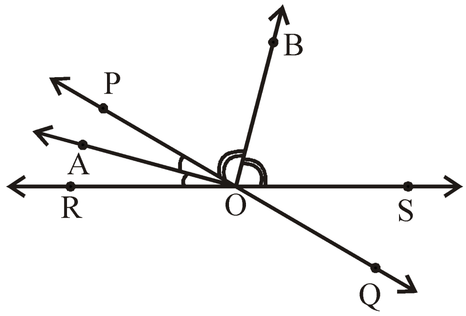
$\Rightarrow 2\angle CAE = 2\angle C$ [$\because \angle CAD$ is bisected by AE]

$\Rightarrow \angle CAE = \angle C \Rightarrow \angle CAE = \angle ACB$

$\Rightarrow$ AE || BC [As alternate interior angles are equal]

SE. 10

In figure, lines PQ and RS intersect each other at point O , ray OA and ray OB bisect $\angle POR$ and $\angle POS$ respectively. If $\angle POA : \angle POB = 2 : 7$, then find $\angle SOQ$ and $\angle BOQ$.



Ans. $\angle POR + \angle POS = 180°$ [Linear pair]

We are given that, ray OA and ray OB bisect $\angle POR$ and $\angle POS$ respectively.

Therefore,

$\angle POA = \frac{1}{2}\angle POR$ and $\angle POB = \frac{1}{2}\angle POS.$

$\Rightarrow \angle POA + \angle POB = \frac{1}{2}(\angle POR + \angle POS)$

$= \frac{1}{2} \times 180° = 90°$

Now, if $\angle POA : \angle POB = 2 : 7$, then, we have

$\angle POA = \frac{2}{9} \times 90° = 20°$ and

$\angle POB = \frac{7}{9} \times 90° = 70°.$

$\angle POR = 2 \times \angle POA = 2 \times 20° = 40°$

$\angle SOQ = \angle POR$ [Vertically opposite angles]

$\therefore \angle SOQ = 40°$

$\angle BOQ = \angle BOS + \angle SOQ = \angle POB + \angle SOQ$

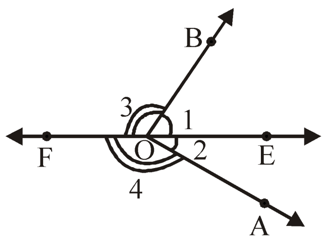
$\left[\angle BOS = \angle POB = \frac{1}{2}\angle POS\right]$

$= 70° + 40° = 110°$

$\therefore \angle BOQ = 110°$

SE. 11

In figure, ray OE bisects $\angle AOB$ and OF is the ray opposite OE. Show that $\angle 3 = \angle 4$.



Ans. Ray OE and OF are opposite. So, FOE is a straight line.

MATRIX OLYMPIAD

$\left\{\begin{array}{l}\therefore \angle 3 + \angle 1 = 180^\circ \text{ (Linear pair) .....(i)} \\ \text{and } \angle 4 + \angle 2 = 180^\circ \text{ (Linear pair) .....(ii)}\end{array}\right\}$

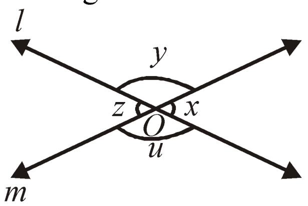
From (i) and (ii), we have $\angle 3 + \angle 1 = \angle 4 + \angle 2$

$\Rightarrow \angle 3 + \angle 1 = \angle 4 + \angle 1$

$(\because$ OE bisect $\angle AOB \Rightarrow \angle 1 = \angle 2) \Rightarrow \sqrt{3} = \sqrt{4}$

SE. 12

In the figure, line l and m intersect at O, forming angles as shown in the figure.



(i) If $x = 45^\circ$, what is z?

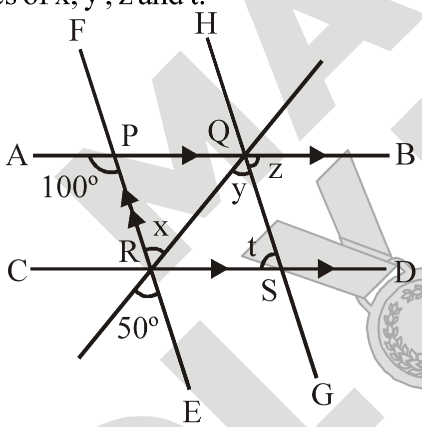
(ii) If $u = 125^\circ$, what is y?

Ans. (i) $z = x = 45^\circ$ (Vertically opposite angles)

(ii) $y = u = 125^\circ$ (Vertically opposite angles)

SE. 13

In the given figure, AB || CD and EF || GH. Find the values of x, y , z and t.



Ans. EF || GH and RQ is a transversal

$\Rightarrow y = 50^\circ$ (Corresponding angles)

$\because$ EF and RQ intersect at R.

$\Rightarrow x = 50^\circ$ (Vertically opposite angles)

Now, EF || GH and PQ is a transversal

$\Rightarrow 100^\circ + z = 180^\circ$ (Co-exterior angles)

$\Rightarrow z = 180^\circ - 100^\circ \Rightarrow z = 80^\circ$ ...(i)

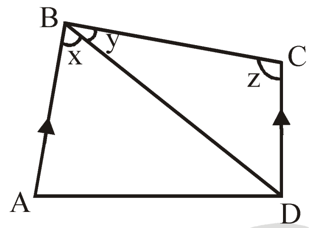
$\because$ AB || CD and QS is a transversal

$\Rightarrow t = z$ (Alternate interior angles)

$\Rightarrow t = 80^\circ$

SE. 14

In the figure, AB || DC. If $x = \frac{4}{3}y$ and $y = \frac{3}{8}z$ find the values of x, y and z.



Ans. AB || DC and BC is a transversal

$\Rightarrow (x + y) + z = 180^\circ$ (co-interior angles)

$\Rightarrow \frac{4}{3}y + y + \frac{8}{3}y = 180^\circ$

$\left(\because x = \frac{4}{3}y \text{ and } y = \frac{3}{8}z \Rightarrow z = \frac{8}{3}y\right)$

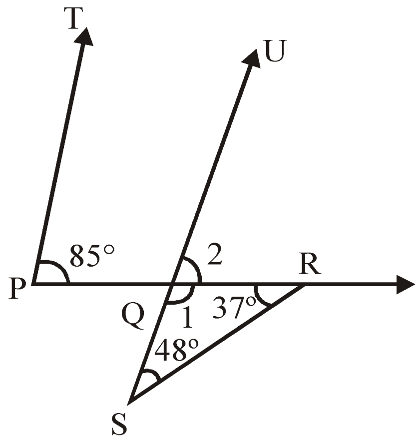
$\Rightarrow \frac{15y}{3} = 180^\circ \Rightarrow y = \frac{180^\circ \times 3}{15} \Rightarrow y = 36^\circ$

Now, $x = \frac{4}{3}y \Rightarrow x = \frac{4}{3} \times 36^\circ \Rightarrow x = 48^\circ$

and, $z = \frac{8}{3}y \Rightarrow z = \frac{8}{3} \times 36^\circ \Rightarrow z = 96^\circ$

SE. 15

In the figure, prove that TP || QU.



Ans. In ΔQRS

$\angle 2 = 48^\circ + 37^\circ$

(Exterior angle property of a traingle)

$\Rightarrow \angle 2 = 85^\circ \Rightarrow \angle UQR = 85^\circ$

So, $\angle TPR = \angle UQR$

TP || UQ (As corresponding angles are equal)

MATRIX OLYMPIAD

EXERCISE – I

ONLY ONE CORRECT TYPE

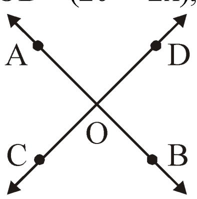
1. An angle is 18° less than its complementary angle. The measure of this angle is

(A) 36° (B) 48°
(C) 83° (D) 81°

2. Supplement of an angle is one fourth of itself. The measure of the angle is :

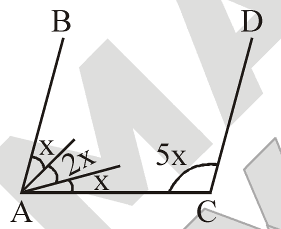
(A) 18° (B) 36°
(C) 144° (D) 72°

3. Line AB and CD intersect at O. If ∠AOC = (3x – 10°) and ∠BOD = (20° – 2x), then the value of x is



(A) 6° (B) 12°
(C) 36° (D) 30°

4. If AB || CD, what is the value of x ?



(A) 18° (B) 15°
(C) 20° (D) 25°

5. If two parallel lines are intersected by a transversal, then each pair of corresponding angles so formed is

(A) Equal (B) Complementary
(C) Supplementry (D) None of these

6. Which one of the following statements is not false?

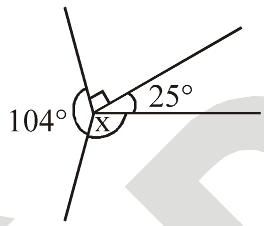
(A) If two angles form a linear pair, then each of these angles is of measure 90°.

(B) Angles forming a linear pair can both be acute angles.

(C) Both of the angles forming a linear pair can be obtuse angles.

(D) Bisectors of the adjacent angles forming a linear pair form a right angle.

7. Calculate the value of x.

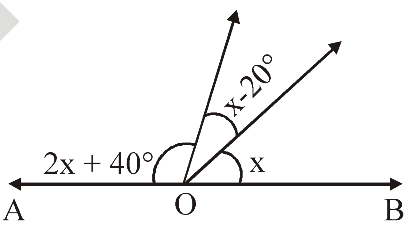


(A) 141° (B) 70°
(C) 105° (D) 45°

8. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 5 : 4, then the greater of the two angles is

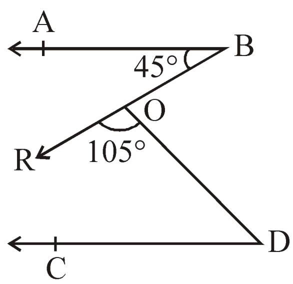
(A) 54° (B) 100°
(C) 120° (D) 136°

9. If AOB is a straight line, then x is



(A) 60° (B) 30°
(C) 90° (D) 40°

10. In figure, if AB || CD. If ∠ABR = 45° and ∠ROD = 105°, then find ∠ODC.

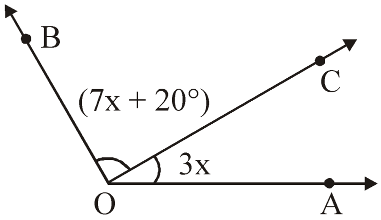


(A) 105° (B) 45°
(C) 30° (D) 65°

MATRIX OLYMPIAD

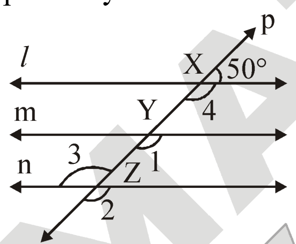
11. If two supplementary angles are in the ratio 4 : 5, then find the angles.
(A) 80°, 100° (B) 40°, 50°
(C) 36°, 108° (D) 108°, 36°

12. In figure, if $\angle BOC = 7x + 20°$ and $\angle COA = 3x$, the magnitude of x which makes AOB a straight line is



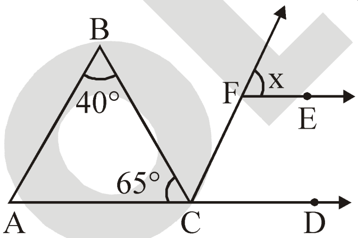
(A) 16° (B) 45°
(C) 32° (D) 49°

13. In given figure, *l*, m and n are parallel lines intersected by a transversal by a transversal p at X, Y and Z respectively. Values of $\angle 2$ and $\angle 3$ are



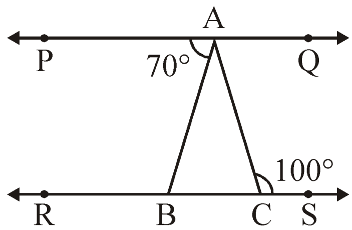
(A) 130°, 130° (B) 130°, 65°
(C) 65°, 130° (D) None of these

14. In the given figure, if AB || CF and CD || FE, then find the value of x.



(A) 105° (B) 60°
(C) 40° (D) 75°

15. In figure, PQ || RS, $\angle PAB = 70°$ and $\angle ACS = 100°$. Determine $\angle CAQ$.

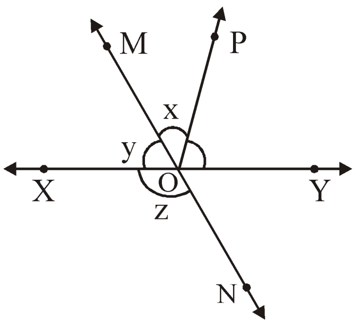


(A) 80° (B) 70°
(C) 30° (D) 100°

16. If angles with measure x and y form a complementary pair, then angles with which of the following measures will form a supplementary pair ?
(A) (x + 47°), (y + 43°)
(B) (x – 23°), (y + 23°)
(C) (x – 43°), (y – 47°)
(D) No such pair is possible

17. If one angle of a triangle is equal to the sum of the other two angles, then triangle is a/an
(A) Acute angled triangle
(B) Obtuse angled triangle
(C) Right angled triangle
(D) None of these

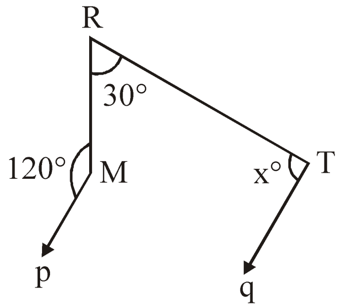
18. In figure, lines XY and MN intersect at O. If $\angle POY = 70°$ and x : y = 3 : 2, find z.



(A) 70° (B) 95°
(C) 136° (D) 120°

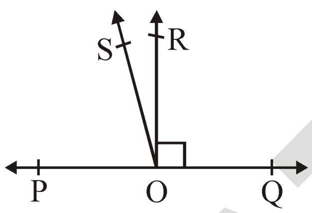
MATRIX OLYMPIAD

19. In the given figure, p || q, find the value of x.



(A) 120° (B) 90°
(C) 30° (D) 150°

20. In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Then ∠ROS =



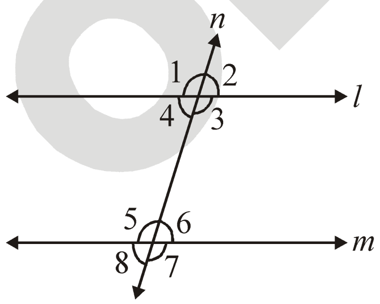
(A) $\frac{1}{2}$ (∠QOS + ∠POS)

(B) $\frac{1}{2}$ (∠QOS – ∠POS)

(C) ∠QOS – ∠POS

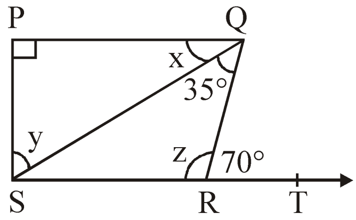
(D) ∠QOS + ∠POS

21. In figure, if l || m and ∠1 = (2x + y)°, ∠4 = (x + 2y)° and ∠6 = (3y + 20)°. The value of ∠7 and ∠4 respectively are



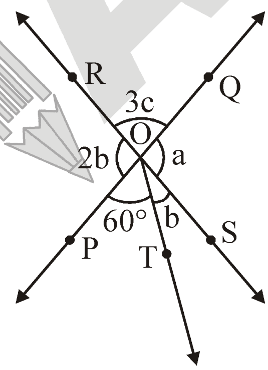
(A) 100°, 80° (B) 20°, 80°
(C) 60°, 40° (D) 80°, 100°

22. In the given figure, if PQ ⊥ PS, PQ || SR, ∠SQR = 35° and ∠QRT = 70°. Find the value of x + y + z.



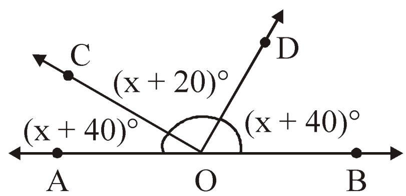
(A) 37° (B) 28°
(C) 65° (D) 200°

23. In the given figure, two straight lines PQ and RS intersect each other at O. If ∠POT = 60°, then find the values of a, b and c respectively.



(A) 80°, 40°, 33.3° (B) 84°, 48°, 21°
(C) 22°, 44°, 84° (D) 48°, 24°, 22°

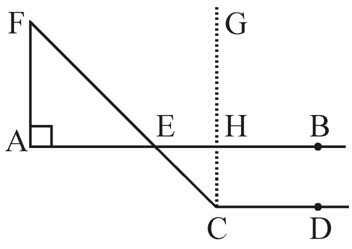
24. In figure, AOB is a straight line, if ∠COA = ∠DOB, then find the value of x.



(A) $\frac{80}{3}$ (B) $\frac{40}{3}$

(C) $\frac{20}{3}$ (D) $\frac{70}{3}$

25. In figure AB || CD and ∠F = 30°. Find ∠ECD.



(A) 150° (B) 60°

(C) 120° (D) 140°

PARAGRAPH TYPE

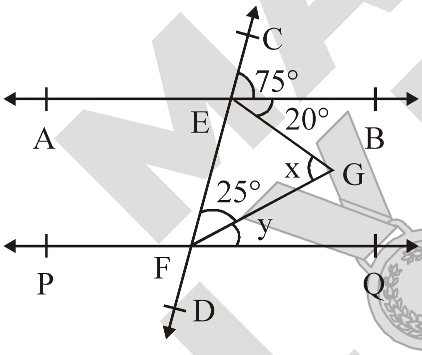
PASSAGE – I : If a transversal intersects any two parallel lines, then

(i) Each pair of corresponding angles are equal

(ii) Each pair of alternate interior angles are equal

(iii) Each pair of co-interior angles are supplementary.

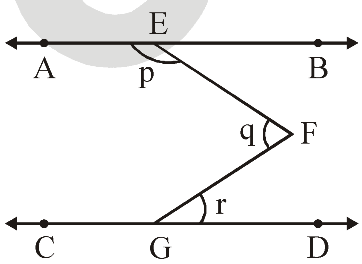
26. In the given figure, AB || PQ. The values of x and y respectively are



(A) 50°, 70° (B) 70°, 50°

(C) 75°, 45° (D) 20°, 75°

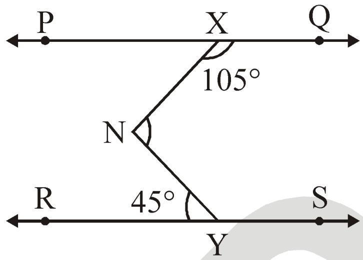
27. In the figure, AB || CD. Then the value of p + q – r is :



(A) 80° (B) 180°

(C) 100° (D) 360°

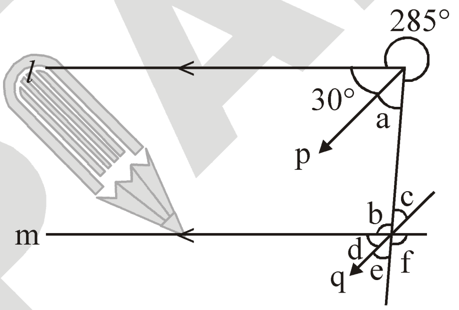
28. In the given figure, PQ || RS and ∠QXN = 105°, ∠RYN = 45°, find ∠XNY.



(A) 45° (B) 30°

(C) 60° (D) 120°

PASSAGE – II : In the given figure, l || m and p || q.



29. What is the product of digits of angle c ?

(A) 10 (B) 30

(C) 20 (D) 40

30. What is the sum of digits of angle d ?

(A) 1 (B) 2

(C) 3 (D) 4

31. Calculate $\frac{f-5^\circ}{25^\circ}$.

(A) 4 (B) 5

(C) 3 (D) 2

MATCH THE COLUMN TYPE

Space for Notes :

32. Match the following :

Column – I	Column – II
(P) When two lines intersect, then the pair of opposite angles so formed is called	(i) 180°
(Q) Complementary angles have sum equal to	(ii) An acute angle
(R) Angle that measures between 0° and 90° is called	(iii) 90°
(S) Supplementary angles have sum equal to	(iv) Vertically opposite angles

(A) P → (iv), (Q) → (ii), (R) → (i), (S) → (iii)
(B) P → (iv), (Q) → (iii), (R) → (ii), (S) → (i)
(C) P → (ii), (Q) → (i), (R) → (iii), (S) → (iv)
(D) P → (iv), (Q) → (iii), (R) → (i), (S) → (ii)

33. Match the following :

Column – I	Column – II
(P) Sum of all three interior angles of a triangle is equal to	(i) The opposite exterior angle
(Q) Sum of the two interior angles of a triangle is equal to	(ii) Equal
(R) Largest angle of a right triangle is	(iii) 180°
(S) When two parallel lines are cut by transversal, then co-interior angles formed are	(iv) 90°
(T) Alternate angles formed when two parallel lines are cut by a transversal are	(v) Supplementary

(A) P→(i), (Q)→(v), (R)→(iv), (S)→(iii), (T)→(ii)
(B) P→(v), (Q)→(iii), (R)→(ii), (S)→(i), (T)→(iv)
(C) P→(iii), (Q)→(i), (R)→(iv), (S)→(v), (T)→(ii)
(D) P→(iv), (Q)→(i), (R)→(v), (S)→(ii), (T)→(iii)

EXERCISE – II

VERY SHORT ANSWER TYPE

1. Convert each DMS form to decimal degree form.

   (i) 126° 12' 27" (ii) 118° 45' 20"

   (iii) 35° 12' 7" (iv) 34° 42' 54"

2. Convert each decimal degree form to DMS form

   (i) 57.24° (ii) 38.427°

   (iii) 24.263° (iv) 48.3625°

3. Solve :

   (i) $\begin{array}{r} 63°17'34" \\ +43°52'28" \\ \hline \\ \hline \end{array}$ (ii) $\begin{array}{r} 23°38'\ 55" \\ +174°42'\ \ 4" \\ \hline \\ \hline \end{array}$

   (iii) $\begin{array}{r} 24°\ 13'\ 41" \\ 6°\ \ 0'\ 58" \\ +32°\ 45'\ 46" \\ \hline \\ \hline \end{array}$ (iv) $\begin{array}{r} 73°\ 25'\ 28" \\ -\ 12°\ 12'\ 34" \\ \hline \\ \hline \end{array}$

   (v) $\begin{array}{r} 74°\ 16'\ \ 5" \\ -\ 26°\ 48'\ 25" \\ \hline \\ \hline \end{array}$ (vi) $\begin{array}{r} 54°\ \ \ \ \ \ 37" \\ -\ 29°\ 32'\ 48" \\ \hline \\ \hline \end{array}$

4. Find the complement of each of the following angles.

   (i) 58° (ii) 16°

   (iii) $\frac{2}{3}$ of right angle (iv) 46°30'

   (v) 52°43'20" (vi) 68°35'45"

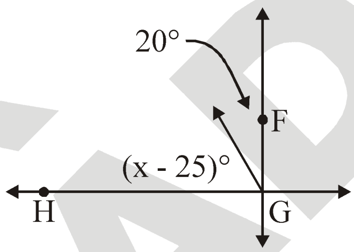
5. Find the supplement of each of the following angles.

   (i) 63° (ii) 138°

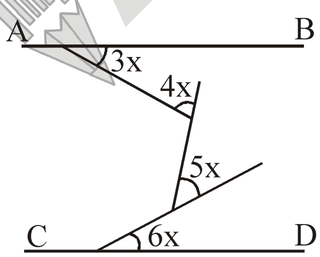
   (iii) $\frac{3}{5}$ of right angle (iv) 75°36'

   (v) 124°20'40" (vi) 108°48'32"

6. In the diagram, $\overrightarrow{FG} \perp \overrightarrow{GH}$. Find the value of x.



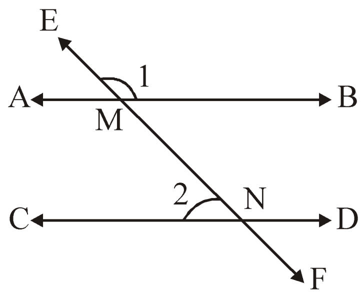
7. In the given figure AB || CD find the value of x ?



8. The ratio of two complementary angles is 2 : 3. Find the angles.

9. Find the angle whose complement is one-third of its supplement.

10. In the given figure, AB || CD and EF is a transversal. If $\angle 1 = 120°$, then find $\angle 2$.

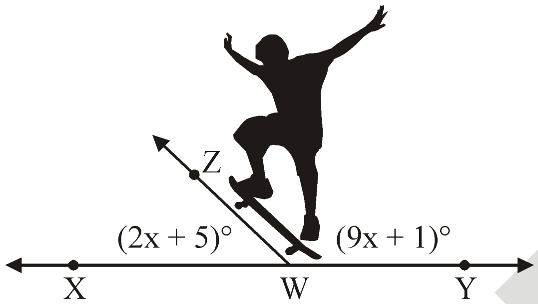


MATRIX OLYMPIAD

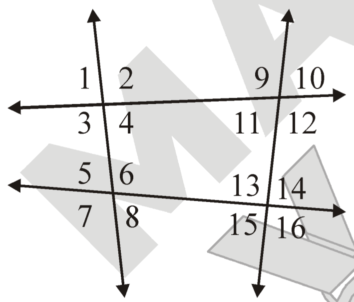
SHORT ANSWER TYPE

1. x and y are two supplementary angles. If supplementary angle of x is equal to half the complementary of y then find x : y ?

2. As shown in the diagram, a skateboarder tilts one end of a skateboard. Find m$\angle$ZWX in degrees.



3. Classify the angle pair as corresponding, alternate interior, alternate exterior, or consecutive interior angles.

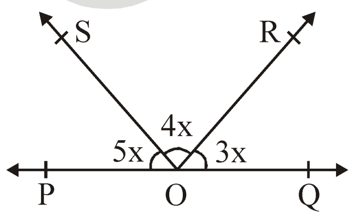


(i) $\angle 5$ and $\angle 1$ (ii) $\angle 11$ and $\angle 13$

(iii) $\angle 6$ and $\angle 13$ (iv) $\angle 10$ and $\angle 15$

(v) $\angle 2$ and $\angle 11$ (vi) $\angle 8$ and $\angle 4$

4. In the given figure, POQ is a straight line and the angles are as shown in the figure.

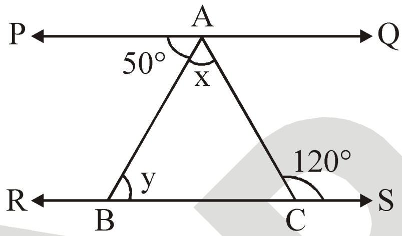


Find :

(i) Value of x (ii) $\angle$POS

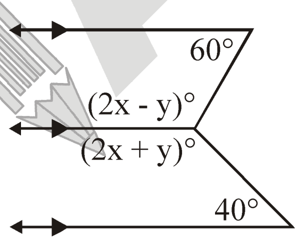
(iii) $\angle$QOS

5. In the given figure, PQ || RS, $\angle PAB = 50°$ and $\angle ACS = 120°$. Find x and y.

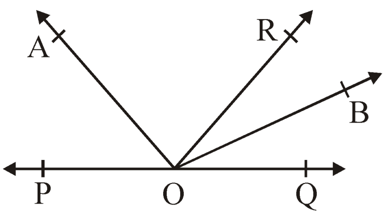


LONG ANSWER TYPE

1. Find the values of x and y.

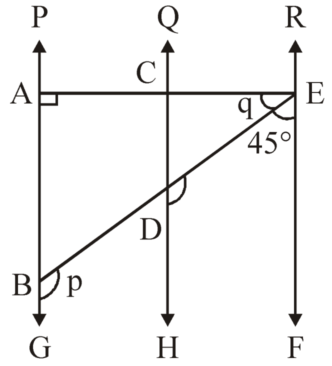


2. In the given figure, POQ is a straight line. $\overline{OA}$ and $\overline{OB}$ are the bisectors of $\angle$POR and $\angle$QOR respectively. Show that $\angle$AOB is a right angle.

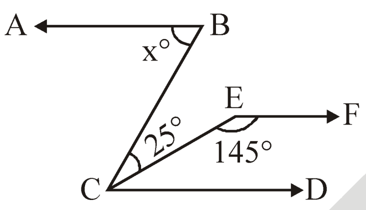


3. In the figure, AB || CD and CD || EF. Also, EA $\perp$ AB and $\angle BEF = 45°$. Find the values of p and q as shown in the figure.

MATRIX OLYMPIAD



4. In the given figure, AB || CD || EF. If ∠BCE = 25° and ∠CEF = 145°, then find the measure of ∠ABC.



5. If four times an angle is 12° more than twice the difference between its supplement and its complement, then find the angle, its complement and its supplement.

TRUE FALSE TYPE

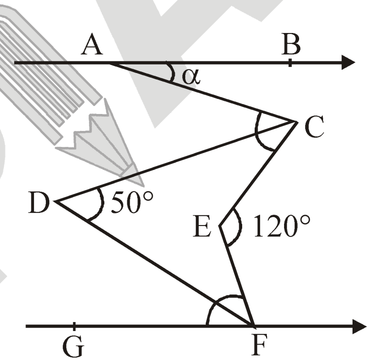
1. Angle is a figure formed by two rays having same end point.
2. Two adjacent angles whose sum is 180° are said to form of linear pair.
3. If two lines intersect each other, then the vertically opposite angles are not equal.
4. If the sum of measure of two angles is 180°, is called supplementary angles.
5. A line which intersects two or more lines at distinct points is called parallel line.

FILL IN THE BLANKS

1. Three or more points lying on the same line are ________ points.
2. Three or more lines passing through a common point are known as ________ lines.
3. When the sum of two angles is ________ then the two angles are called supplementary angles.
4. Angle measuring less than 90° and more than 0° ________.
5. Vertically opposite angles are ________ .

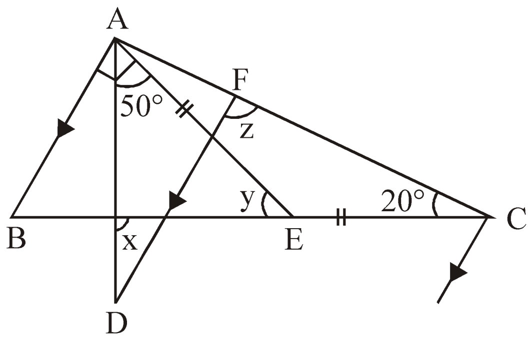
ANALYTICAL PROBLEMS & BRAIN TEASER

1. AB || FG, CD and FD are angle bisector as shown in figure. Find ∠BAC



(A) 20° (B) 25°
(C) 30° (D) 35°

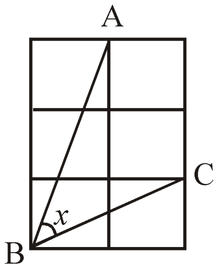
2. In the given figure (not drawn to scale), find the value of x, y and z respectively.



(A) 90°, 50°, 110° (B) 90°, 40°, 110°
(C) 40°, 90°, 110° (D) 110°, 40°, 90°

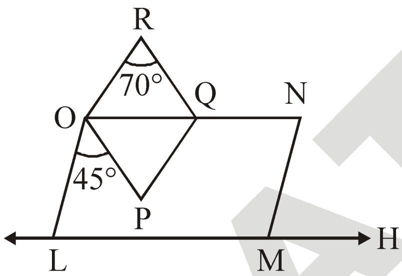
MATRIX OLYMPIAD

3. Six congruent squares are assembled as shown figure. find ∠ABC.



(A) 30º (B) 25º
(C) 45º (D) 60º

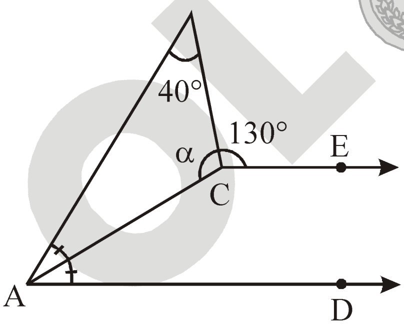
4. In the given figure (not drawn to scale), LMNO is a parallelogram and OPQR is a rhombus. Find ∠NMH given that LMH is a straight line.



(A) 80° (B) 60°

(C) 70° (D) 50°

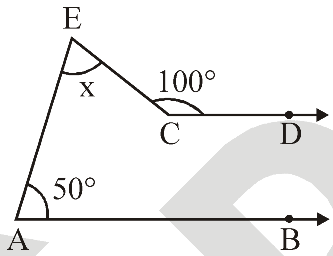
5. In given figure. AD || CE, ∠BAC = ∠CAD. Find ∠ACB



(A) 90º (B) 95º
(C) 100º (D) 105º

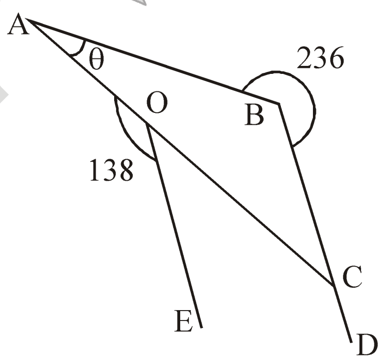
NUMERICAL PROBLEMS

1. In the given figure, AB || CD. Find the value of $\frac{x}{25°}$.



2. If an angle x is supplement of itself. Then value of $\frac{x - 60°}{6°}$ is.

3. In the shown figure OE || BD, find the value $\frac{\theta}{2°}$?



4. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio of 1 : 4. Then what will be the result if difference of the angles is divided by smaller angle ?

5. If ∠A is right angle and ∠B is straight angle, then what is the value of $\frac{2\angle B - 3\angle A}{15°}$?

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	A	C	A	D	A	B	D	C	A	A	A	D	A
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	C	C	B	B	A	D	A	A	C	B	B	D	C	C
31	32	33												
A	B	C												

EXERCISE II

VERY SHORT ANSWER TYPE

1. (i) 126.2075°, (ii) 118.7556°, (iii) 35.20194°, (iv) 34.715°

2. (i) 57° 14' 24", (ii) 38° 25' 37", (iii) 24° 15' 46", (iv) 48° 21' 45"

3. (i) 107° 10' 02", (ii) 198° 20' 59", (iii) 63° 25", (iv) 61° 12' 54", (v) 47° 27' 40", (vi) 24° 27' 49"

4. (i) 32°, (ii) 74°, (iii) 30°, (iv) 43° 30',(v) 37° 16' 40", (vi) 21° 24' 15"

5. (i) 117°, (ii) 42°, (iii) 126°, (iv) 104° 24', (v) 55° 39' 20", (vi) 71° 11' 28"

6. 95° **7.** 10° **8.** 36°, 54° **9.** 45° **10.** 60°

SHORT ANSWER TYPE

1. 5 : 1 **2.** 37°

3. (i) Corresponding angles, (ii) Consecutive interior angles, (iii) Consecutive interior angles (iv) Alternate exterior angles, (v) Alternate interior angles, (vi) Corresponding angles

4. (i) 15°, (ii) 75°, (iii) 105° **5.** 70°, 50°

LONG ANSWER TYPE

1. x = 65, y = 10 **3.** p = 135°, q = 45° **4.** 60° **5.** 48°, 42°, 132°

TRUE FALSE TYPE

1. T **2.** T **3.** F **4.** T **5.** F

FILL IN THE BLANKS

1. Collinear **2.** Concurrent **3.** 180° **4.** Acute **5.** Equal

ANALYTICAL PROBLEMS

1. A **2.** B **3.** C **4.** A **5.** B

NUMERICAL PROBLEMS

1. 2 **2.** 5 **3.** 7 **4.** 3 **5.** 6

MATRIX OLYMPIAD

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : LINES AND ANGLES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

MATRIX OLYMPIAD

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

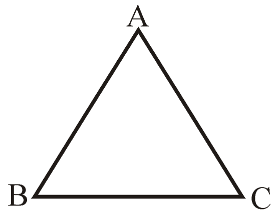
Space for Notes :

TRIANGLES 6

Concepts

INTRODUCTION

A plane figure bounded by three line segments is called a triangle. Consider ΔABC. It has



(i) three vertices i.e. A, B and C (ii) three sides i. e. AB, BC and CA

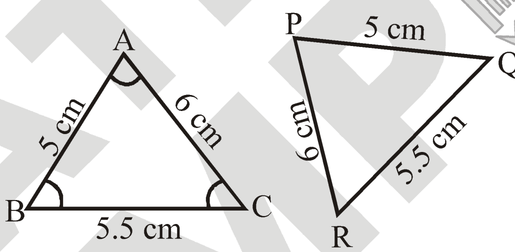
(iii) three angles i. e. ∠A, ∠B and ∠C.

1. CONGRUENCE OF TRIANGLES

Now let us consider two triangles ABC and PQR.

We observe that AB = PQ = 5 cm, AC = PR = 6 cm, BC = QR = 5.5 cm.

i.e., sides of ΔPQR fall on corresponding sides of ΔABC. PQ covers AB, QR covers BC and RP covers CA, ∠P covers ∠A, ∠Q covers ∠B and ∠R covers ∠C. Also there is a one-one correspondence between vertices P ↔ A, Q ↔ B, R ↔ C. i.e., ΔPQR is congruent to ΔABC i.e., ΔPQR ≅ ΔABC.

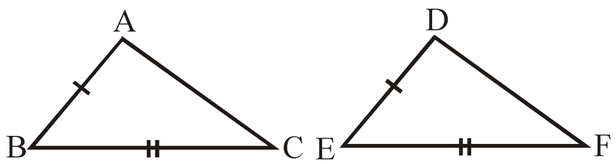


Note : In congruent triangles corresponding parts are equal and we write in short 'C.P.C.T'. for Corresponding Parts of congruent Triangles.

2. CRITERIA FOR CONGRUENCE OF TRIANGLES

2.1 SAS CONGRUENCE RULE

Statement : Two triangles are said to be congruent if two sides and the included angle of one triangle are equal to the corresponding sides and the included angle of the other triangle.



Given : ΔABC and ΔDEF in which AB = DE, BC = EF and ∠B = ∠E.

To Prove: ΔABC ≅ ΔDEF.

Proof: Place ΔABC over ΔDEF such that A falls on D and AB falls along DE.

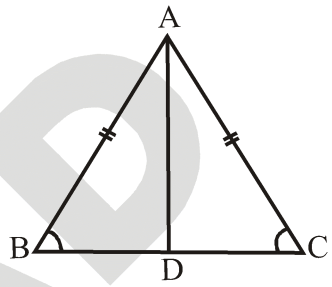
Since AB = DE, so B falls on E. Since $\angle B = \angle E$, so BC will fall along EF.

Since, BC = EF, so C will fall on F. Therefore, BC will coincide with EF. And, thus, AC will coincide with DF.

$\Rightarrow \Delta ABC$ coincides with ΔDEF. Hence, $\Delta ABC \cong \Delta DEF$.

Note : The medians of an equilateral triangle are equal.

Theorem - 1



Statement : The angles opposite to two equal sides of a triangle are equal.

Given : ΔABC in which AB = AC.

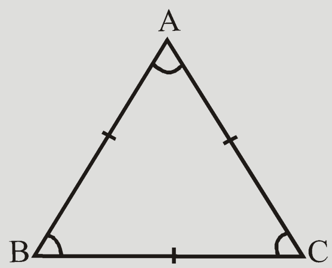
To Prove : $\angle B = \angle C$.

Construction : Draw AD, the bisector of $\angle A$, to meet BC at D.

Proof : In ΔABD and ΔACD, we have

AB = AC	[Given]
$\angle BAD = \angle CAD$	[By construction]
AD = AD	[Common]
$\therefore\ \Delta ABD \cong \Delta ACD$	[By SAS congruency]
Hence, $\angle B = \angle C$	[by C.P.C.T]

BUILD THE CONCEPT



- **Corollary :** Each angle of an equilateral triangle is of measure 60°.
- **Proof :** ABC is an equilateral triangle.

$\Rightarrow AB = AC = BC$.

Now, AB = AC gives that, $\angle B = \angle C$ ...(1)

[Angles opposite to equal sides of a triangle are equal]

Similarly, BA = BC gives that $\angle A = \angle C$ ...(2)

From (1) and (2), $\angle A = \angle B = \angle C$ ...(3)

Also, we have $\angle A + \angle B + \angle C = 180°$ ...(4) [Angle sum property of a triangle]

From (3) and (4), we have $\angle A = \angle B = \angle C = \frac{1}{3} \times 180° = 60°$.

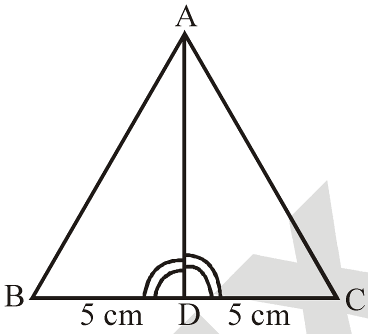
MATRIX OLYMPIAD

Example 1

Prove that ΔABD ≅ ΔACD given that BD = CD = 5 cm and ∠ADB = ∠ADC.

Solution :

Given : ΔABC in which BD = CD = 5 cm and ∠ADB = ∠ADC



To Prove : ΔABD ≅ ΔACD

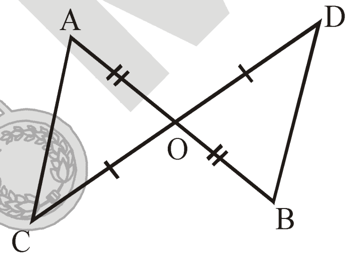
Proof : In ΔABD and ΔACD,

we have

BD = CD = 5 cm	[Given]
∠ADB = ∠ADC	[Given]
AD = AD	[Common side]
∴ ΔABD ≅ ΔACD	[By SAS congruency]

Example 2

In figure , O is the mid-point of AB and CD. Prove the AC = BD and AC || BD.



Solution :

In ΔAOC and ΔBOD, we have

AO = BO	[O is the mid-point of AB]
∠AOC = ∠BOD	[Vertically opposite angles]
CO = DO	[O is the mid-point of CD]
ΔAOC ≅ ΔBOD	[By SAS Congruency]
⇒ AC = BD	[By C.P.C.T]
and ∠CAO = ∠DBO	[By C.P.C.T]

Now, AC and BD are two lines intersected by a transversal AB such the ∠CAO = ∠DBO i.e. alternate angles are equal.

Therefore, AC || BD

2.2 ASA CONGRUENCE RULE

Statement : Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.

Given : Two triangles ABC and DEF such that ∠B = ∠E, ∠C = ∠F and BC = EF.

To Prove : ΔABC ≅ ΔDEF.

Proof : Let us consider, two possible situations as below:

Case 1 : Let AB = DE as shown in figure.

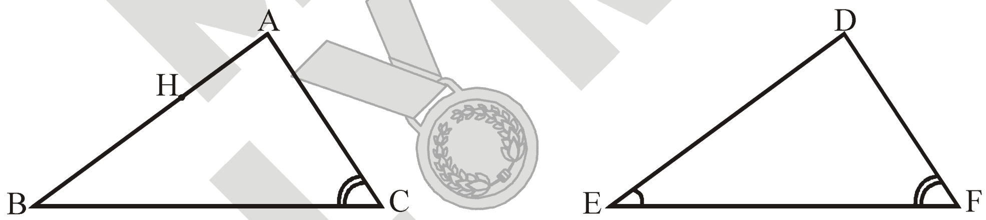
Now, we have AB = DE [Supposed]

∠B = ∠E [Given]

BC = EF [Given]

Thus, we have ΔABC ≅ ΔDEF [By SAS congruency]

Case 2 : Let if possible AB ≠ DE, in particular AB > DE.



In figure we take a point H on AB such that HB = DE.

In ΔHBC and ΔDEF, we have

HB = DE [By Construction]

∠B = ∠E [Given]

BC = EF [Given]

So, ΔHBC ≅ ΔDEF [By SAS Congruency]

⇒ ∠HCB = ∠DFE ...(1) [By C.P.C.T.]

Also, ∠ACB = ∠DFE ...(2) [Given]

From (1) and (2), we have

$\angle HCB = \angle ACB$

It can only be possible if H coincides with A. In other words, AB = DE.

So, $\Delta ABC \cong \Delta DEF$ [By SAS Congruency]

Case 3 : If AB < DE, we can choose a point M on DE such that ME = AB and repeating the arguments as given in case (2), we can conclude that AB = DE and So, $\Delta ABC \cong \Delta DEF$

2.3 AAS CONGRUENCE OF TRIANGLES

Statement : Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

Proof : The AAS congruence rule can be derived with the help of ASA theorem as below

In the above figure, two triangles, ABC and DEF have one pair of sides equal i.e., BC = EF and also, $\angle A = \angle D$ and $\angle B = \angle E$.

$\Rightarrow \angle A + \angle B = \angle D + \angle E$.

$\Rightarrow 180° - (\angle A + \angle B) = 180° - (\angle D + \angle E)$. [Angle sum property of triangle]

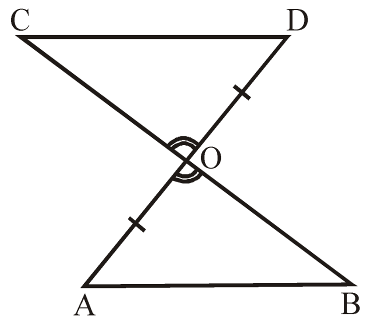
$\Rightarrow \angle C = \angle F$.

In case, we are given that , BC = EF, we write $\angle B = \angle E$, BC = EF and $\angle C = \angle F$.

Therefore, by ASA congruence rule, we have $\Delta ABC \cong \Delta DEF$.

Example 3

In the figure AB || CD, AD and BC intersect at O and O is mid-point of AD. Show that (i) $\Delta AOB \cong \Delta DOC$ and (ii) OB = OC.



Solution :

In ΔAOB and ΔDOC, we have

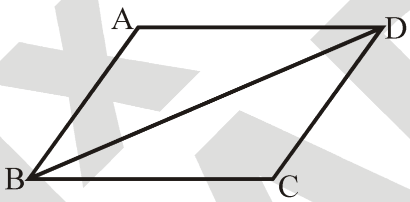
∠AOB = ∠DOC	[Vertically opposite angles]
OA = OD	[O is mid-Point of AD]
∠OAB = ∠ODC	[Alternate angles, as AB ‖ CD]
∴ ΔAOB ≅ ΔDOC	[By ASA congruency]
Also, OB = OC	[By C.P.C.T.]

Example 4

In quadrilateral ABCD, AB ‖ CD and BC ‖ AD.

Show that

(i) ΔABD ≅ ΔCDB and (ii) AB = CD; AD = CB.



Solution :

AB ‖ CD and AD ‖ BC [Given]

⇒ ∠ABD = ∠CDB and ∠ADB = ∠CBD [Alternate angles]

In ΔABD and ΔCDB, we have

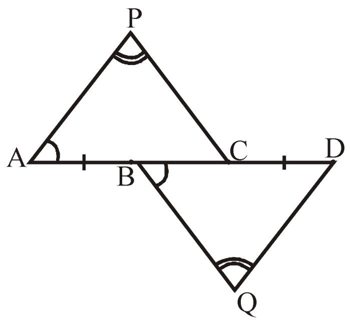
∠ABD = ∠CDB	[Proved]
BD = DB	[Common side]
∠ADB = ∠CBD	[Proved]
Therefore, ΔABD ≅ ΔCDB	[By ASA congruency]
and AB = CD; AD = CB	[By C.P.C.T.]

Example 5

In figure, AB = CD,

∠APC = ∠BQD and ∠PAC = ∠QBD,

Prove that ΔAPC ≅ ΔBQD.



Solution :

AB = CD

⇒ AB + BC = BC + CD

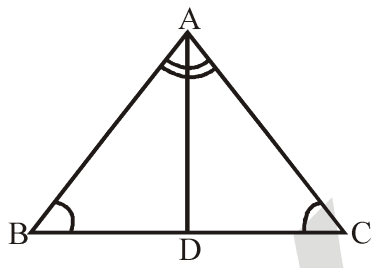
⇒ AC = BD

In ΔAPC and ΔBQD, we have

AC = BD	[Proved]
∠APC = ∠BQD	[Given]
and ∠PAC = ∠QBD	[Given]
∴ ΔAPC ≅ ΔBQD	[By AAS Congruency]

Example 6

In figure, ∠ABD = ∠ACD, AD is bisector of ∠BAC and AD meets BC at D. Prove that, D is mid-Point of BC.



Solution :

In ΔABD and ΔACD, we have

∠ABD = ∠ACD	[Given]
∠BAD = ∠CAD	[AD is bisector of ∠A]
AD = AD	[Common side]
Therefore, ΔABD ≅ ΔACD	[By AAS congruency]
⇒ BD = CD	[By C.P.C.T.]

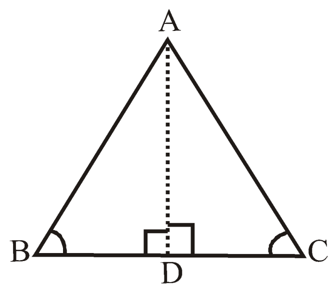
Therefore, D is mid-point of BC.

Theorem - 2

Statement : The sides opposite to equal angles of a triangle are equal.

Given : In ΔABC, ∠B = ∠C

To Prove : AB = AC



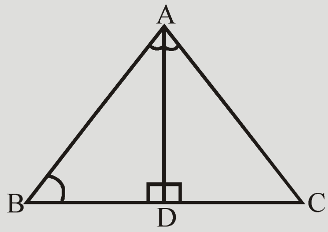
Construction : We construct AD ⊥ BC, AD meets BC at D.

Proof : In ΔABD and ΔACD, we have

∠ABD = ∠ACD	[Given]
∠ADB = ∠ADC	[Each 90°]
AD = AD	[Common side]
Therefore, ΔABD ≅ ΔACD	[By AAS congruency]
So, AB = AC.	[By C.P.C.T]

BUILD THE CONCEPT

- In ΔABC, the bisector AD of $\angle A$ is perpendicular to side BC (see figure) than ΔABC is isosceles.



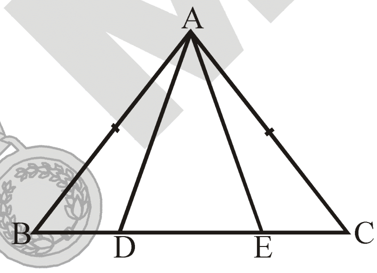
In ΔABD and ΔACD,

$\angle BAD = \angle CAD$	[Given]
$AD = AD$	[Common]
$\angle ADB = \angle ADC = 90°$	[Given]
$\therefore \Delta ADB \cong \Delta ADC$	[By ASA congruency]
So, $AB = AC$	[By C.P.C.T]

or, ΔABC is isosceles.

Example 7

In an Isosceles triangle ABC with AB = AC, D and E are points on BC such that BE = CD (see figure). Show that AD = AE.



Solution :

$BE = CD$ [Given]

So, $BE - DE = CD - DE$

$\Rightarrow BD = CE$

In ΔABD and ΔACE, we have

$AB = AC$	[Given]
$\angle B = \angle C$	[Angles opposite to equal sides]
$BD = CE$	[Proved above]
So, $\Delta ABD \cong \Delta ACE$	[By SAS congruency]
$\Rightarrow AD = AE$	[By C.P.C.T]

2.4 SSS CONGRUENCE RULE

Statement : If the three sides of one triangle are equal to the corresponding three sides of another triangle then the two triangles are congruent.

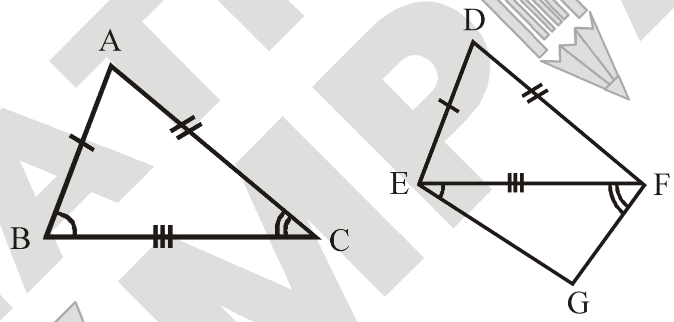
Given : ΔABC and ΔDEF in which AB = DE, AC = DF and BC = EF.

To Prove : ΔABC ≅ ΔDEF.

Construction : Suppose BC is the longest side of ΔABC. Draw EG and FG such that ∠FEG = ∠CBA and ∠EFG = ∠BCA. Join DG.

Proof : In ΔABC and ΔGEF, we have

BC = EF	[Given]
∠CBA = ∠FEG	[By construction]
∠BCA = ∠EFG	[By construction]
∴ ΔABC ≅ ΔGEF	[By ASA congruency]



So, ∠BAC = ∠EGF, AB = GE and AC = GF [By C.P.CT]

⇒ ∠BAC = ∠EGF, DE = GE and DF = GF [AB = DE and AC = DF]

Now, in ΔEGD, DE = GE ⇒ ∠EGD = ∠EDG ......(1)

And, in ΔFGD, DF = GF ⇒ ∠FGD = ∠FDG ......(2)

Adding eq. (1) and (2), we get ∠EGF = ∠EDF

⇒ ∠A = ∠EDF

[∵ ∠EGF = ∠A]

⇒ ∠A = ∠D

Now, in ΔABC and ΔDEF, we have

AB = DE, AC = DF and ∠A = ∠D.

∴ ∠ABC ≅ ∠DEF [By SAS congruency].

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2.5 RHS CONGRUENCE RULE

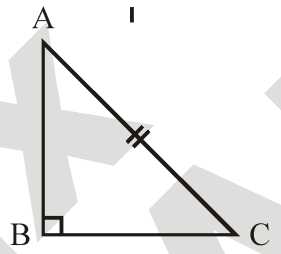
Statement : Two right – angled triangles are congruent if one side and the hypotenuse of the one is respectively equal to the corresponding side and the hypotenuse of the other.

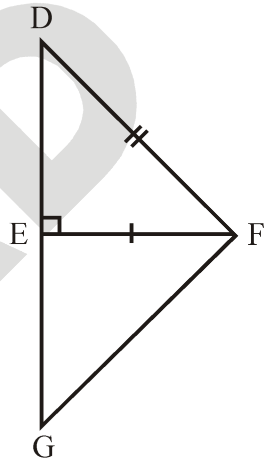
Given : Two right – angled triangles ΔABC and ΔDEF in which ∠B = ∠E = 90º, BC = EF and AC = DF.

To Prove : ΔABC ≅ ΔDEF

Construction : Produce DE to G such that GE = AB. Join GF.

Proof : in ΔABC and ΔGEF, we have





AB = GE [Construction]

BC = EF [Given]

∠B = ∠FEG = 90º [Given]

∴ ΔABC ≅ ΔGEF [By SAS congruency]

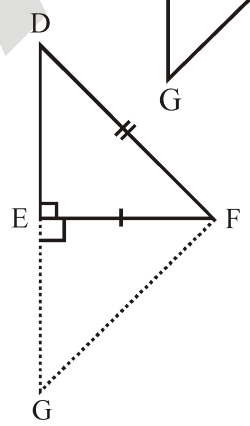
∴ ∠A = ∠G and AC = GF [By C.P.CT]

Now, AC = GF and AC = DF

⇒ GF = DF

⇒ ∠G = ∠D

⇒ ∠A = ∠D [∵ ∠G = ∠A]



Now, ∠A = ∠D, ∠B = ∠E

Thus, in ΔABC and ΔDEF, we have

BC = EF, AC = DF and ∠C = ∠F [By SAS Congruency]

∴ ΔABC ≅ ΔDEF

3. INEQUALITIES IN A TRIANGLE

- The angle opposite to longer side of a triangle is larger.
- The side opposite to the larger angle of a triangle is longer.
- The sum of any two sides of a triangle is always greater than the third side.
- The difference of any two sides of a triangle is always less than the third side.

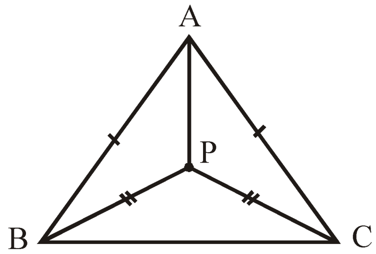
Note : A triangle is isosceles if and only if any two altitude are equal.

Example 8

ΔABC and ΔPBC are two isosceles triangles on the same base BC and vertices A and P are on the same side of BC. A and P are joined, show that (i) ΔABP ≅ ΔACP and (ii) ∠AP bisects A of ΔABC.

Solution :

(i) We are given that ΔABC and ΔPBC are isosceles.



⇒ AB = AC .....(i)

and PB = PC .....(ii)

Now, in ΔABP and ΔACP, we have

AB = AC [By (1)]

PB = PC [By (2)]

and AP = AP [Common side]

∴ ΔABP ≅ ΔACP [By SSS congruency]

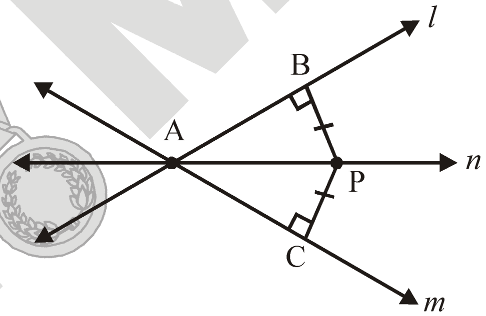
(ii) Now, ΔABP ≅ ΔACP

⇒ ∠BAP = ∠CAP [By C.P.C.T.]

⇒ AP bisects ∠BAC i.e., AP bisects ∠A.

Example 9

In figure, P is a point equidistant from the lines *l* and *m* intersecting at point A. Show that the line *n* (along AP) bisects the angle between *l* and *m*.



Solution :

Let us consider ΔPAB and ΔPAC (as shown in figure).

Here, we have PB = PC [Given]

∠PBA = ∠PCA [Each 90°]

PA = PA [Common]

∴ ΔPAB ≅ ΔPAC [By RHS congruency]

So, ∠BAP = ∠CAP. [By C.P.C.T.]

Therefore, the line AP i.e., the line n is the bisector of the angle between *l* and *m*.

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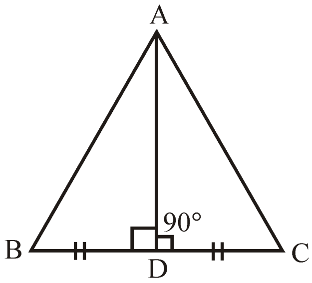
SOLVED EXAMPLES

SE. 1

Prove that ΔABC is isosceles if median AD is perpendicular to BC.



Ans.

Given : A ΔABC in which median $AD \perp BC$.

To Prove : ΔABC is isosceles.

Proof : In ΔABD and ΔACD

$BD = CD$ [$\because$ AD is median]

$\angle ADB = \angle ADC$ [Each 90º]

$AD = AD$ [Common side]

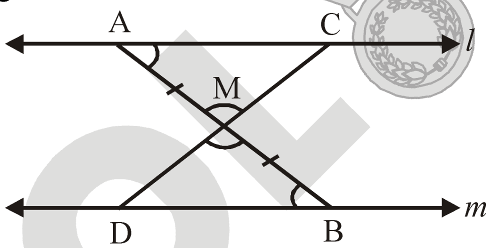
$\therefore\ \Delta ABD \cong \Delta ACD$ [By SAS congruency]

$\Rightarrow AB = AC$ [By C.P.C.T.]

$\therefore\ \Delta ABC$ is isosceles.

SE. 2

In figure, $l \parallel m$ and M is the mid–point of the line segment AB. Prove that M is also the mid–point of line segemnt CD.



Ans. In ΔAMC and ΔBMD

$\angle MAC = \angle MBD$ [Alternate interior anlges]

$\angle AMC = \angle BMD$ [Vertically opposite angles]

$AM = BM$ [Given]

$\therefore\ \Delta AMC \cong \Delta BMD$ [By ASA congruency]

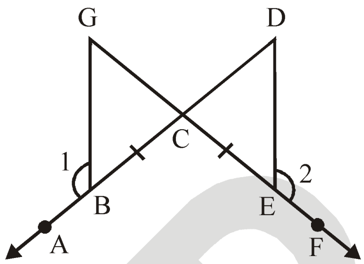
$\Rightarrow CM = DM$ [By C.P.C.T.]

Hence, M is the mid–point of CD.

SE. 3

In figure it is given that BC = CE and $\angle 1 = \angle 2$.

Prove that $\Delta GCB \cong \Delta DCE$.



Ans. We have, $\angle 1 + \angle GBC = 180º$ [Linear pair]

and, $\angle 2 + \angle DEC = 180º$ [Linear pair]

$\therefore\ \angle 1 + \angle GBC = \angle 2 + \angle DEC$

$\Rightarrow \angle GBC = \angle DEC$ ....(i) [$\because \angle 1 = \angle 2$ (Given)]

Now, in ΔGCB and ΔDCE, we have

$\angle GBC = \angle DEC$ [From (i)]

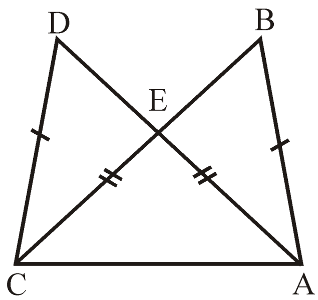
$BC = EC$ [Given]

$\angle GCB = \angle DCE$ [Vertically opposite angles]

$\therefore\ \Delta GCB \cong \Delta DCE$ [By ASA congruency].

SE. 4

In the figure it is given that AB = CD and AD = BC. Prove that $\Delta ABC \cong \Delta CDA$.



Ans. In ΔABC and ΔCDA, we have

$AB = CD$ [Given]

$BC = DA$ [Given]

$AC = CA$ [Common side]

$\therefore\ \Delta ABC \cong \Delta CDA$ [By SSS congruency]

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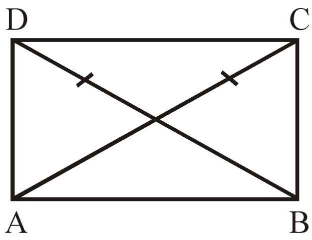
SE. 5

ABCD is a parallelogram, if the two diagonals are equal, find the measure of $\angle ABC$.

Ans.



Since ABCD is a parallelogram, therefore

AB = CD and AD = BC ...(1)

[Opposite sides of a parallelogram are equal]

Now, in ΔABD and ΔBAC, we have

AD = BC [From (1)]

BD = AC [Given]

AB = BA [Common]

$\therefore \Delta ABD \cong \Delta BAC$ [By SSS congruency]

$\Rightarrow \angle BAD = \angle ABC$ ...(2) [By C.P.C.T]

Now, AD || BC and transversal AB intersects them at A and B respectively.

Therefore, $\angle BAD + \angle ABC = 180°$.

[Co–interior angles]

$\Rightarrow \angle ABC + \angle ABC = 180°$ [From (2)]

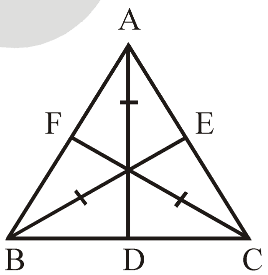
$\Rightarrow 2\angle ABC = 180° \Rightarrow \angle ABC = 90°$

Hence, the measure of $\angle ABC$ is 90°.

SE. 6

AD, BE and CF, the altitude of ΔABC are equal. Prove that ABC is an equilateral triangle.

Ans.



In right triangles BCE and CBF,

BC = CB [Common hypotenuse]

BE = CF [Given]

$\angle BEC = \angle CFB$ [Each 90°]

$\therefore \Delta BCE \cong \Delta CBF$ [By R.H.S. congruency]

$\Rightarrow \angle BCE = \angle CBF$ [By C.P.C.T]

$\Rightarrow \angle C = \angle B$

$\Rightarrow AB = AC$ ...(1)

[Sides opposite to equal angles of a Δ are equal]

Similarly, $\Delta ABD \cong \Delta BAE$

$\Rightarrow \angle B = \angle A$ [By C.P.C.T.]

$\Rightarrow AC = BC$ ...(2)

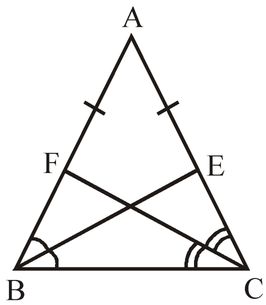
[Sides opposite to equal angles of a Δ are equal]

From (1) and (2), we get AB = BC = AC.

Hence, ABC is an equilateral triangle.

SE. 7

In the figure AB = AC, BE and CF are respectively the bisectors of $\angle B$ and $\angle C$. Prove that $\Delta EBC \cong \Delta FCB$.



Ans. In ΔABC, AB = AC [Given]

$\Rightarrow \angle ACB = \angle ABC \Rightarrow \angle ECB = \angle FBC$ ...(1)

Also, $\angle ACB = \angle ABC$

$$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$$

$\Rightarrow \angle FCB = \angle EBC$ ...(2)

[CF and BE are bisectors of $\angle ACB$ and $\angle ABC$

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respectively]

In ΔEBC and ΔFCB, we have

$\angle ECB = \angle FBC$ [From 1]

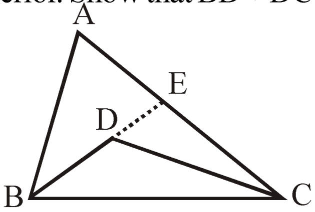
$BC = CB$ [Common]

$\angle EBC = \angle FCB$ [From 2]

$\therefore\ \Delta EBC \cong \Delta FCB$ [By ASA congruency]

SE. 8

In the given figure, ABC is a triangle and D is any point in its interior. Show that $BD + DC < AB + AC$.



Ans. Produce BD.

Sum of two sides is greater than the third

In ΔABE, $AB + AE > BE$

$\Rightarrow AB + AE > BD + DE$

In ΔCDE, $DE + EC > DC$

Adding (i) and (ii), we get

$AB + AE + DE + EC > BD + DE + DC$

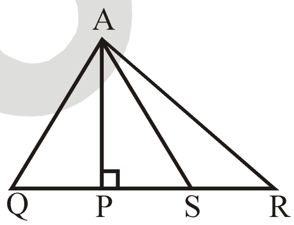
$\Rightarrow AB + (AE + EC) > BD + DC$

$\Rightarrow AB + AC > BD + DC.$

Hence, $BD + DC < AB + AC$.

SE. 9

In the given figure, $AP \perp QR$, $PR > PQ$ and $PS = PQ$. Show that $AR > AQ$.



Ans. In ΔAPQ and ΔAPS, we have

$PQ = PS$ [Given]

$AP = AP$ [Common]

$\angle APQ \cong \angle APS$ [Each 90°]

$\therefore \Delta APQ = \Delta APS$ [By SAS congruency]

$\therefore \angle AQP = \angle ASP$ [By C.P.C.T.]

or $\angle AQS = \angle ASQ$ ...(1)

But, $\angle ASQ > \angle ARS$

$\therefore \angle AQS > \angle ARS$ [From (1)]

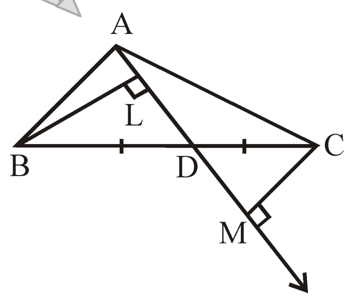
$\Rightarrow \angle AQR > \angle ARQ$

$\therefore$ AR > AQ.

[Side opposite to greater angle is longer]

SE. 10

In the figure AD is a median and BL, CM are perpendiculars drawn from B and C respectively on AD and AD is produced to M. Prove that BL = CM.



Ans. In ΔBDL and ΔCDM, we have

$\angle BLD = \angle CMD$ [Each 90°]

$\angle BDL = \angle CDM$ [Vertically opp. angles]

$BD = CD$ [D is the mid–point of BC]

$\Delta BDL \cong \Delta CDM$ [By AAS congruency]

$\Rightarrow BL = CM$ [By C.P.C.T.]

SE. 11

Suppose line segments AB and CD intersect at O in such a way that AO = OD and OB = OC. Prove that AC = BD but AC may not be parallel to BD.

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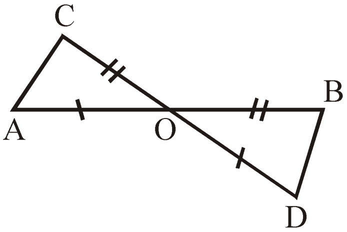
Ans. In Δs AOC and DOB, we have



AO = OD [Given]

∠AOC = ∠DOB [Vertically opposite angles]

and, OC = OB [Given]

ΔAOC ≅ ΔDOB [By SAS congruency]

⇒ ∠OAC = ∠ODB

and, ∠OCA = ∠OBD [By C.P.C.T.]

Clearly, AC will be parallel to BD only when ∠OAC = ∠OBD.

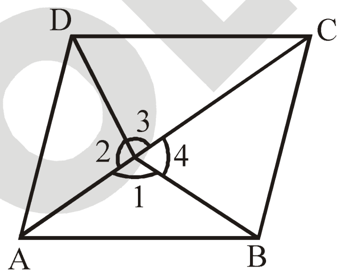
But ∠OAC may not be equal to ∠OBD. So, AC may not be parallel to BD.

SE. 12

A point O is taken inside an equilateral four sided figure ABCD such that its distances from the angular points D and B are equal. Show that AOC is a stratight line.

Ans. Given : Point O is taken inside equilateral quad. ABCD such that BO = OD.

To prove : AOC is a straighe line.



Proof : In ΔAOD and ΔAOB, we have

AD = AB [Given]

AO = AO [Common side]

OD = OB [Given]

∴ ΔAOD ≅ ΔAOB (By SSS congruency)

∴ ∠1 = ∠2 [By C.P.C.T]

Similarly, ΔDOC ≅ ΔBOC

∴ ∠3 = ∠4

But ∠1 + ∠2 + ∠3 + ∠4 = 360°

[∵ Sum of ∠s at a point = 360°]

or 2∠2 + 2∠3 = 360° (∵ ∠1 = ∠2 and ∠3 = ∠4)

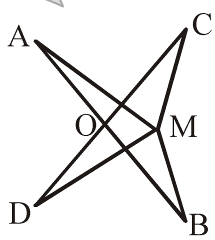
∴ ∠2 + ∠3 = 180°

Hence, AOC is a straight line. [Axiom of linear pair]

SE. 13

In figure, prove that

[AM + BM + CM + DM) > (AO + BO + CO + DO]



Ans. In ΔAMB, we have AM + BM > AB

[Sum of any two sides of a Δ is greater than the third side]

Now, AB = AO + BO

∴ AM + BM > AO + BO ...(1)

In ΔCMD, we have CM + DM > CD

CD = CO + DO

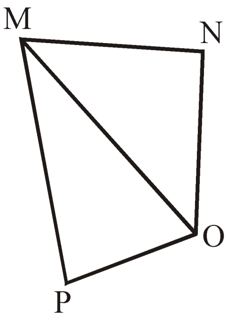
∴ CM + DM > CO + DO ...(2)

Adding (1) and (2), we get

AM + BM + CM + DM > AO + BO + CO + DO

SE. 14

In figure, prove that



(i) MN + NO + OP + PM > 2MO

(ii) MN + NO + OP > PM.

Ans. (i) In ΔMON, we have

MN + NO > MO ...(1)

[∵ Sum of any two sides of a Δ is greater than the third side]

Also in ΔMOP, OP + PM > MO. ...(2)

Hence from (1) and (2), we get

MN + NO + OP + PM > 2MO.

(ii) In ΔMON, MN + NO > MO ...(1)

Again in ΔMOP, MO + OP > PM ...(2)

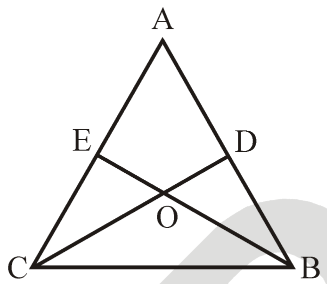
Adding (1) and (2), we have

MN + NO + MO + OP > MO + PM

∴ MN + NO + OP > PM

SE. 15

In figure, it is given that AE = AD and BD = CE. Prove that ΔAEB ≅ ΔADC.



Ans. We have, AE = AD and CE = BD

⇒ AE + CE = AD + BD ⇒ AC = AB ...(i)

Now, in ΔAEB and ΔADC, we have

AE = AD [Given]

∠EAB = ∠DAC [Common]

AB = AC [From (i)]

∴ ΔAEB ≅ ΔADC. [By SAS Congruency]

ONLY ONE CORRECT TYPE

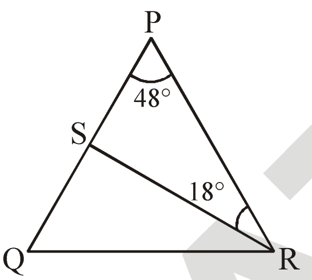
1. In a ΔABC, AB = 5 cm, AC = 5 cm and $\angle A = 50°$, then $\angle B$ =

(A) 35° (B) 65°
(C) 80° (D) 40°

2. If two sides of a triangle are unequal then opposite angle of larger side is :

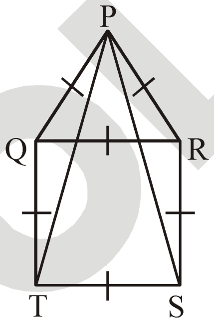
(A) Greater (B) Less
(C) Equal (D) Half

3. In the given figure PQ = QR, $\angle QPR = 48°$, $\angle SRP = 18°$, then $\angle PQR$ =



(A) 48° (B) 84°
(C) 30° (D) 36°

4. In the given figure, PQR is an equilateral triangle and QRST is a square. Then $\angle PSR$ =



(A) 30° (B) 15°
(C) 90° (D) 60°

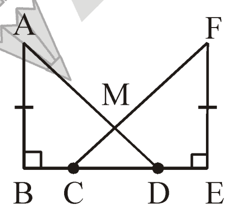
5. Can we draw a triangle ABC with AB = 3 cm, BC = 3.5 cm and CA = 6.5 cm ?

(A) Yes
(B) No
(C) Can't be determined
(D) None of these

6. Which of the following is not a criterion for congruence of triangles ?

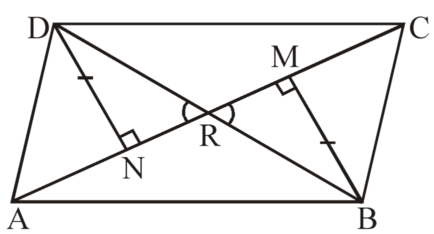
(A) SSA (B) SAS
(C) ASA (D) SSS

7. In the given figure AB $\perp$ BE and EF $\perp$ BE. Also BC = DE and AB = EF. Then :



(A) $\Delta ABD \cong \Delta FEC$ (B) $\Delta ABD \cong \Delta EFC$
(C) $\Delta ABD \cong \Delta CMD$ (D) $\Delta ABD \cong \Delta CEF$

8. In quadrilateral ABCD, BM and DN are drawn perpendicular to AC such that BM = DN. If BR = 8 cm, then BD is :



(A) 4 cm (B) 2 cm
(C) 12 cm (D) 16 cm

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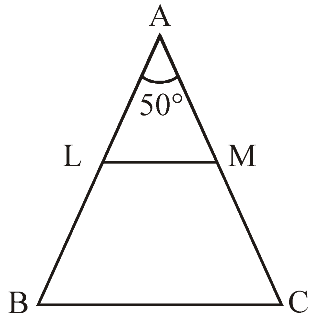
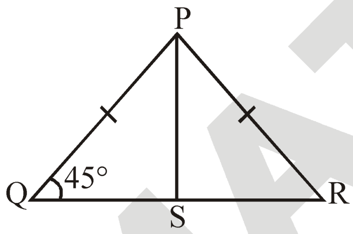
9. In the figure, ABC is an isosceles triangle in which AB = AC and LM is parallel to BC. If ∠A = 50°, find ∠LMC.



(A) 60° (B) 100°

(C) 115° (D) None of these

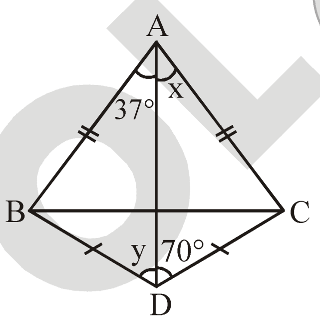
10. In the given figure, PS is the median, bisecting angle P, then ∠QPS is :



(A) 110° (B) 70°

(C) 45° (D) 55°

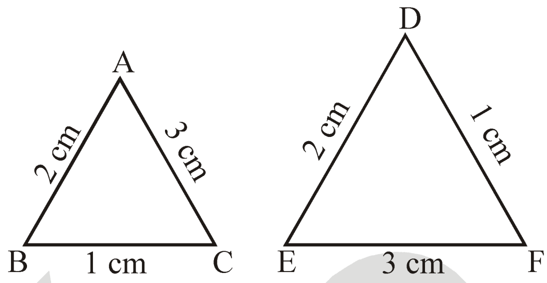
11. In the given figure, x and y are :



(A) x = 70°, y = 37° (B) x = 37°, y = 70°

(C) x + y = 117° (D) x – y = 100°

12. For the given triangles, write the correspondence, if they are congruent.



(A) ΔABC ≅ ΔDEF (B) ΔABC ≅ ΔEDF

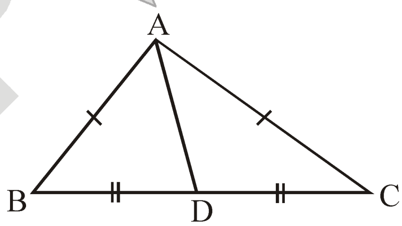
(C) ΔABC ≅ ΔFDE (D) Not congruent

13. In ΔABC, if ∠B < ∠A, then :

(A) BC > CA (B) BC < CA

(C) BC > AB + CA (D) AB < CA

14. In ΔABC, if AB = AC and BD = DC (see figure), then ∠ADC =



(A) 60° (B) 45°

(C) 120° (D) 90°

15. Which of the following is a correct statement :

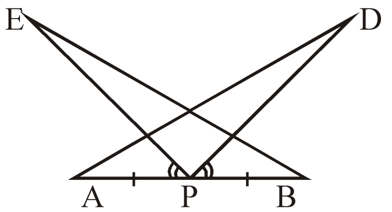
(A) Two triangles having same shape are congruent

(B) If two sides of a triangle are equal to the corresponding sides of another triangle, then the two triangles are congruent.

(C) If the hypotenuse and one side of one right triangle are equal to the hypotenuse and one side of the other triangle, then the triangles are not congruent.

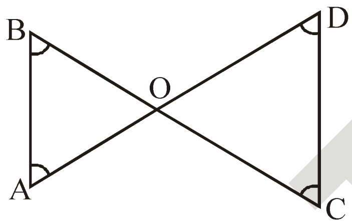
(D) None of these

16. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see figure). Which of the following is true ?



(A) ΔAPD ≅ ΔBPE (B) ∠APE = ∠DPE
(C) AP = BE (D) None of these

17. In figure, ∠B < ∠A and ∠C < ∠D then



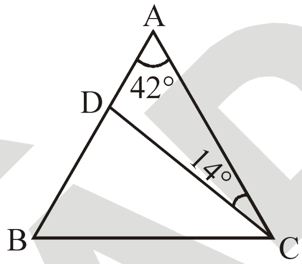
(A) AD < BC (B) OD > OC
(C) OB < OA (D) None of these

18. Which of the following is a correct statement ?
(A) In an isosceles triangle, the angle opposite to equal sides are equal
(B) If the hypotenuse and an acute angle of the right-angled triangle are not equal to the hypotenuse and the corresponding acute angle of another triangle, then the triangles are congruent
(C) The bisector of the vertical angle of an isosceles triangle bisects the base at acute angles
(D) All of these

19. In triangles ABC and RQP, if AB = AC, ∠C = ∠P and ∠B = ∠Q, then two triangles are :
(A) Isosceles but not necessarily congruent
(B) Isosceles and congruent
(C) Congruent but not isosceles
(D) Neither congruent nor isosceles

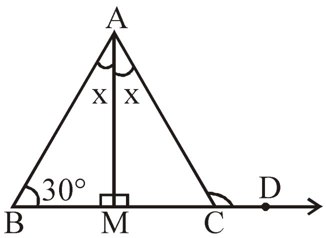
20. In ΔABC, side AB is produced to D so that BD = BC. If ∠B = 60° and ∠A = 70°, then
(A) AD > CD (B) ∠ADC = 90°
(C) AD < CD (D) ∠CAD = 30°

21. In the given figure, AB = AC, ∠A = 42° and ∠ACD = 14°. ∠BCD is equal to :



(A) 55° (B) 69°
(C) 45° (D) 50°

22. In the given figure, find the measure of ∠ACD.



(A) 150° (B) 120°
(C) 140° (D) 160°

23. If S is any point on the side QR of a ΔPQR, then
(A) PQ + QR + RP > 2PS
(B) PQ + QR + RP < 2PS
(C) PQ + QR + RP = 3PS
(D) None of these

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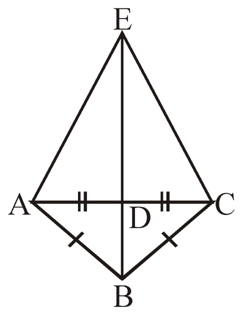
24. In the given figure, AB = BC, AD = CD. Then, which of the following is true ?



(A) ∠ADE = 90° (B) AE = EC

(C) Both (A) and (B) (D) AE = BC

25. ABC is a triangle in which ∠B = 2∠C. D is a point on BC such that AD bisects ∠BAC and AB = CD, then ∠BAC =

(A) 144° (B) 36°

(C) 72° (D) 98°

26. Which of the following pairs of triangles is congruent ?

(A) ΔABC : AC=2cm, BC=3cm and ∠C=72°,
ΔDEF : DE=2cm, DF=3cm and ∠D=72°

(B) ΔABC : AB=4cm, AC=8cm and ∠A=90°,
ΔPQR : PQ=4cm, QR=8cm and ∠Q=90°

(C) ΔABC and ΔDEF in which BC=EF, ∠A=90°, ∠B=∠E = 50° and ∠F = 40°

(D) None of these

27. In a quadrilateral ABCD, AC bisects ∠C and BC = CD, then which of the following statement is false ?

(A) AB = AD

(B) AC is the perpendicular bisector of BD

(C) ΔDCO ≅ ΔBCO

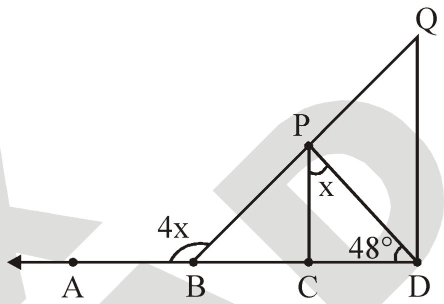
(D) None of these

28. In a ΔABC, if AB = AC and ∠B = 70°, find ∠A.

(A) 40° (B) 70°

(C) 60° (D) 90°

29. In the given figure, ABCD and BPQ are straight lines. If BP = BC and DQ is parallel to CP. Find ∠BDQ.



(A) 48° (B) 45°

(C) 90° (D) 96°

30. The vertical angle of an isosceles triangle is 100°. Find its base angles.

(A) 100° (B) 40°

(C) 80° (D) 90°

31. ABCD is a square and ABE is an equilateral triangle outside the square then :

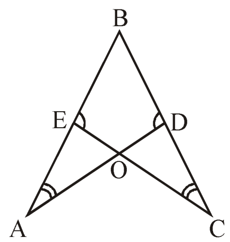
(A) $\angle ACE = \frac{1}{2}\angle ABE$

(B) ∠ACE = ∠ABE

(C) ∠ACE = 2∠ABE

(D) None of these

32. In given figure, ∠A = ∠C and AB = BC. Then which of following is correct ?



(A) ∠OEB = ∠ODB (B) ΔABD ≅ ΔCBE

(C) ∠AEO = ∠CDO (D) All of these

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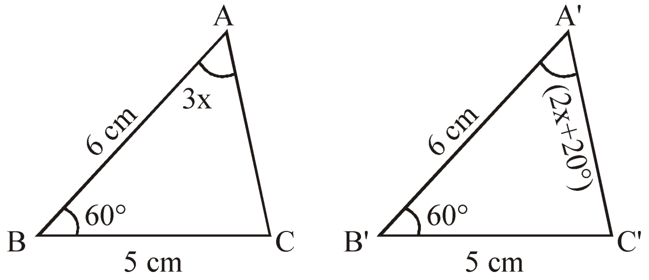
33. In given figures, the measure of ∠B'A'C' is :



(A) 50° (B) 60°

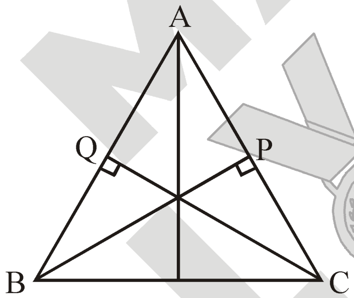
(C) 70° (D) 80°

34. In a ΔABC, if 2∠A = 3∠B = 6∠C, the measure of ∠A, ∠B, ∠C, respectively are :

(A) 90°, 60°, 30° (B) 45°, 60°, 85°

(C) 30°, 60°, 90° (D) 35°, 55°, 90°

35. If in ΔABC, AB = AC (see figure) ; BP and CQ are the altitudes from the vertices to their opposite sides, then



(A) BP = CQ (B) AP = AQ

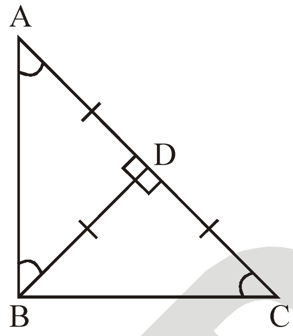
(C) ∠ABC = ∠ACB (D) All of these

36. The sum of altitudes of a triangle is _____ than the perimeter of the triangle.

(A) Greater (B) Equal

(C) Half (D) Less

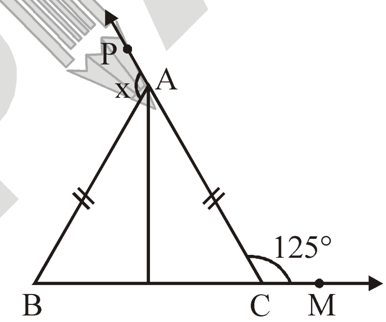
37. In the given figure, BD ⊥ AC, the measure of ∠ABC is :



(A) 60° (B) 30°

(C) 45° (D) 90°

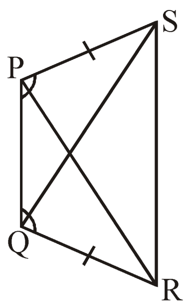
38. In figure, AB = AC, ∠ACM = 125° and ∠PAB = x. Find the value of x.



(A) 130° (B) 110°

(C) 100° (D) 120°

39. In given figure, PS = QR and ∠SPQ = ∠RQP. If QS = 8 cm, then PR =



(A) 8 m (B) 4 m

(C) 16 m (D) None of these

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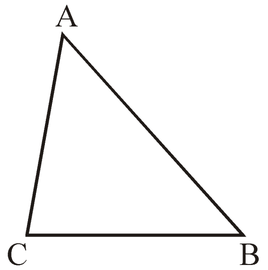
40. In ΔABC, if AB is the greatest side, then



(A) ∠C > 60° (B) ∠B > 60°

(C) ∠A > 60° (D) ∠C < 60°

41. If a triangle ABC is an isosceles triangle, then which of the following conditions hold ?

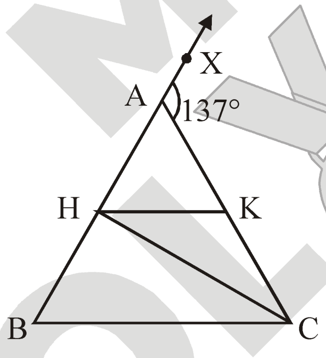
(A) Altitude AD bisects ∠BAC

(B) Bisector of ∠BAC is perpendicular to the base BC.

(C) Both (A) and (B)

(D) None of these

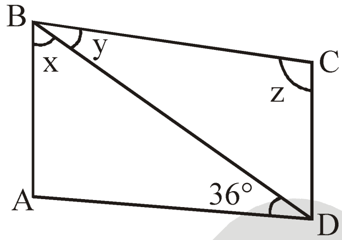
42. In figure, AB = AC, CH = CB and HK || BC. If ∠CAX = 137°, then find ∠CHK.



(A) 68.5° (B) 43°

(C) 137° (D) 68.5°

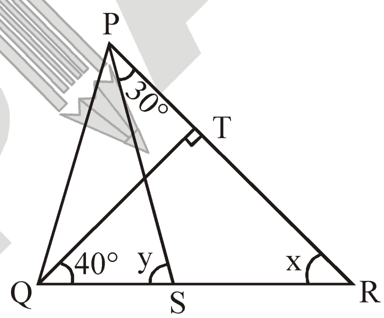
43. In figure, if AB || DC. If $x = \frac{4y}{3}$ and $y = \frac{3z}{8}$ then values of ∠BCD and ∠BAD respectively are



(A) 96°, 96° (B) 48°, 96°

(C) 96°, 48° (D) None of these

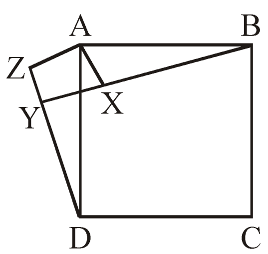
44. In figure, if QT ⊥ PR, ∠TQR = 40° and ∠SPR = 30°, find value of y – x.



(A) 80° (B) 50°

(C) 30° (D) 130°

45. In the given figure, X is a point in the interior of square ABCD. AXYZ is also a square. If DY = 3 cm and AZ = 2 cm, then BY =



(A) 5 cm (B) 6 cm

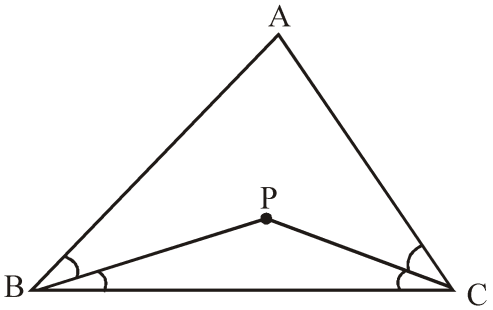
(C) 7 cm (D) 8 cm

MATRIX OLYMPIAD

PARAGRAPH TYPE

PASSAGE – I : In any triangle, the side opposite to the greater angle is longer.

46. In ΔABC, $AB > AC$, PB and PC are internal bisectors of $\angle B$ and $\angle C$ respectively :



(A) $PC < PB$ (B) $PC > PB$
(C) $PC = PB$ (D) None of these

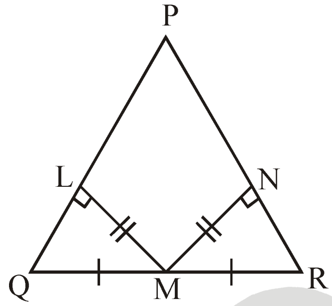
47. In ΔABC if $\angle C > \angle B$, then :
(A) $BC > AC$ (B) $AB > AC$
(C) $AB < AC$ (D) $BC < AC$

48. In a ΔABC, if $\angle A = 45°$, $\angle B = 70°$. The largest side of a triangle is :
(A) BC (B) AB
(C) AC (D) None of these

PASSAGE – II : If in two right triangles, the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

49. If the altitude from two vertices of a triangle to the opposite sides are equal, then the triangle is
(A) Isosceles
(B) Scalene
(C) Right-angled
(D) Equilateral

50. In the figure, it is given that $LM = MN$, $QM = MR$, $ML \perp PQ$ and $MN \perp PR$. Then



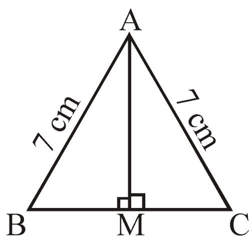
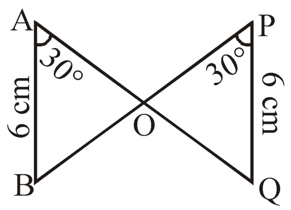
(A) $PQ < PR$ (B) $PQ > PR$
(C) $PQ = PR$ (D) None of these

51. PL is an altitude from P of ΔPQR on QR such that $QL = LR$. Then,
(A) $\angle Q < \angle R$ (B) $\angle Q = \angle R$
(C) $\angle Q > \angle R$ (D) $\angle P = \angle R$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column – I and Column – II are given as options (A), (B), (C) and (D) out of which one is correct.

52. Match the following :

Column – I	Column – II
(P)  $\Delta AMB \cong \Delta AMC$ by	(i) SAS Rule
(Q)  $\Delta AOB \cong \Delta POQ$ by	(ii) RHS Rule

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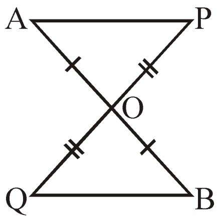
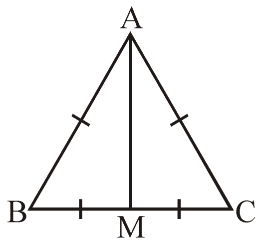
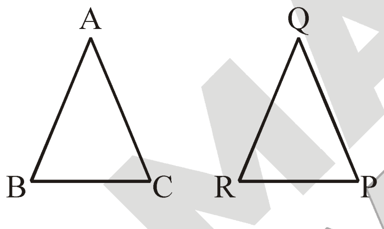
(R)  (iii) SSS Rule

$\Delta AOP \cong \Delta BOQ$ by

(S)  (iv) AAS Rule

$\Delta AMB \cong \Delta AMC$ by

(A) (P) $\rightarrow$ (ii), (Q) $\rightarrow$ (iv), (R) $\rightarrow$ (i), (S) $\rightarrow$ (iii)
(B) (P) $\rightarrow$ (iv), (Q) $\rightarrow$ (ii), (R) $\rightarrow$ (i), (S) $\rightarrow$ (iii)
(C) (P) $\rightarrow$ (i), (Q) $\rightarrow$ (ii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (iii)
(D) (P) $\rightarrow$ (ii), (Q) $\rightarrow$ (i), (R) $\rightarrow$ (iii), (S) $\rightarrow$ (iv)

53. 

$\Delta ABC \cong \Delta QRP$. Match the following

Column – I	**Column – II**
(P) AB	(i) $\angle Q$
(Q) BC	(ii) QP
(R) AC	(iii) QR
(S) $\angle A$	(iv) RP

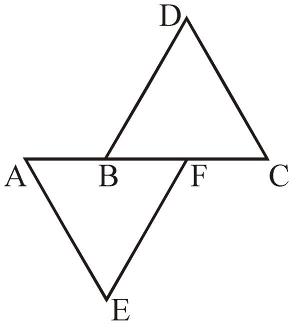
(A) (P) $\rightarrow$ (i), (Q) $\rightarrow$ (ii), (R) $\rightarrow$ (iii), (S) $\rightarrow$ (iv)
(B) (P) $\rightarrow$ (iii), (Q) $\rightarrow$ (ii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (i)
(C) (P) $\rightarrow$ (iii), (Q) $\rightarrow$ (iv), (R) $\rightarrow$ (ii), (S) $\rightarrow$ (i)
(D) (P) $\rightarrow$ (ii), (Q) $\rightarrow$ (iii), (R) $\rightarrow$ (iv), (S) $\rightarrow$ (i)

Space for Notes :

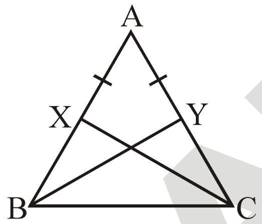
EXERCISE – II

VERY SHORT ANSWER TYPE

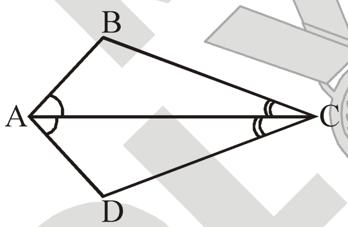
1. In the figure, given that AB = CF, EF = BD and $\angle AFE = \angle DBC$. Prove that $\Delta AFE \cong \Delta CBD$.



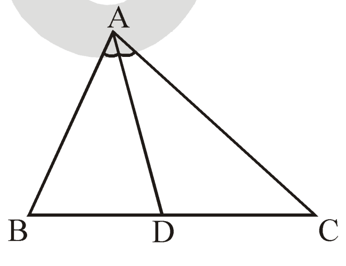
2. In the figure, X and Y are two points on equal sides AB and AC of a ΔABC such that AX = AY. Prove that XC = YB.



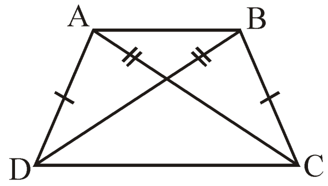
3. In the figure, diagonal AC of a quadrilateral ABCD bisects the angles A and C. Prove that AB = AD and CB = CD.



4. In ΔABC, if AD is the bisector of $\angle A$, show that AB > BD and AC > DC.

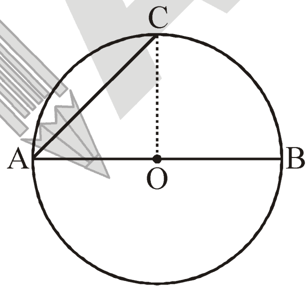


5. In the figure, AD = BC and BD = CA. Prove that $\angle ADB = \angle BCA$ and $\angle DAB = \angle CBA$.

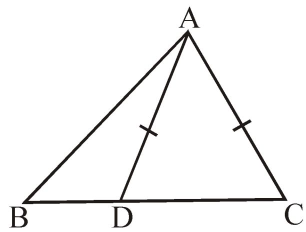


6. In a ΔABC, if $\angle A = 40°$ and $\angle B = 60°$ then which side of the triangle is longest and which side is shortest?

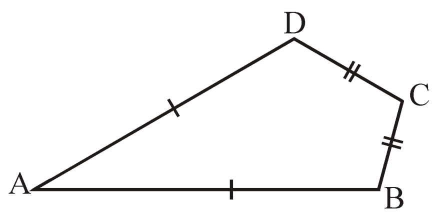
7. In the given figure, O is centre of circle and AB is a diameter. If AC is any other chord of the circle, then show that AB > AC.



8. D is a point on side BC of ΔABC such that AD = AC (see figure). Show that AB > AD.



9. In quadrilateral ABCD, AB = AD and BC = CD. Show that $\angle ABC = \angle ADC$.



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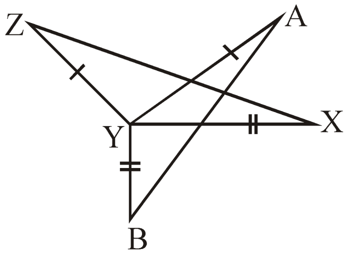
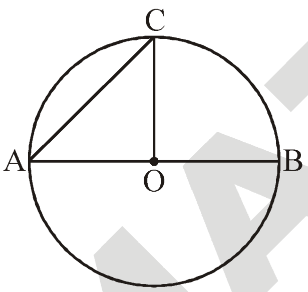
SHORT ANSWER TYPE

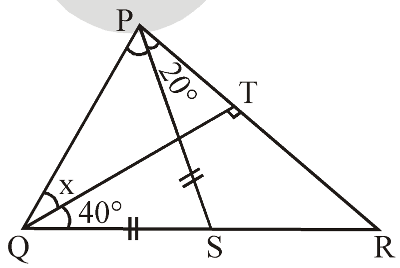
1. If the bisector of the vertical angle of a tirangle bisects the base, prove that the triangle is isosceles.
2. In the given figure, $AY \perp ZY$ and $BY \perp XY$ such that $AY = ZY$ and $BY = XY$. Prove that $AB = ZX$.



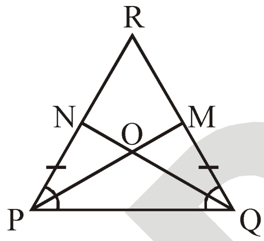
3. In the figure, O is centre of circle and AB is a diameter. If AC is a chord, then show that $\angle A = \frac{1}{2}\angle COB$.



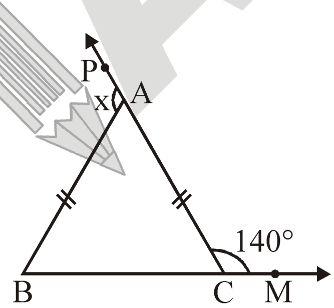
4. AB is a line segment, AX and BY are two equal line segments drawn on opposite sides of line AB such that AX || BY. If AB and XY intersect each other at P, prove that
(i) $\Delta APX \cong \Delta BPY$
(ii) AB and XY bisect each other.
5. In the given figure, $QT \perp PR$ and $QS = PS$. If $\angle TQR = 40°$ and $\angle RPS = 20°$ then find value of x.



6. In the figure, $\angle QPR = \angle PQR$ and M and N are respectively on sides QR and PR of ΔPQR such that $QM = PN$. Prove that $OP = OQ$, where O is the point intersection of PM and QN.

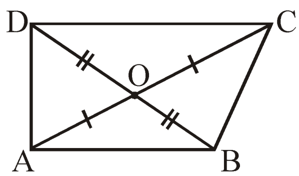


7. In the figure, $AB = AC$, $\angle ACM = 140°$ and $\angle PAB = x$. Find the value of x.



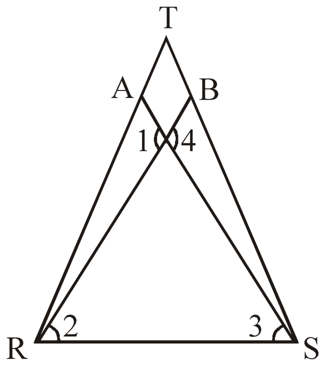
8. In the figure, show that

$2(AC + BD) > AB + BC + CD + DA$.



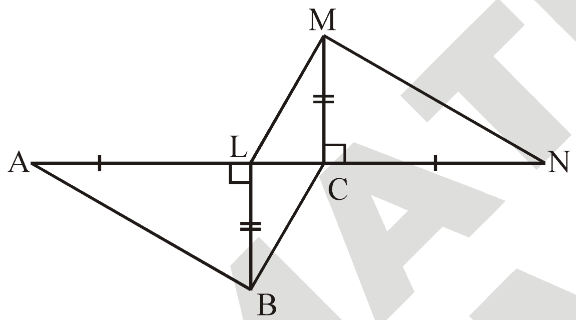
9. In a right angled triangle, one acute angle is double the other. Prove that the hypotenuse is double the smallest side.

10. In figure, it is given that RT = TS, ∠1 = 2∠2 and ∠4 = 2∠3. Prove that ΔRBT ≅ ΔSAT.



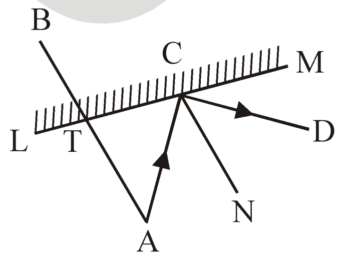
LONG ANSWER TYPE

1. In the figure, BL ⊥ AC, MC ⊥ LN, AL = CN and BL = CM. Prove that ΔABC ≅ ΔNML.

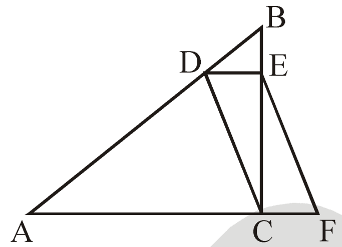


2. If two isosceles triangles have a common base, prove that the line segment joining their vertices bisects the common base at right angles.

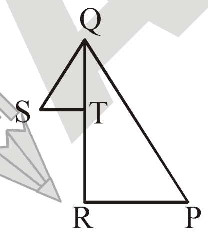
3. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in figure. Prove that the image is as far behind the mirror as the object is in front of the mirror.



4. In figure, ∠ACB is a right angle and AC = CD and CDEF is a parallelogram. If ∠FEC = 10°, then calculate ∠BDE.



5. In shown figure, T is a point on side QR of Δ PQR and S is a point such that RT = ST. Prove that PQ + PR > QS.



TRUE / FALSE TYPE

1. If two triangles are constructed which have all corresponding angles equal but have unequal corresponding sides, then two triangles cannot be congruent to each other.

2. In ΔABC and ΔPQR, AB = PQ, AC = PR and ∠ABC = ∠PQR ∴ ΔABC ≅ ΔPQR.

3. Two triangles are congruent if two sides and the included angle of one trianlge is equal to the corresponding two sides and included angle of the other.

4. If two angles and included side of a triangle are equal to the corresponding angles and side of the other triangle then the triangles are congruent by ASA congruence criteria.

5. In a triangle, angles opposite to equal sides are unequal.

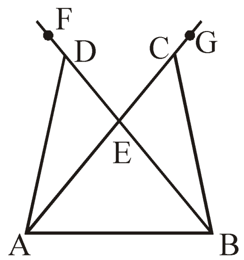
ANALYTICAL PROBLEMS & BRAIN TEASER

1. If ΔABC is an obtuse angled triangle in which $\angle C = 110°$, then which one of the following is true ?

(A) AB = AC (B) AB < AC

(C) AB > AC (D) AB < BC

2. In the given figure, if ED = EC and $\angle ADF = \angle BCG$, then ΔABE is a/an.



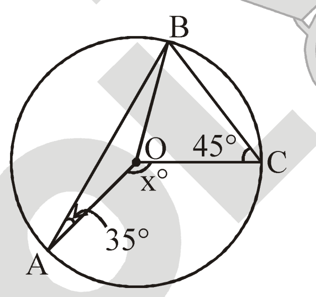
(A) Equilateral triangle

(B) Isosceles triangle

(C) Scalene triangle

(D) Non-isosceles right angled triangle

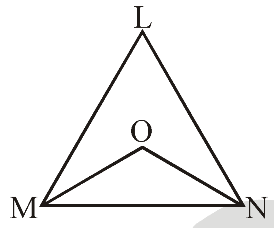
3. In the given figure, O is the centre of circle. If $\angle BAO = 35°$ and $\angle BCO = 45°$ then the value of x will be



(A) 160 (B) 170

(C) 80 (D) 140

4. In the given figure, $\angle L = 62°$, $\angle LMN = 54°$, If MO and NO are bisectors of $\angle LMN$ and $\angle LNM$ respectively of ΔLMN, find $\angle ONM$ and $\angle MON$.



(A) 27°, 121° (B) 64°, 32°

(C) 64°, 121° (D) 32°, 121°

5. In a ΔABC, the bisectors of $\angle B$ and $\angle C$ intersect each other at a point O then $\angle BOC$ =

(A) 90° (B) $90° - \frac{\angle A}{2}$

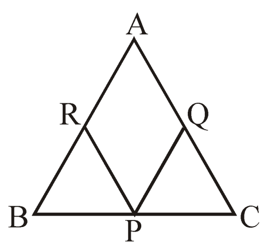
(C) $90° + \frac{\angle A}{2}$ (D) None of these

6. If ΔABC is an isosceles triangle where AB = AC, D and E are points on BC, such that BE = CD. Then which of the following relation is true ?

(A) AD = AB (B) AE = DE

(C) AD = DE (D) AD = AE

7. In a ΔABC, P, Q and R are the mid-points of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm. The perimeter of the quadrilateral ARPQ is



(A) 91 cm (B) 60 cm

(C) 51 cm (D) 70 cm

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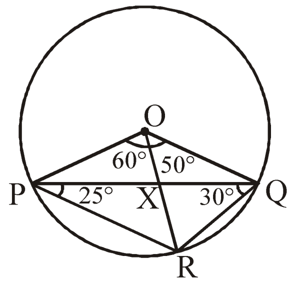
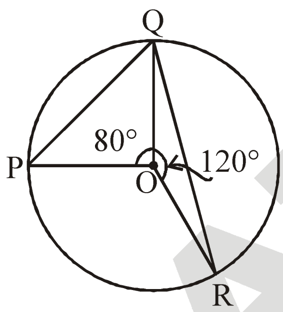
8. In the given figure, the value of ∠PXR is



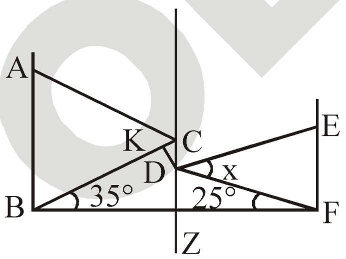
(A) 85° (B) 100°
(C) 95° (D) 120°

9. In the given figure, if O is centre of circle, then ∠PQR =



(A) 60° (B) 80°
(C) 100° (D) 120°

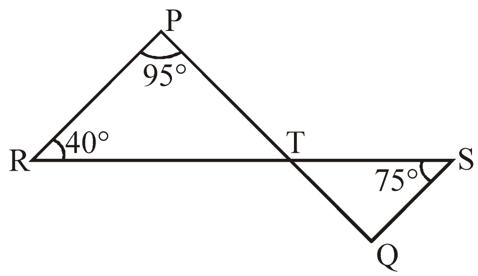
10. In the given figure, it is given that
(i) AB ⊥ BF and EF ⊥ BF
(ii) AC = BC
(iii) KD is perpendicular to BC and DE.



Find the measure of x.
(A) 75° (B) 30°
(C) 60° (D) 45°

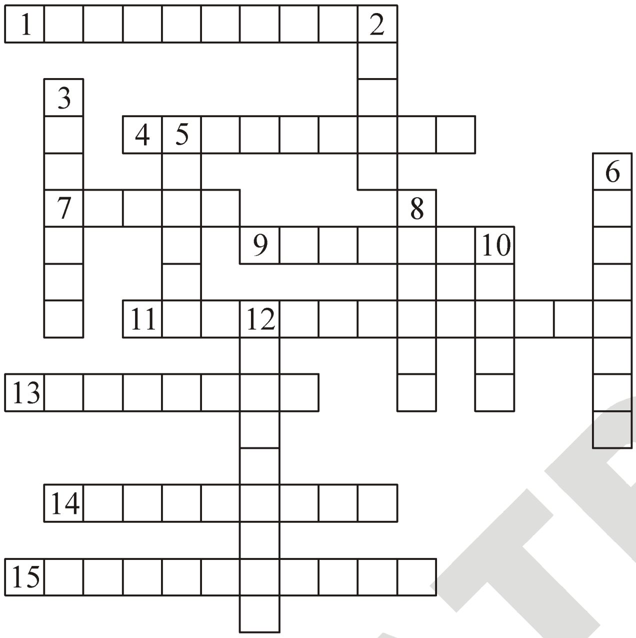
NUMERICAL PROBLEMS

1. The vertical angle of an isosceles triangle is 110°. What is value of product of the digits in the measure of one of the equal angles.
2. How many criterion of congruency of triangles are there ?
3. Sum of the angles of a triangle is equal to how many right angles ?
4. Two angles of a triangle are 65° and 85°. What is the value of third angle when multiplied by 2 degrees?
5. The angles of a triangle are in ratio 7 : 6 : 2. What is unit place digit of the smallest angle ?
6. Of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Sum of the digits of the smallest angle is
7. In a ΔABC, if ∠A = 45° and ∠B = 70°. Determine the sum of the digits of ∠C.
8. The angles of a triangle are (x – 40°), (x – 20°) and $\left(\frac{1}{2}x - 10°\right)$. Find the product of the digits of x.
9. An exterior angle of a triangle is 110°, and one of the interior opposite angle is 30°. Then the other interior angle is K × 40°. Find K.
10. In figure, if lines PQ and RS intersect at a point T such that ∠PRT = 40°, ∠RPT = 95° and ∠TSQ = 75°, ∠SQT = K × 60°. Find the value of K.



CROSS WORD PUZZLE

Complete the following word puzzle with the help of clues given below :



Across

1. The altitudes of a triangle are ______ . [10]
4. Two geometrical figures having exactly same shape and size are known as ______ figure. [9]
7. A triangle in which all the angles are acute is called a/an ____ angled triangle. [5]
9. A triangle whose all sides are of different lengths is called as ____ triangle. [7]
11. Circumcentre is the point of intersection of all the three _____ bisectors of sides of the triangle. [13]
13. The point of intersection of the bisectors of the interior angles of a triangle is called the ____. [8]
14. A triangle having two sides equal is called a/an _____ triangle. [9]
15. The point of intersection of all the three altitudes of a triangle is called the ______ . [11]

Down

2. Sum of all the ______ interior of a triangle is 180 degrees. [5]
3. In a triangle, ______ angle has longer side opposite to it. [7]
5. A triangle in which one angle is obtuse is called a/an ____ angled triangle. [6]
6. If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of two opposite _____ angles. [8]
8. Line segment joining the mid-point of the side of the triangle with the opposite vertex is called a _____ . [6]
10. Angles opposite to the two equal sides of a triangle are _____. [5]
12. The sum of the lengths of the sides of a triangle is called _____. [9]

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Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
B	A	B	B	B	A	A	D	C	C	B	B	A	D	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	A	A	A	A	A	A	A	C	C	C	D	A	D	B
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A	B	B	A	D	D	D	B	A	A	C	B	A	C	C
46	47	48	49	50	51	52	53							
A	B	C	A	C	B	A	C							

EXERCISE II

VERY SHORT ANSWER TYPE

6. BC is the shortest side and AB is longest

SHORT ANSWER TYPE

5. 15° **7.** 80°

LONG ANSWER TYPE

4. 50°

TRUE/FALSE TYPE

1. True **2.** True **3.** True **4.** True **5.** False

NUMERICAL PROBLEMS

1. 15 **2.** 5 **3.** 2 **4.** 60 **5.** 4 **6.** 3 **7.** 11

8. 0 **9.** 2 **10.** 1

ANALYTICAL PROBLEMS AND BRAIN TEASER

1. C **2.** B **3.** A **4.** D **5.** C **6.** D **7.** C

8. C **9.** B **10.** C

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : TRIANGLES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

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CIRCLES 7

Concepts

INTRODUCTION

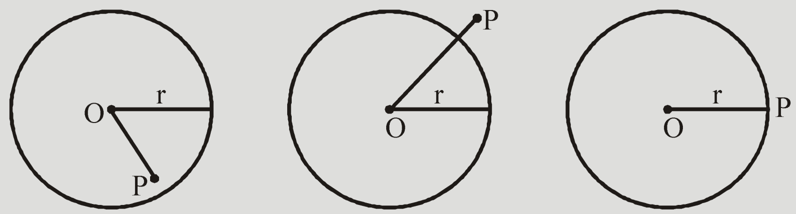
A collection of all the points in a plane which are at a fixed distance from a fixed point in the plane is called a circle.

♦ The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle. Here O is the centre and length OX = r is the radius of the circle.

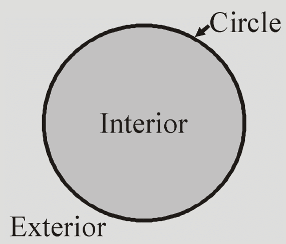
♦ A circle is the locus of the point which moves in a plane in such a way that its distance from a fixed point in the plane is always constant.

Focus Point

• **POSITION OF A POINT WITH RESPECT TO A CIRCLE**



(i) (ii) (iii)



♦ A point P is said to lie inside, outside or on the circle according as OP < r, OP > r or OP = r, where O is the centre of the circle.

♦ In the given figure, all points lying inside the circle are called its interior points and those points which lie outside it are called its exterior point.

♦ The circle and its interior make up the circular region or circular disc *i.e.*, the region consisting of all points which are either on the circle or lie inside the circle.

1. TERMS RELATED TO A CIRCLE

1.1 RADIUS

A line segment joining the centre and a point on the circle is called its radius.

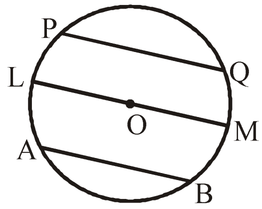
1.2 CIRCUMFERENCE

The perimeter of a circle is called its circumference.

Circumference = $2\pi r$; Where r is the radius of circle

1.3 CHORD

A chord of a circle is a line segment joining any two points on the circle.
In the given figure, PQ, LM and AB are chords of the circle.



1.4 DIAMETER

A diameter is a chord of a circle passing through the centre of the circle. Thus, LOM is the diameter of circle with centre O. Diameter = 2 × Radius

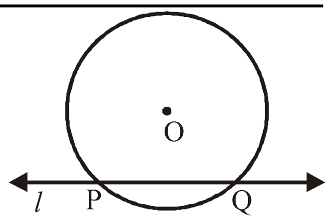
Focus Point

- Of any two chords of a circle, the one which is longer is nearer to the center.
- Of any two chords of a circle, the one which is nearer to the centre is longer
- The diameter is the longest chord of circle.

1.5 SECANT

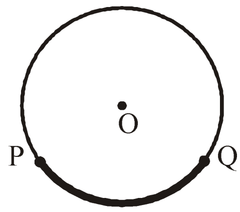
A line which intersects a circle at two distinct points is called a secant of the circle drawn from the external points

In the given figure, the line l cuts the circle at the points P and Q. So, l is the secant of the circle.



1.6 ARC OF THE CIRCLE

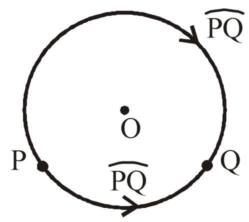
A continuous piece of a circle between two points is called an arc of the circle. Let P and Q be two points on the circle. We denote the arc from P to Q in anti-clockwise direction by $\overset{\frown}{PQ}$ and the arc from Q to P in clockwise direction by $\overset{\frown}{QP}$.



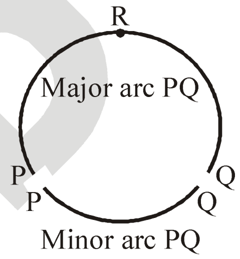
1.7 LENGTH OF AN ARC

The length of an arc by $\overset{\frown}{PQ}$ is the length of the fine thread which just covers the arc completely.

♦ We denote the length of arc $\overset{\frown}{PQ}$ by $l\,(\overset{\frown}{PQ})$



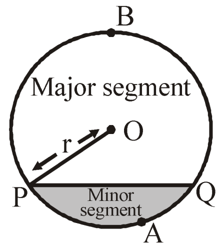
♦ The longer one is called the major arc PQ denote as $\overset{\frown}{PRQ}$ and the shorter one is called the minor arc PQ denote as $\overset{\frown}{PQ}$.



♦ When P and Q are ends of a diameter, then both arcs are equal and each is called a semi circle.

1.8 SEGMENT OF A CIRCLE

The part of the circular region between a chord and either of its arcs is called segment of a circle. The segment containing the minor arc is called the minor segment. In the given figure, PAQP is the minor segment of the circle.

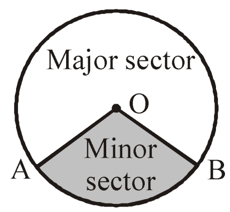


♦ The segment containing the major arc is called the major segment of the circle. In the given figure, PBQP is the major segment of the circle.

Note : The minor and major segments of a circle are called the alternate segments of each other.

1.9 SECTOR OF A CIRCLE

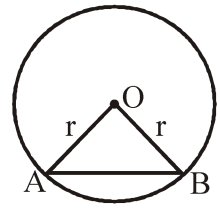
The region between an arc and its two bounding radii is called a sector of the circle. The minor arc corresponds to the minor sector and the major arc corresponds to the major sector. In the given figure, the region OAB is the minor sector and the remaining part of the circular region is called the major sector.



1.10 CONGRUENT CIRCLES

Two circles are said to be congruent if and only if either of them can be superimposed on the other so as to cover it exactly. Two circles are said to be congruent if and only if their radii are equal.

2. ANGLE SUBTENDED BY A CHORD



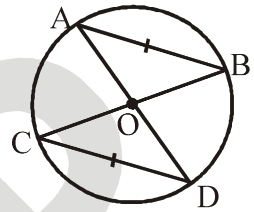
From figure, ∠AOB is the angle subtended by the chord AB at centre.

Theorem - 1

Statement : Equal chords of a circle subtend equal angles at the centre.

Given : A circle with centre O in which chord AB = chord CD.

To Prove : ∠AOB = ∠COD.



Proof : In ΔAOB and ΔCOD, we have

OA = OC	[Radii of circle]
OB = OD	[Radii of circle]
AB = CD	[Given]
∴ ΔAOB ≅ ΔCOD	[By SSS congruence criteria]
Hence, ∠AOB = ∠COD	[C.P.C.T.]

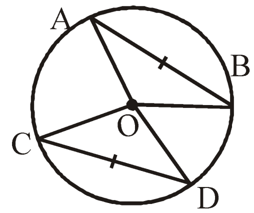
Theorem - 2

(Converse of Theorem 1)

If the angles subtended by two chords at the centre of a circle are equal then the chords are equal.

Given : A circle with centre O in which AB and CD are the chords such that ∠AOB = ∠COD.

To Prove : AB = CD

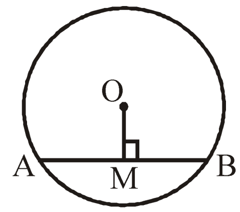


Prove : In ΔAOB and ΔCOD, we have

OA = OC	[Radii of circle]
OB = OD	[Radii of circle]
∠AOB = ∠COD	[Given]
∴ ΔAOB ≅ ΔCOD	[By SAS congruence criteria]
Hence, AB = CD	[C.P.C.T.]

3. PERPENDICULAR FROM THE CENTRE TO A CHORD

Let AB be the chord of the circle with centre O then OM is the perpendicular from the centre to the chord AB.



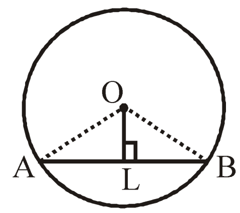
Theorem - 3

Statement : The perpendicular from the centre of a circle to a chord bisects the chord.

Given : A chord AB of a circle with centre O and OL ⊥ AB.

To Prove : LA = LB

Construction : Join OA and OB



Proof : In the right triangles OLA and OLB, we have

OA = OB [Radii of circle]

OL = OL [Common]

∠OLA = ∠OLB [Equal to 90º]

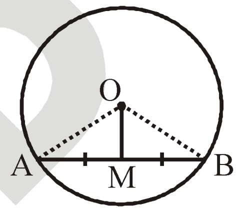
∴ ΔOLA ≅ ΔOLB [By RHS congruence criteria]

Hence, LA = LB [C.P.C.T.]

Theorem - 4

Statement (converse of theorem - 3) : The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Given : M is the midpoint of the chord AB of a circle with centre O.



To Prove : OM ⊥ AB

Construction : Join OA and OB.

Proof : In ΔOMA and ΔOMB, we have

OA = OB [Radii of circle]

OM = OM [Common]

MA = MB [Given]

∴ ΔOMA ≅ OMB [By SSS congruence criteria]

∴ ∠OMA = ∠OMB [C.P.C.T.]

Now, ∠OMA + ∠OMB = 180º [Linear pair]

⇒ 2 ∠OMA = 180º [By (1)]

⇒ ∠OMA = 90º

Hence, OM ⊥ AB

4. CIRCLE THROUGH THREE POINTS

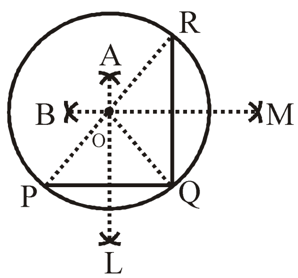
Theorem - 5

Statement : There is one and only one circle passing through three non-collinear points.

Given : Three non-collinear points P, Q and R.

To Prove : There is one and only one circle passing through P, Q and R.

Construction : Join PQ and QR. Draw perpendicular bisectors AL and BM of PQ and RQ respectively. Since P, Q, R are not collinear. Therefore, the prependicular bisectors AL and BM are not parallel. Let AL and BM intersect at O. Join OP, OQ and OR.



Proof : Since O lies on the perpendicular bisector of PQ. Therefore,

OP = OQ

Again, O lies on the perpendicular bisector of QR. Therefore,

OQ = OR

Thus, OP = OQ = OR = r (say)

Taking O as the centre, draw a circle of radius r. Clearly, the circle passes throught P, Q and R. This proves that there is a circle passing through the points P, Q and R.

5. EQUAL CHORDS AND THEIR DISTANCES FROM THE CENTRE

Theorem - 6

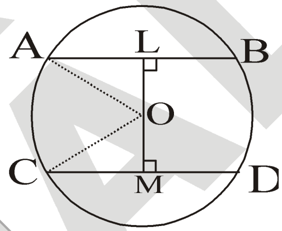
Statement : Equal chords of a circle are equidistant from the centre.

Given : A circle with centre O in which chord AB = chord CD, OL ⊥ AB and OM ⊥ CD.

To Prove : OL = OM

Construction : Join OA and OC

Proof : We know that the perpendicular from the centre of a circle to a chord bisects the chord.



∴ $AL = \frac{1}{2}AB$ and $CM = \frac{1}{2}CD$.

Since, AB = CD (Given)

⇒ $\frac{1}{2}AB = \frac{1}{2}CD$ ⇒ AL = CM ......(1)

Now, in ΔOLA and ΔOMC, we have

AL = CM [By equation (1)]

OA = OC [Radii of circle]

∠OLA = ∠OMC [Each equal to 90°]

∴ ΔOLA ≅ ΔOMC [By RHS congruence criteria]

So, OL = OM [C.P.C.T.]

Hence, AB and CD are equidistant from O.

Theorem - 7

Statement : Chords equidistant from the centre of a circle are equal in length.

Given : Circle with centre O, OL ⊥ AB, OM ⊥ CD, OL = OM

To Prove : AB = CD

Proof :

- **Case -I** AB || CD

In fig. (1), AB || CD. Join OA and OC

In ΔOAL and ΔOCM, we have

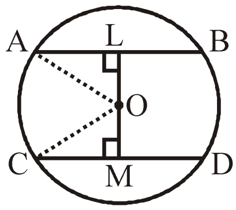


Fig. (1)

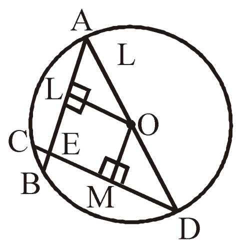


Fig. (2)

OA = OC [Radii of circle]

OL = OM [Given]

∠OLA = ∠OMC [Each 90°]

⇒ ΔOAL ≅ ΔOCM [By RHS congruency]

⇒ AL = CM [C.P.C.T.] ......(1)

Since perpendicular from centre bisects the chord

$\therefore \quad \left.\begin{aligned} 2AL &= AB \\ \text{and } 2CM &= CD \end{aligned}\right\}$ ......(2)

From equations (1) and (2), we get

AB = CD

• **Case-II :** AB and CD are intersecting at E.

Join OA and OD

$\therefore \quad \Delta OAL \cong \Delta ODM$ [Same as case I]

$\Rightarrow \quad AL = DM$ [By C.P.C.T.] ....(3)

Since perpendicular from centre bisects the chord

$\therefore \quad \left.\begin{aligned} 2AL &= AB \\ \text{and } 2DM &= CD \end{aligned}\right\}$ ....(4)

(3) and (4), we get

AB = CD

$\therefore$ From both cases, we get

AB = CD

Example 1

Find the length of the chord which is at a distance of 5 cm from the centre a circle of radius 13 cm.

Solution :

Let AB the be chord of a circle with centre O and radius 13 cm. Draw OL ⊥ AB.

Join OA. Clearly, OL = 5 cm and OA = 13 cm

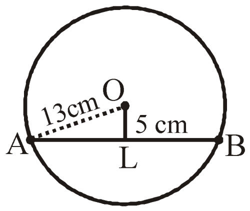
In the right triangle OLA, we have

$OA^2 = OL^2 + AL^2$ [Pythagoras theorem]

$\Rightarrow \quad 13^2 = 5^2 + AL^2$

$\Rightarrow \quad AL^2 = 144$

$\Rightarrow \quad AL = 12$ cm



Since the perpendicular from the centre to a chord bisects the chord.

Therefore, AB = 2AL = (2 × 12) cm = 24 cm.

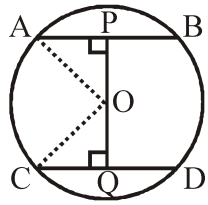
Example 2

In the given figrure, O is the centre of the circle of radius 5 cm.

OP ⊥ AB, OQ ⊥ CD, AB || CD,

AB = 6 cm and CD = 8 cm.

Determine PQ.



Solution :

Join OA and OC.

Since the perpendicular from centre of the circle to a chord bisects the chord. Therefore, P and Q are midpoint of AB and CD respectively.

Now, $AP = PB = \frac{1}{2}AB = 3\,cm$

and $CQ = QD = \frac{1}{2}CD = 4\,cm$

In right triangles OAP and OCQ, we have

$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$

$\Rightarrow\quad 5^2 = OP^2 + 3^2$ and $5^2 = OQ^2 + 4^2$

$\Rightarrow\quad OP^2 = 5^2 - 3^2$ and $OQ^2 = 5^2 - 4^2$

$\Rightarrow\quad OP^2 = 16$ and $OQ^2 = 9$

$\Rightarrow\quad OP = 4$ cm and $OQ = 3$ cm

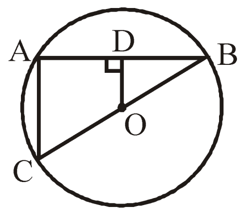
$\therefore\quad PQ = OP + OQ = (4 + 3)$ cm $= 7$ cm

Example 3

In the given figure, OD is perpendicular to the chord AB of a circle whose centre is O.

If BC is a diameter, show that CA = 2OD.

Solution :



Since $OD \perp AB$ and the perpendicular drawn from the centre to a chord bisects the chord.

$\therefore\quad$ D is the midpoint of AB

Also, O being the centre, is the mid-point of BC.

Thus, in ΔABC, D and O are the midpoints of AB and BC respectively.

$\therefore\quad OD \parallel AC$ and, $OD = \frac{1}{2}CA$

[By midpoint theorem *i.e.*, segment joining the mid-point of two sides of a triangle is half of the third side]

$\Rightarrow\quad CA = 2OD$

Example 4

An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.

Solution :

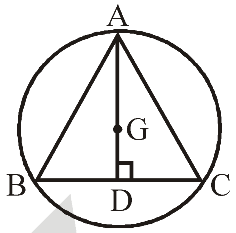
Let ABC be an equilateral triangle of side 9 cm and let AD be one of its medians. Let G be the centroid of ΔABC. Then, AG : GD = 2 : 1.

We know that in an equilateral triangle centroid coincides with the circumcentre. Therefore, G is the centre of the circumcircle with circumradius GA.

Also, G is the centre and GD ⊥ BC.

Therefore,

BD = CD = 4.5 cm



In right triangle ADB, we have

$AB^2 = AD^2 + DB^2$

$\Rightarrow \quad 9^2 = AD^2 + (4.5)^2$

$\Rightarrow \quad AD = \sqrt{81 - \frac{81}{4}} = \frac{9\sqrt{3}}{2}$ cm

$\therefore \quad$ Radius $AG = \frac{2}{3}AD = 3\sqrt{3}$ cm

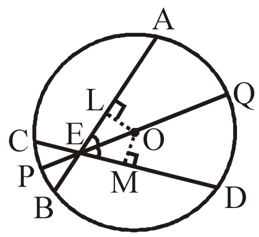
Example 5

If two intesecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

Solution :

Given that AB and CD are two chords of a circle, with centre O, intersecting at a point E. PQ is a diameter through E, such that ∠AEQ = ∠DEQ. We have to prove that AB = CD. Draw perpendicular OL and OM on chords AB and CD respectively.

In triangles OLE and OME,



∠LEO = ∠MEO	[Given]
∠OLE = ∠OME	[Each 90°]
EO = EO	[Common]
Therefore, ΔOLE ≅ ΔOME	[By AAS rule]
∴ OL = OM	[C.P.C.T.]

So, AB = CD

[Chords equidistant from centre are equal]

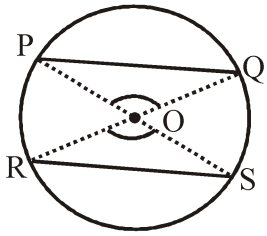
6. ANGLE SUBTENDED BY AN ARC OF A CIRCLE

Theorem - 8

Statement : If two arcs of a circles are congruent, then their corresponding chords are equal.

Given : A circle with centre O in which $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$.

To Prove : Chord PQ = Chord RS



Proof :

Case - I : When $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are minor arcs.

Join OP, OQ, OR and OS.

In ΔPOQ and ΔROS, we have

OP = OR	[Radii of circle]
OQ = OS	[Radii of circle]
∠POQ = ∠ROS	$[\overset{\frown}{PQ} \cong \overset{\frown}{RS} \Rightarrow m(\overset{\frown}{PQ}) = m(\overset{\frown}{RS})]$
∴ ΔPOQ ≅ ΔROS	[By SAS congruence criteria]
So, PQ = RS	[C.P.C.T.]

Case-II : When $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are major arcs. In this case, $\overset{\frown}{QP}$ and $\overset{\frown}{SR}$ are minor arcs.

∴ $\overset{\frown}{PQ} \cong \overset{\frown}{RS} \Rightarrow \overset{\frown}{QP} \cong \overset{\frown}{SR}$

⇒ QP = SR (Case I) ⇒ PQ = RS

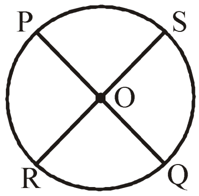
Hence, in both the cases, we get PQ = RS.

Theorem - 9

Statement (Converse of Theorem 8) : If two chords of a circle are equal, then their corresponding arcs (semi-circular, minor or major) are congruent.

Given : A circle with centre O in which chord PQ = chord RS

To Prove : $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$, where both $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are either semicircular, minor or major arcs.



Proof :

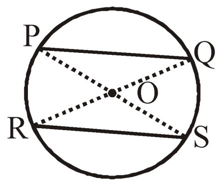
Case-I : When PQ and RS are diameters.

In this case, $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are semicircles with the same radii.

So, $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$. Thus, PQ = RS ⇒ $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$,

Case-II : When chord PQ = chord RS, where $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are minor arcs. Join OP, OQ, OR, OS.

In ΔPOQ and ΔROS, we have



PQ = RS	[Given]
OP = OR	[Radii of circle]
OQ = OS	[Radii of circle]
∴ ΔPOQ ≅ ΔROS	[By SSS congruence criteria]
So, ∠POQ = ∠ROS	[C.P.C.T.]

$\Rightarrow\ m(\widehat{PQ}) = m(\widehat{RS})\ \Rightarrow\ \widehat{PQ} \cong \widehat{RS}$

Case-III : When chord PQ = chord RS, where $\widehat{PQ}$ and $\widehat{RS}$ are major arcs.

In this case, $\widehat{QP}$ and $\widehat{SR}$ are minor arcs.

$\therefore\ PQ = RS\ \Rightarrow\ QP = SR$

$\Rightarrow\ \widehat{QP} \cong \widehat{SR} \Rightarrow m(\widehat{QP}) = m(\widehat{SR})\ \Rightarrow m(\widehat{PQ}) = m(\widehat{RS})\ \Rightarrow\ \widehat{PQ} \cong \widehat{RS}$

Hence, in all the cases, $PQ = RS\ \Rightarrow\ \widehat{PQ} \cong \widehat{RS}$

Theorem - 10

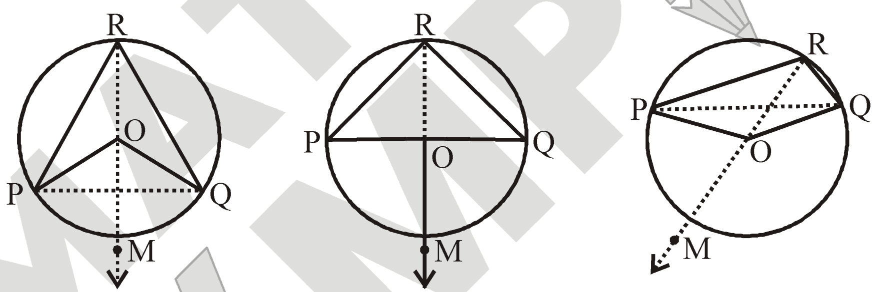
Statement : The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Given : An arc PQ of a circle with centre O and radius r and a point R on the remaining part of the circle *i.e.*, arc QP.

To Prove : $\angle POQ = 2\angle PRQ$

Construction : Join RO and produce it to a point M outside the circle.

Proof : We shall consider the following three different cases.



(i) (ii) (iii)

Case-I : When $\widehat{PQ}$ is a minor arc (see given figure)

We know that an exterior angle of a triangle is equal to the sum of the interior opposite angles.

In ΔPOR, $\angle POM$ is the exterior angle.

$\therefore\ \angle POM = \angle OPR + \angle ORP$

$\Rightarrow\ \angle POM = \angle ORP + \angle ORP$ [$OP = OR = r \Rightarrow \angle OPR = \angle ORP$]

$\Rightarrow\ \angle POM = 2\angle ORP$ .....(1)

In $\angle QOR$, $\angle QOM$ is the exterior angle.

$\therefore\ \angle QOM = \angle OQR + \angle ORQ$

$\Rightarrow\ \angle QOM = \angle ORQ + \angle ORQ$ [$OQ = OR = r\ \Rightarrow \angle ORQ = \angle OQR$]

$\Rightarrow\ \angle QOM = 2\angle ORQ$ .....(2)

Adding equations (1) and (2), we get

∠POM + ∠QOM = 2∠ORP + 2∠ORQ

⇒ ∠POM + ∠QOM = 2(∠ORP + ∠ORQ)

⇒ ∠POQ = 2∠PRQ ......(3)

Case-II : When PQ is a diameter (Proof is same as case I)

Case-III : PQ is a major arc so, equation (3) is replaced by

Reflex ∠POQ = 2∠PRQ.

Theorem - 11

Statement : Angles in the same segment of a circle are equal.

Given : A circle with centre O, an arc PQ and two angles ∠PRQ and ∠PSQ in the same segment of the circle.

To Prove : ∠PRQ = ∠PSQ.

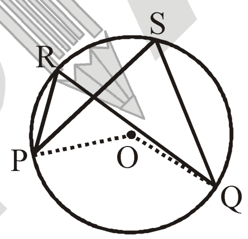
Construction : Join OP and OQ.

Proof : We know that the angle subtended by an arc at the centre is double the angle subtended by the arc at any point in the remaining part of the circle. So in the figure (i), we have

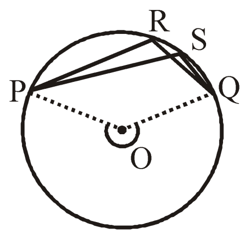
∠POQ = 2∠PRQ and ∠POQ = 2∠PSQ

⇒ 2∠PRQ = 2∠PSQ

⇒ ∠PRQ = ∠PSQ



(i)



(ii)

In the figure (ii), we have

Reflex ∠POQ = 2∠PRQ and Reflex ∠POQ = 2∠PSQ

⇒ 2∠PRQ = 2∠PSQ

⇒ ∠PRQ = ∠PSQ

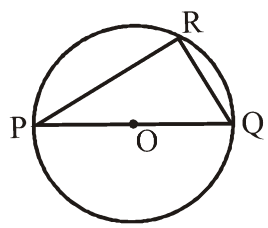
Thus, in both the cases, we have ∠PRQ = ∠PSQ

Theorem - 12

Statement : The angle in a semi-circle is a right angle.

Given : PQ is a diameter of a circle with centre O and ∠PRQ is an angle in semi-circle.

To Prove : ∠PRQ = 90°



Proof : We know that the angle subtended by an arc of a circle at its centre is twice the angle formed by same arc at a point on the circle. So, we have

∠POQ = 2∠PRQ

⇒ 180° = 2∠PRQ [POQ is a straight line]

⇒ ∠PRQ = 90°

Theorem - 13

Statement : If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the segment then the four points are concyclic, *i.e.*, lie on the same circle.

Given : AB is a line segment and C, D are two points lying on the same side of AB such that ∠ACB = ∠ADB.

To Prove : A, B, C, D lie on the same circle.

Construction : Draw the circle through three non-collinear points A, B and C.

Proof : If D lies on the circle passing through A, B and C then clearly the result follows.

If possible, suppose D does not lie on this circle.

Then, this circle will intersect AD [Fig. (i)] or AD produced in D′ [Fig. (ii)].

Join D′B.

Now, $\angle ACB = \angle ADB$ [Given]

and $\angle ACB = \angle AD'B$ [Angles in the same segment]

$\therefore \quad \angle ADB = \angle AD'B$

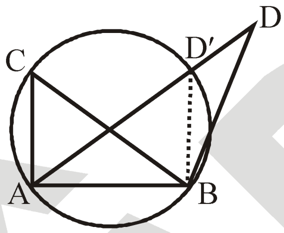


Fig. (i)

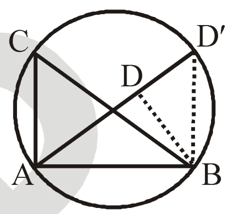


Fig. (ii)

But, an exterior angle of a triangle can never be equal to its interior opposite angle.

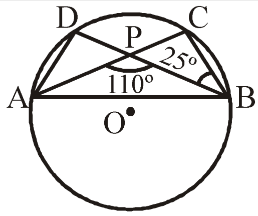
So, $\angle ADB = \angle AD'B$ is true only when D′ coincides with D.

Thus, D lies on the circle passing through A, B and C.

Hence, the points A, B, C, D are concylic.

Example 6

In the given figure, O is the centre of circle. Find the value of $\angle ADB$



Solution :

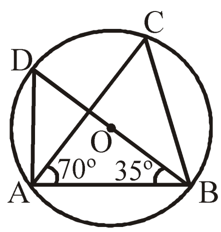
$\angle BPC = (180^\circ - 110^\circ) = 70^\circ$

$\angle ACB = \angle PCB = 180^\circ - (70^\circ + 25^\circ) = 85^\circ$

$\angle ADB = \angle ACB = 85^\circ$ [Angles in the same segment]

Example 7

In the given figure, O is the centre of the circle. Find $\angle ACB$.



Solution :

$\angle BAD = 90^\circ$ [Angle in semicircle]

$\therefore \quad \angle ADB = 180^\circ - (90^\circ + 35^\circ) = 55^\circ$

$\Rightarrow \quad \angle ACB = \angle ADB = 55^\circ$ [Angles in same segment]

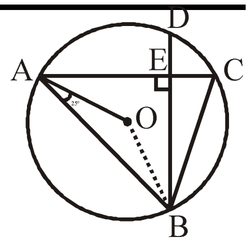
Example 8

In the given figure, chords AC and BD of a circle with centre O, intersect at right angles at E.

If $\angle OAB = 25^\circ$, find $\angle EBC$.

Solution :

Join OB, Now, OA = OB [Radii of circle]



$\Rightarrow$ $\angle OBA = \angle OAB = 25^\circ$

$\therefore$ $\angle OAB + \angle OBA + \angle AOB = 180^\circ$

$\Rightarrow$ $\angle AOB = 130^\circ$

$\therefore$ $\angle ACB = \frac{1}{2}\angle AOB = 65^\circ$

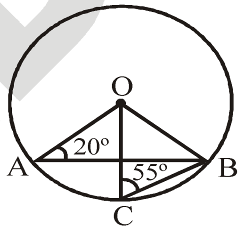
$\therefore$ $\angle ECB = 65^\circ$

Now, $\angle EBC + \angle BEC + \angle ECB = 180^\circ$

$\Rightarrow$ $\angle EBC + 90^\circ + 65^\circ = 180^\circ$ $\Rightarrow$ $\angle EBC = 25^\circ$

Example 9

In the given figure, O is the centre of the circle. Find (i) $\angle BOC$ (ii) $\angle AOC$



Solution :

$OB = OC \Rightarrow \angle OBC = \angle OCB = 55^\circ$.

$\therefore$ $\angle BOC = 180^\circ - (\angle OCB + \angle OBC) = [180^\circ - (55^\circ + 55^\circ)] = 70^\circ$

$OA = OB \Rightarrow \angle OBA = \angle OAB = 20^\circ$

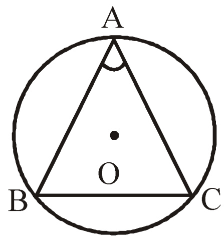
$\Rightarrow$ $\angle AOB = 180^\circ - (\angle OAB + \angle OBA)$

$\Rightarrow$ $\angle AOB = 180^\circ - (20^\circ + 20^\circ) = 140^\circ$

$\therefore$ $\angle AOC = \angle AOB - \angle BOC = (140^\circ - 70^\circ) = 70^\circ$

Example 10

In the given figure, $\angle BAC = 30^\circ$. Show that BC is equal to the radius of the circumcircle of ΔABC whose centre is O.



Solution :

Join OB and OC.

$\angle BOC = 2\angle BAC = 2 \times 30^\circ = 60^\circ$

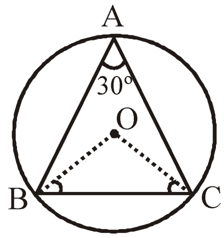
$OB = OC \Rightarrow \angle OBC = \angle OCB = x$ (say)

In ΔOBC, we have $\angle BOC + \angle OBC + \angle OCB = 180^\circ$

$\Rightarrow$ $60^\circ + 2x = 180^\circ$

$\Rightarrow$ $x = 60^\circ$

Thus, each angle of ΔBOC is 60°.



$\therefore$ ΔBOC is an equilateral triangle.

$\therefore$ $OB = OC = BC$.

Hence, BC is the radius of the circumcircle

MATRIX OLYMPIAD

7. CYCLIC QUADRILATERALS

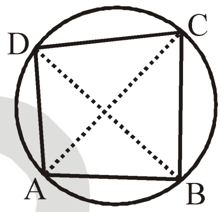
The quadrilateral obtained by joining any four points on the circle is called cyclic quadrilateral.

Theorem - 14

Statement : The sum of either pair of the opposite angles of a cyclic quadrilateral is 180º.

or

The opposite angles of a cyclic quadrilateral are supplementary.



Given : A cyclic quadrilateral ABCD.

To Prove : $\angle A + \angle C = 180^\circ$ and $\angle B + \angle C = 180^\circ$

Construction : Join AC and BD.

Proof : We have

$\angle ACB = \angle ADB$ ...(1) [Angles in the same segment]

$\angle BAC = \angle BDC$ ...(2) [Angles in the same segment]

Adding eqautions (1) and (2), we get

$\angle ACB + \angle BAC = \angle ADB + \angle BDC$

$\Rightarrow \angle ACB + \angle BAC = \angle ADC$

$\Rightarrow \angle ACB + \angle BAC + \angle ABC = \angle ADC + \angle ABC$ [Adding $\angle ABC$ on both sides]

$\Rightarrow \angle ADC + \angle ABC = 180^\circ$

$\Rightarrow \angle B + \angle D = 180^\circ$ ...(3)

Now, $\angle A + \angle B + \angle C + \angle D = 360^\circ$ [Sum of the angles of a quadrilateral is 360º]

$\Rightarrow \angle A + \angle C = 360^\circ - (\angle B + \angle D) = (360^\circ - 180^\circ) = 180^\circ$ [Using eqaution (3)]

Hence, $\angle A + \angle C = 180^\circ$ and $\angle B + \angle D = 180^\circ$

Theorem - 15

Statement (Converse of Theorem 14) : If a pair of opposite angles of quadrilateral is supplementary then the quadrilateral is cyclic.

Given : A quadrilateral ABCD in which $\angle B + \angle D = 180^\circ$.

To Prove : ABCD is a cyclic quadrilateral.

Construction : If possible, let ABCD be not a cyclic quadrilateral. Draw a circle, passing through three non-collinear ponts A, B, C. Let this circle intersect CD (Fig. (i)) or CD produced in D′ (Fig (ii)). Join D′ A.

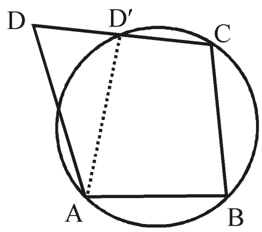


Fig. (i)

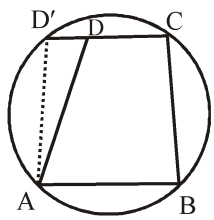


Fig. (ii)

Proof : $\angle ABC + \angle ADC = 180^\circ$ [Given]

$\angle ABC + \angle AD'C = 180^\circ$ [Opposite angles of a cyclic quadrilateral]

$\therefore$ $\angle ABC + \angle ADC = \angle ABC + \angle AD'C$

$\Rightarrow$ $\angle ADC = \angle AD'C$

This is not possible, since an exterior angle of a triangle can never be equal to its interior opposite angle.

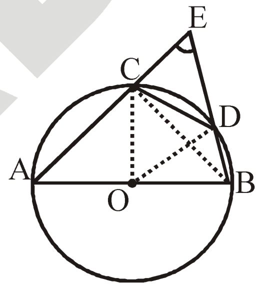
So, $\angle ADC = \angle AD'C$ is possible only when D′ coincides with D.

Hence, the circle passing through A, B, C must pass through D also.

Hence, ABCD is a cyclic quadrilateral.

Example 11

In the given figure, AB is the diameter of the circle, CD is the chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Prove that $\angle AEB = 60^\circ$.



Solution :

Join OC, OD and BC

Now, OC = OD = CD

$\therefore$ ΔODC is equilateral

$\Rightarrow$ $\angle COD = 60^\circ$

$\because$ Angle subtended by an arc at centre is double the angle subtended by it at any point on remaining part of the circle.

$\therefore$ $\angle CBD = \frac{1}{2}\angle COD = 30^\circ$

Again, $\angle ACB = 90^\circ$ [Angle in semi-circle]

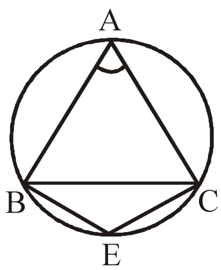
So, $\angle BCE = 180^\circ - \angle ACB = 90^\circ$

In ΔCBE, $\angle CEB = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

i.e. $\angle AEB = 60^\circ$

Example 12

In the given figure, ΔABC is equilateral triangle. Find $\angle BEC$.



Solution :

Since ΔABC is equilateral

$\therefore \quad \angle A = \angle B = \angle C = 60^\circ$

Now, ABEC is a cyclic quadrilateral.

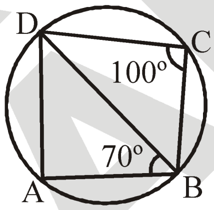
$\therefore \quad \angle A + \angle E = 180^\circ$

$\Rightarrow \quad \angle E = 120^\circ$

$\Rightarrow \quad \angle BEC = 120^\circ \qquad [\angle A = 60^\circ]$

Example 13

In the given figure, ABCD is a cyclic quadrilateral. Find $\angle ADB$.



Solution :

We have, $\angle A + \angle C = 180^\circ$

$\Rightarrow \quad \angle A = 80^\circ \qquad [\angle C = 100^\circ]$

In ΔADB, we have

$\angle A = 80^\circ, \angle ABD = 70^\circ$

$\therefore \quad \angle ADB = 180^\circ - (80^\circ + 70^\circ) = 30^\circ$

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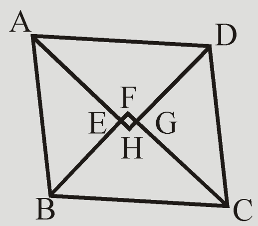
BUILD THE CONCEPT

- The quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral cyclic is a quadrilateral.



In the given figure, ABCD is a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles A, B, C and D respectively form a quadrilateral EFGH.

Now, $\angle FEH = \angle AEB = 180^\circ - \angle EAB - \angle EBA$

$$= 180^\circ - \frac{1}{2}(\angle A + \angle B)$$

and $\angle FGH = \angle CGD = 180^\circ - \angle GCD - \angle GDC$

$$= 180^\circ - \frac{1}{2}(\angle C + \angle D)$$

Therefore, $\angle FEH + \angle FGH$

$$= 180^\circ - \frac{1}{2}(\angle A + \angle B) + 180^\circ - \frac{1}{2}(\angle C + \angle D)$$

$$= 360^\circ - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$$

$$= 360^\circ - \frac{1}{2} \times 360^\circ$$

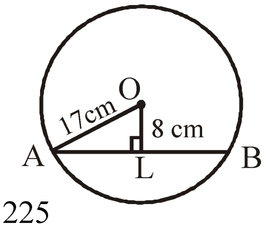
Hence, EFGH is a cyclic quadrilateral.

SE. 1

Find the length of a chord which is at a distance of 8 cm from the centre of the circle of radius 17 cm.

Ans. Let AB be the chord of circle with centre O and radius 17 cm.

Draw OL ⊥ AB. Join OA. Then, OL = 8 cm and OA = 17 cm



From ΔOLA, we have

$OA^2 = OL^2 + AL^2$

$\Rightarrow \quad AL^2 = (OA^2 - OL^2)$

$= [(17)^2 - (8)^2] = 225$

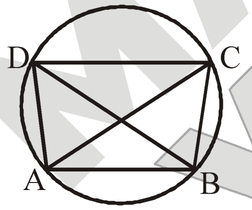
$\Rightarrow \quad AL = \sqrt{225}\text{ cm} = 15\text{ cm}$

Since, the perpendicular from the centre of a circle to a chord bisects the chord, we have

AB = 2 × AL = (2 × 15) cm = 30 cm.

SE. 2

If two sides of a cyclic quadrilateral are parallel, prove that the remaining two sides are equal and the diagonals are also equal.



Ans. **Given :** A cyclic quadrilateral ABCD in which AB || DC.

To Prove : (i) AD = BC (ii) AC = BD

Proof : In order to prove the desired resultes, it is sufficient to show that ΔADC ≅ ΔBCD. Since ABCD is a cyclic quadrilateral and sum of opposite pairs of anlges in a cyclic quadrilateral is 180º.

$\therefore \quad \angle B + \angle D = 180^\circ$ ....(i)

Since AB || DC and BC is a transversal and sum of the interior angles on the same side of a transversal is 180º

$\therefore \quad \angle ABC + \angle BCD = 180^\circ$ .....(ii)

$\Rightarrow \quad \angle B + \angle C = 180^\circ$

From (i) and (ii), we get

$\angle B + \angle D = \angle B + \angle C$

$\Rightarrow \quad \angle C = \angle D$ ....(iii)

In ΔADC and ΔBCD, we have

∠ADC = ∠BCD [From (iii)]

DC = DC [Common]

and ∠DAC = ∠CBD [Angles in same segment]

So, by AAS-criterion of congruence, we have

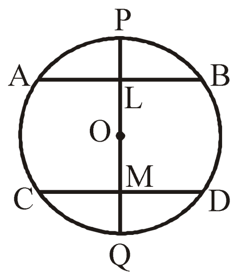
ΔADC ≅ ΔBCD

⇒ AD = BC and AC = BD

SE. 3

If a diameter of a circle bisects each of the two chords of a circle, prove that the chords are parallel.

Ans. Let AB and CD be two chords of the circle whose centre is O and let PQ be the diameter bisecting chords AB and CD at L and M respectively. Since PQ is the diameter. So, it passes through the centre O of the circle.



Now, L is the midpoint of AB

⇒ OL ⊥ AB

[The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord]

$\Rightarrow \quad \angle ALO = 90^\circ$

Similarly, $\angle CMO = 90^\circ \Rightarrow \angle DMO = 90^\circ$

$\therefore \quad \angle ALO = \angle DMO$

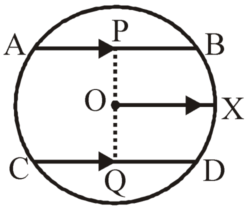
But, these are alternate interior angles.

So, AB || CD

SE. 4

Prove that the line joining the midpoints of two parallel chords of a circle passes through the centre.

Ans. Let AB and CD be two parallel chords having P and Q as their mid-points respectively. Let O be the centre of the circle. Join OP and OQ and draw OX || AB or CD. Now, P is the midpoint of AB.



$\Rightarrow$ OP $\perp$ AB $\Rightarrow$ $\angle BPO = 90^\circ$

But, OX || AB $\therefore$ $\angle POX + \angle BPO = 180^\circ$

[Sum of interior angles on the same side of transversal]

$\Rightarrow$ $\angle POX = 90^\circ$

Similarly, Q is the midpoint of CD.

$\Rightarrow$ OQ $\perp$ CD

$\Rightarrow$ $\angle DQO = 90^\circ$

But, OX || CD

$\therefore$ $\angle XOQ + \angle DQO = 180^\circ$

[Sum of interior angles on the same side of transversal]

$\Rightarrow$ $\angle XOQ = 90^\circ$

$\Rightarrow$ $\angle POX + \angle XOQ = 90^\circ + 90^\circ = 180^\circ$

$\Rightarrow$ POQ is a straight line.

Hence, PQ is a straight line passing through the centre of the circle.

SE. 5

Prove that the perpendicular bisector of the chords of a circle always passes throught its centre.

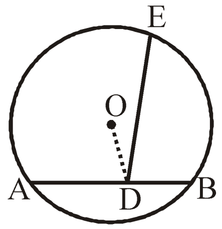
Ans. Let DE be the perpendicular bisector of the given chords AB of a circle with center O

Then, AD = DB and $\angle ADE = 90^\circ$. ....(1)

Now, we have to show that DE passes through O.

If possible, suppose DE does not pass through O.

Join OD.



We know that the line joining the centre of a circle to the midpoint of a chord is always perpendicular to the chord.

$\therefore$ OD $\perp$ AB

$\Rightarrow$ $\angle ADO = 90^\circ$ ....(2)

From (1) and (2), we get $\angle ADE = \angle ADO$.

This is a contradiction, since $\angle ADO$ is a part of $\angle ADE$.

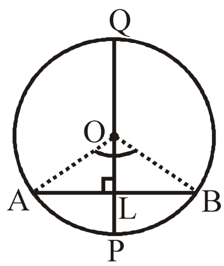
The contradiction arises by assuming that DE does not pass through O.

Hence, perpendicular bisector of the chord of a circle always passes through its centre.

SE. 6

Prove that the right bisector of a chord of a circle bisects the corresponding minor arc of the circle.

Ans. Let AB be the chord of circle with centre O. Let PQ be the perpendicular bisector of the chord AB, intersecting it at L and the circle at P and Q. Since the right bisector of a chord always passes through the centre of the circle, PQ must pass through O.



Join OA and OB.

In ΔOLA and ΔOLB, we have

$OA = OB$ [Radii of circle]

$\angle OLA = \angle OLB$ [Each equal to 90°]

$OL = OL$ [Common]

$\therefore\ \Delta OLA \cong \Delta OLB$ [By RHS congruence criteria]

$\Rightarrow\ \angle AOL = \angle BOL$ [C.P.C.T]

$\Rightarrow\ \angle AOP = \angle BOP$

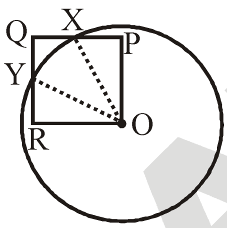
$\Rightarrow\ AP = BP \ \Rightarrow\ m(\widehat{AP}) = m(\widehat{BP})$

$\Rightarrow\ \widehat{AP} \cong \widehat{BP}$

Hence, the right bisector PQ of a chord of a circle bisects the minor arc AB.

SE. 7

In the given figure, OPQR is square. A circle drawn with centre O cuts the square at X and Y. Prove that QX = QY.



Ans. In ΔOXP and ΔOYR, we have

$OP = OR$ [Sides of square]

$OX = OY$ [Radii of circle]

and $\angle OPX = \angle ORY = 90°$

$\Rightarrow \Delta OXP \cong \Delta OYR$ [By RHC congruence criteria]

$\Rightarrow\ PX = RY$ [C.P.C.T.]

$\Rightarrow\ PQ - QX = QR - QY$

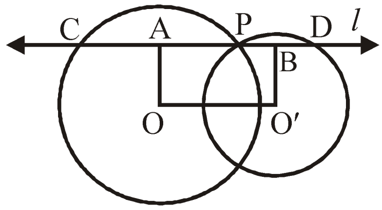
$\Rightarrow\ PQ - QX = PQ - QY$ [$\because$ PQ = QR]

$\Rightarrow\ -QX = -QY \ \Rightarrow\ QX = QY$

SE. 8

Two circles whose centres are O and O′ intersect at P. Through P, a line *l* parallel to OO′ intesecting the circles at C and D is drawn. Prove that CD = 2OO′.

Ans. Draw $OA \perp l$ and $OB \perp l$.



Now, $OA \perp l$

$\Rightarrow\ OA \perp CP$

$\Rightarrow\ CA = AP$

$\Rightarrow\ CP = 2AP$

and $O'B \perp l$ .....(i)

$\Rightarrow\ O'B \perp DP$

$\Rightarrow\ BP = BD$

$\Rightarrow\ PD = 2PB$

$\therefore\ CD = CP + PD$

$\Rightarrow\ CD = 2AP + 2PB$

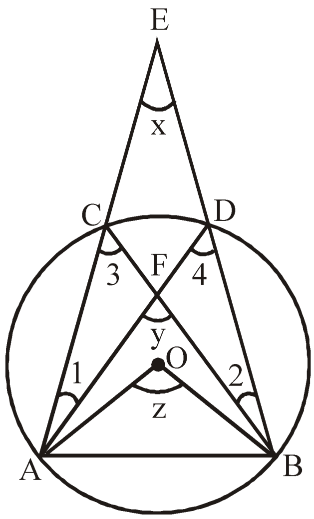
[Using equations (i) and (ii)]

$\Rightarrow\ CD = 2(AP + PB) = 2AB = 2OO'$

[ABO′O is a rectangle]

SE. 9

In the given figure, O is the centre of a circle. Prove that $\angle x + \angle y = \angle z$.



Ans. In ΔACF, side CF is produced to B.

$\therefore\ \angle Y = \angle 1 + \angle 3$ ....(1)

[Exterior angle is sum of interior opposite angles]

In ΔAED, side ED is produced to B.

$\therefore \quad \angle 1 + \angle x = \angle 4$ .....(2)

Adding (1) and (2), we get

$\angle 1 + \angle x + \angle y = \angle 1 + \angle 3 + \angle 4$

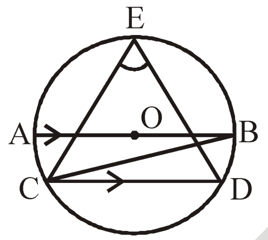
$\Rightarrow \quad \angle x + \angle y = \angle 3 + \angle 4 = 2\angle 3$

[$\angle 4 = \angle 3$ as angles in the same segment are equal]

$\Rightarrow \quad \angle x + \angle y = \angle z$ [$\because \angle AOB = 2\angle ACB$]

SE. 10

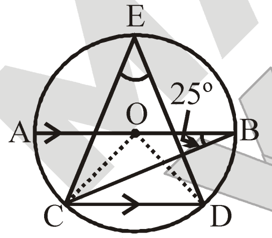
In the given figure, O is the centre of the circle. Chord CD is parallel to diameter AB. If $\angle ABC = 25^\circ$, calculate $\angle CED$.



Ans. Join CO and DO.

$\angle BCD = \angle ABC = 25^\circ$ [Alternate Interior angles]

Also, $\angle BOD = 2\angle BCD = 50^\circ$



[Arc BD makes $\angle BOD$ at the centre and $\angle BCD$ at a point on the circle]

Similarly, $\angle AOC = 2\angle ABC = 50^\circ$

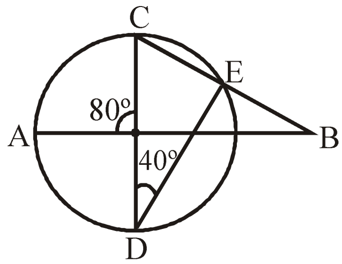
Now, $\angle AOC + \angle COD + \angle BOD = 180^\circ$

$\Rightarrow \quad \angle COD = 80^\circ \quad \Rightarrow \angle CED = \frac{1}{2}\angle COD = 40^\circ$

SE. 11

In the given figure, AB and CD are straight lines through the centre O of the circle. Find

(i) $\angle DCE$ (ii) $\angle ABC$



Ans. $\angle CED = 90^\circ$ [Angle in a semi-circle]

$\therefore \quad \angle DCE = 180^\circ - (40^\circ + 90^\circ) = 50^\circ$

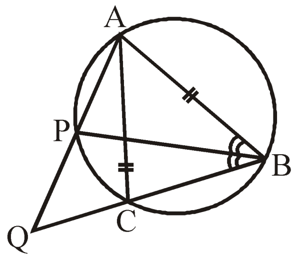
$\angle BOC = (180^\circ - 80^\circ) = 100$

$\therefore \quad \angle OBC = 180^\circ - (\angle OCB + \angle BOC)$

$= 180^\circ - (100^\circ + 50^\circ) = 30^\circ = \angle ABC$

SE. 12

The bisector of $\angle B$ of an isosceles triangle ABC with AB = AC meets the circumcircle of ΔABC at P as shown in the figure. If AP and BC are produced to meet at Q, prove that CQ = CA.



Ans. In ΔAQC, we have

$\angle ACB = \angle AQC + \angle QAC$

[Exterior angle is equal to the sum of interior opposite angles]

$\Rightarrow \quad \angle ABC = \angle AQC + \angle QAC$

[$AB = AC \therefore \angle ACB = \angle ABC$]

$\Rightarrow \quad 2\angle PBC = \angle AQC + \angle PBC$

[BP is the bisector of $\angle B \therefore \angle ABC = 2\angle PBC$]

$\Rightarrow \angle PBC = \angle AQC \Rightarrow \angle PAC = \angle AQC$

[$\angle PBC = \angle PAC$ (Angles in the same segment)]

$\Rightarrow \quad \angle QAC = \angle AQC \quad \Rightarrow \quad CQ = CA$

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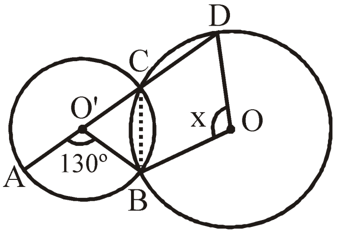
SE. 13

In the given figure, O and O′ are centres of two circles intersecting at B and C. If ACD is a straight line, find x.



Ans. We have, $\angle ACB = \frac{1}{2}\angle AO'B = 65°$

[Central angle property]

Now, $\angle DCB + \angle ACB = 180°$

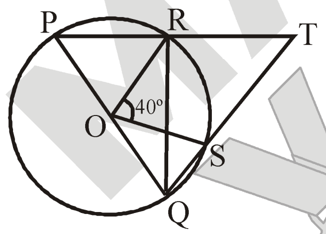
$\Rightarrow \angle DCB = 180° - 65° = 115°$

Also, reflex $\angle BOD = 2\angle BCD$

$\Rightarrow 360° - x = 2 \times 115° \Rightarrow x = 130°$

SE. 14

In the given figure, O is the centre of circle and PQ is the diameter. If $\angle ROS = 40°$, find $\angle RTS$.



PQ is diameter

$\Rightarrow \angle PRQ = 90°$ [Angle in semi circle]

$\Rightarrow \angle TRQ = 90°$ [Linear pair]

We have, $\angle RQS = \frac{1}{2}\angle ROS = 20°$

In right triangle TRQ, we have

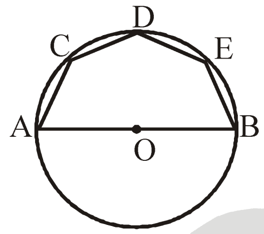
$\angle QRT + \angle RQS + \angle RTQ = 180°$

$\Rightarrow 90° + 20° + \angle RTQ = 180°$

$\Rightarrow \angle RTQ = 70° \Rightarrow \angle RTS = 70°$

SE. 15

In the given figure, AOB is the diameter of circle with centre O. Find the numerical value of $\angle ACD + \angle DEB$.



Ans. Join BC.

Then, $\angle ACB = 90°$ ....(i) [Angle in semicircle]

Now, DCBE is a cyclic quadrilateral.

$\therefore \angle BCD + \angle DEB = 180°$ ....(ii)

[Opposite angles of cyclic quadrilateral are supplementary]

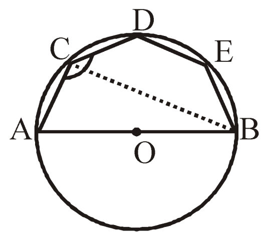
Adding equtions (i) and (ii), we get

$\angle ACB + \angle BCD + \angle DEB = 90° + 180°$

$\Rightarrow \angle ACD + \angle DEB = 270°$

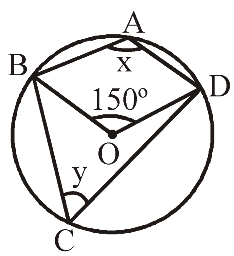
[$\angle ACB + \angle BCD = \angle ACD$]

$\therefore$ The numerical value of $(\angle ACD + \angle DEB)$ is 270°



SE. 16

In the given figure,O is the cenre of circle. Find the value of x and y.



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Ans. Reflex ∠BOD = (360° – 150°) = 210°

$x = \frac{1}{2}$ reflex ∠BOD = 105°

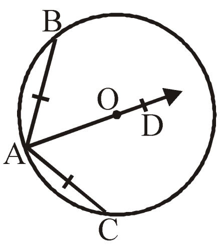
Now, x + y = 180°

[ABCD is a cyclic quadrilateral]

⇒ y = 75°

SE. 17

Two chords AB and AC of a circle are equal. Prove that the centre of the circle lies on the angle bisector of ∠BAC.



Ans. Join BC, meeting AD at M.

In ΔBAM and ΔCAM, we have

AB = AC [Given]

∠BAM = ∠CAM [AM is bisector of ∠BAC]

AM = AM [Common]

∴ ΔBAM ≅ ΔCAM [By SAS congruence criteria]

⇒ BM = CM and ∠BMA = ∠CMA [C.P.C.T]

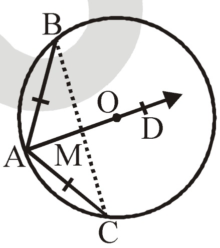
⇒ BM = CM and ∠BMA = ∠CMA = 90°

[∠BMA + ∠CMA = 180° and ∠BMA = ∠CMA]

⇒ ∠BMA = ∠CMA = 90°

⇒ AM is the perpendicular bisector of chord BC.

⇒ AD is the prependicular bisector of the chord BC.

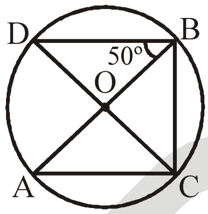


As, the perpendicular bisector of a chord always passes through the centre of the circle.

∴ AD passes through the centre O of the circle *i.e.* O lies on AD.

SE. 18

AB and CD are diameters of the circle with centre O. Find ∠AOC.



Ans. Clearly, arc AD subtends ∠ABD = 50° at B and ∠AOD at the centre.

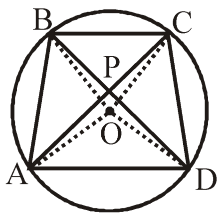
∴ ∠AOD = 2∠ABD = 100°

Since, CD is a straight line

∴ ∠AOC = 80°

SE. 19

ABCD is a cyclic quadrilateral whose diagonals AC and BD intersect at P. If AB = DC, then show that ΔPAB ≅ ΔPDC



Ans. In triangles PAB and PDC, we have

∠ABD = ∠ACD [Angles in the same segment]

∴ ∠ABP = ∠DCP

∠BAP = ∠CDP

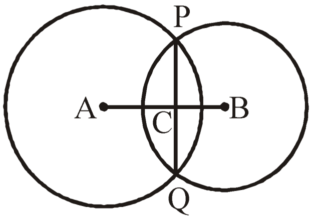
[∠BAC = ∠BDC, Angle in the same segment]

and AB = CD [Given]

∴ ΔPAB ≅ ΔPDC [By ASA congruence criteria]

SE. 20

In the given figure, A and B are the centres of two circles having radii 5 cm and 3 cm respectively and intersecting at points P and Q. If PQ = 4 cm, then find the length of AB.



Ans. We know that the line joining the centres of two intersecting circles is the perpendicular bisector of the common chord. Join AP and BP.

Then, AP = 5 cm, BP = 3 cm and PQ = 4

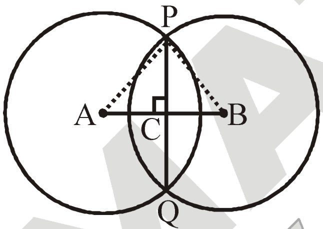
And, $PC = \frac{1}{2} PQ = \frac{1}{2} \times 4 = 2$ cm

In right triangle ΔPAC, we have

$AP^2 = PC^2 + AC^2$

$\Rightarrow \quad 5^2 = 2^2 + AC^2 \qquad \Rightarrow AC^2 = 25 - 4 = 21$

$\Rightarrow \quad AC = \sqrt{21}$ cm



Now, in right triangle ΔPBC, we have

$PB^2 = PC^2 + BC^2$

$\Rightarrow \quad 3^2 = 2^2 + BC^2 \qquad \Rightarrow BC^2 = 9 - 4 = 5$

$\Rightarrow \quad BC = \sqrt{5}$ cm

$\therefore \quad AB = AC + BC = (\sqrt{21} + \sqrt{5})$ cm

Space for Notes :

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NCERT SOLUTIONS

EXERCISE - 10.1

NS. 1

Fill in the blanks :

(i) The centre of a circle lies in______of the circle. (exterior/interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in____of the circle. (exterior/interior)

(iii) The longest chord of a circle is a_____of the circle

(iv) An arc is a_____when its ends are the ends of a diameter

(v) Segment of a circle is the region between an arc and ____of the circle.

(vi) A circle divides the plane, on which it lies, in_____parts.

Ans. (i) interior (ii) exterior

(iii) diameter (iv) semicircle

(v) the chord (vi) three

NS. 2

Write True or False. Give reasons for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divides into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter or the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure

Ans. (i) True [All points on the circle are equidistant from the centre]

(ii) False [A circle can have an infinite number of equal chords]

(iii) False [Each part will be less than a semicircle]

(iv) True [Diameter = 2 × Radius]

(v) False [The region between the chord and its corresponding arc is a segment]

(vi) True [A circle is drawn on a plane]

EXERCISE - 10.2

NS. 1

Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Ans. **Given :** Two congruent circles with centres O and O′ and radii r which have chords AB and CD respectively such that AB = CD.

To Prove : $\angle AOB = \angle CO'D$

Proof : In ΔAOB and $\Delta CO'D$, we have

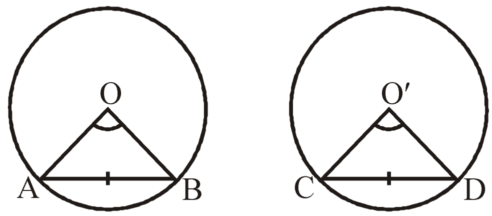
AB = CD [Given]

OA = O′C [Each equal to r]

OB = O′B [Each equal to r]

$\therefore$ $\Delta AOB \cong \Delta CO'D$ [By SSS congruence criteria]

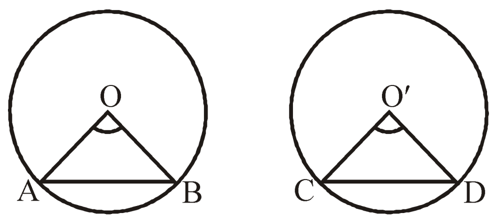
$\Rightarrow$ $\angle AOB = \angle CO'D$ [C.P.C.T.]



NS. 2

Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Ans. **Given :** Two congruent circles with centres O & O′ and radii r which have chords AB and CD respectively such that $\angle AOB = \angle CO'D$.



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To Prove : AB = CD

Proof : In ΔAOB and ΔCO′D, we have

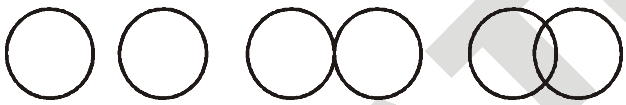
OA = O′C	[Each equal to r]
OB = O′D	[Each equal to r]
∠AOB = ∠CO′D	[Given]
∴ ΔAOB ≅ ΔCO′[D	[By SAS congruence criterion]
Hence, AB = CD	[C.P.C.T.]

EXERCISE - 10.3

NS. 1

Draw different pairs of circles. How many points does each pair have in common ? What is the maximum number of common points.

Ans. Let us draw different pairs of circles as shown below.



(i) (ii) (iii)

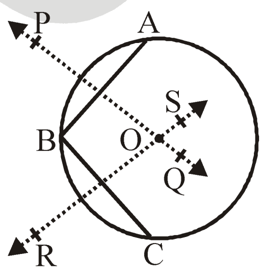
We have

In figure	Maximum number of common points
(i)	nil
(ii)	one
(iii)	two

Thus, two circles can have at the most two points in common.

NS. 2

Suppose you are given a circle. Give a construction to find its centre.



Ans. Steps of construction :

I. Take any three points on the given circle. Let these points be A, B and C.

II. Join AB and BC.

III. Draw the perpendicular bisector PQ of AB.

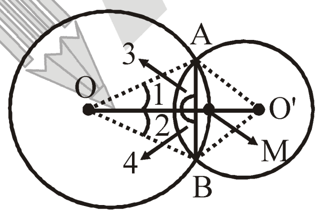
IV. Draw the perpendicular bisector RS of BC such that it intersects PQ at O.

Thus, 'O' is the required centre of the given circle.

NS. 3

If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Ans. We have two circles with centres O and O′, intersecting at A and B.



∴ AB is the common chord of two circles and OO′ is the line segment joining their centres. Let OO′ and AB intersect each other at M.

∴ To prove that OO′ is the perpendicular bisector of AB, we join OA, OB, O′A and O'B.

Now, in ΔOAO′ and ΔOBO′, we have

OA = OB	[Radii of the same circle]
O′A = O′B	[Radii of the same circle]
OO′ = OO′	[Common]

∴ ΔOAO′ ≅ ΔOBO′ [By SSS congruence criteria]

⇒ ∠1 = ∠2 [C.P.C.T.]

Now, in ΔAOM and ΔBOM, we have

OA = OB	[Radii of the same circle]
OM = OM	[Common]
∠1 = ∠2	[Proved above]
∴ ΔAOM ≅ ΔBOM	[SAS criterion]
⇒ ∠3 = ∠4	[C.P.C.T.]

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But $\angle 3 + \angle 4 = 180°$ [Linear pair]

$\therefore$ $\angle 3 = \angle 4 = 90°$ $\Rightarrow$ $AM \perp OO'$

Also AM = BM [C.P.C.T.]

$\Rightarrow$ M is the midpoint of the AB.

Thus, OO′ is the perpendicular bisecrtor of AB.

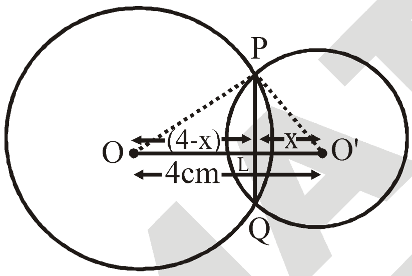
EXERCISE - 10.4

NS. 1

Two circles of radi 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Ans. We have two intersecting circles with centres at O and O′ respectively. Let PQ be the common chord.

$\because$ In two intersecting circles, the line joining their centres is perpendicular bisector of the common chord.



$\therefore$ $\angle OLP = \angle OLQ = 90°$ and PL = LQ

Now, in right ΔOLP, we have

$PL^2 + OL^2 = OP^2$ $\Rightarrow$ $PL^2 + (4 - x)^2 = 5^2$

$\Rightarrow$ $PL^2 = 5^2 - (4 - x)^2$

$\Rightarrow$ $PL^2 = 25 - 16 - x^2 + 8x$

$\Rightarrow$ $PL^2 = 9 - x^2 + 8x$ ....(1)

Again, in $\Delta O'LP$,

$PL^2 = PO'^2 - LO'^2 = 3^2 - x^2 = 9 - x^2$ .....(2)

From equations (1) and (2), we have

$9 - x^2 + 8x = 9 - x^2$

$\Rightarrow$ $8x = 0$ $\Rightarrow$ $x = 0$

$\Rightarrow$ L and O′ coincide.

$\therefore$ PQ is a diameter of the smaller circle.

$\Rightarrow$ PL = 3 cm

But PL = LQ $\therefore$ LQ = 3 cm

$\therefore$ PQ = PL + LQ = 3 cm + 3 cm = 6 cm

$\therefore$ Length of the common chord = 6 cm.

NS. 2

If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chords.

Ans. **Given :** A circle with centre O. Equal chords AB and CD intersect at E.

To Prove : AE = DE and CE = BE,

Construction : Draw $OM \perp AB$ and $ON \perp CD$. Join OE

Proof : Since AB = CD [Given]

$\therefore$ OM = ON [Equal chords are equidistant from the centre]

Now, in ΔOME and ΔONE, we have

$\angle OME = \angle ONE$ [Each equal to 90°]

OM = ON [Proved above]

OE = OE [Common]

$\therefore$ $\Delta OME \cong \Delta ONE$ [By RHS congruence criteria]

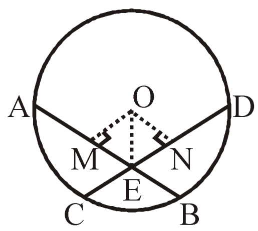
$\Rightarrow$ ME = NE [C.P.C.T.]

Adding AM both sides, we get

AM + ME = AM + NE

$\Rightarrow$ AE = DN + NE = DE

$[AB = CD \Rightarrow \frac{1}{2}AB = \frac{1}{2}DC \Rightarrow AM = DN]$



$\Rightarrow$ AE = DE ....(1)

$\Rightarrow$ AB – BE = CD – CE

$\Rightarrow$ BE = CE [AB = CD] ....(2)

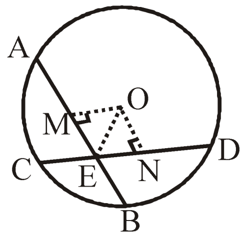
From equations (1) and (2), we have

AE = DE and CE = BE

NS. 3

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersecting to the centre makes equal angles with the chords.

Ans. **Given :** A circle with centre O and equal chords AB and CD are intersecting at E.



To Prove : $\angle OEA = \angle OED$

Construction : Draw $OM \perp AB$ and $ON \perp CD$.

Proof : In right ΔOME and right ΔONE, $OM = ON$

[Equal chords are equidistant from the centre]

$OE = OE$ [Common]

$\angle OME = \angle ONE$ [Each equal to 90°]

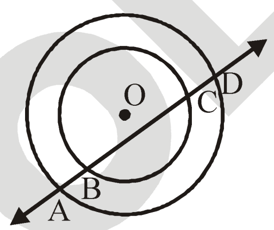
$\therefore$ $\Delta OME \cong \Delta ONE$ [By RHS congruence]

$\therefore$ $\angle OEM = \angle OEN$ [C.P.C.T.]

$\Rightarrow$ $\angle OEA = \angle OED$

NS. 4

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure).



Ans. **Given :** Two circles with the common centre O. A line '*l*' intersects the outer circle at A and D and the inner circle at B and C.

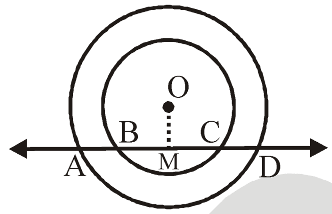
To Prove : AB = CD.

Construction : Draw $OM \perp l$.

Proof : For the outer circl, $OM \perp l$

$\therefore$ $AM = MD$ ....(1)

[Perpendicular from the centre to the chord bisects the chord]



For the inner circle, $OM \perp l$

$\therefore$ $BM = MC$ .....(2)

[Perpendicular from the centre to the chord bisects the chord]

Subtracting equation (2) from equation (1), we have

$AM - BM = MD - MC$

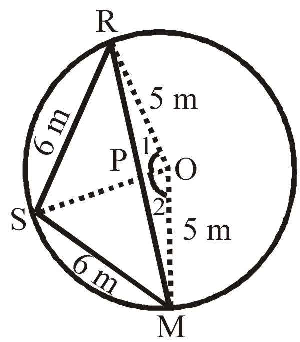
$\Rightarrow$ $AB = CD$

NS. 5

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip.

Ans. Let the three girls Reshma, Salma and Mandip be positioned at R, S and M respectively on the circle of radius 5 cm.

RS = SM = 6 m [Given]



$\because$ Equal chords of a circle subtend equal angles at the centre

$\therefore \quad \angle 1 = \angle 2 \qquad$ ....(1)

In ΔPOR and ΔPOM, we have

$OP = OP \qquad$ [Common]

$OR = OM \qquad$ [Radii of the same circle]

$\angle 1 = \angle 2 \qquad$ [By (1)]

$\therefore \quad \Delta POR \cong \Delta POM$, [By SAS congruence criteria]

$\therefore \quad PR = PM$ and $\angle OPR = \angle OPM \quad$ [C.P.C.T.]

$\because \quad \angle OPR + \angle OPM = 180^\circ \quad$ [Linear pair]

$\therefore \quad \angle OPR = \angle OPM = 90^\circ$

$\Rightarrow \quad OP \perp RM$

Now, in ΔRSP and ΔMSP, we have

$RS = MS \qquad$ [6 m each]

$SP = SP \qquad$ [Common]

$PR = PM \qquad$ [Proved above]

$\therefore \quad \Delta RSP \cong \Delta MSP \quad$ [By SSS congruence criteria]

$\Rightarrow \quad \angle RPS = \angle MPS \quad$ [C.P.C.T.]

But $\angle RPS + \angle MPS = 180^\circ$

$\Rightarrow \quad \angle RPS = \angle MPS = 90^\circ$

$\therefore$ SP passes through O.

Let $OP = x$ m $\quad \therefore \; SP = (5 - x)$m

Now, in right ΔOPR, we have

$x^2 + RP^2 = 5^2 \quad \Rightarrow \; RP^2 = 5^2 - x^2 \qquad$ .....(2)

In right ΔSPR, we have

$(5 - x)^2 + RP^2 = 6^2$

$\Rightarrow \quad RP^2 = 6^2 - (5 - x)^2 \qquad$ .....(3)

From (1) and (2), we have

$5^2 - x^2 = 6^2 - (5 - x)^2$

$\Rightarrow \quad 25 - x^2 = 36 - [25 - 10x + x^2]$

$\Rightarrow \quad -10x + 14 = 0 \quad \Rightarrow \; 10x = 14$

$\Rightarrow \quad x = \frac{14}{10} = 1.4$

Now, $RP^2 = 5^2 - x^2 \quad \Rightarrow \; RP^2 = 25 - (1.4)^2$

$\Rightarrow \quad RP^2 = 25 - 1.96 = 23.04$ m

$\therefore \quad RP = \sqrt{23.04} = 4.8$ m

$\therefore \quad RM = 2RP = 2 \times 4.8 \text{ m} = 9.6$ m

Thus, distance between Reshma and Mandip is 9.60 m.

NS. 5

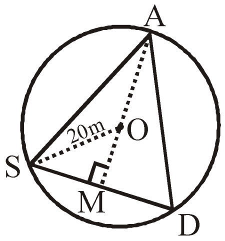
A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Ans. Let Ankur Syed and David are sitting at A, S and D respectively such that AS = SD = AD

i.e. ΔASD is an equilateral triangle.

Let the length of each side of the equilateral triangle be 2x metres.

Draw $AM \perp SD$.



Since ΔASD is equilateral,

$\therefore \quad$ AM passes through O.

$\Rightarrow \quad SM = \frac{1}{2}SD = \frac{1}{2}(2x)$

$\Rightarrow \quad SM = x$

Now, in right ΔASM, we have

$AM^2 + SM^2 = AS^2$

$\Rightarrow \quad AM^2 = AS^2 - SM^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

$\Rightarrow \quad AM = \sqrt{3}x$

Now, $OM = AM - OA = (\sqrt{3}x - 20)$ m

Again, in right ΔOSM, we have

$SO^2 = SM^2 + OM^2$

$20^2 = x^2 + (\sqrt{3}x - 20)^2$

$\Rightarrow \quad 400 = x^2 + 3x^2 - 40\sqrt{3}x + 400$

$\Rightarrow \quad 4x^2 = 40\sqrt{3}x \quad \Rightarrow \; 4x = 40\sqrt{3}$

$\Rightarrow \quad x = 10\sqrt{3}$ m

Now, $SD = 2x = 2 \times 10\sqrt{3} \text{ m} = 20\sqrt{3}$ m

Thus, the length of the string of each phone $= 20\sqrt{3}$ m

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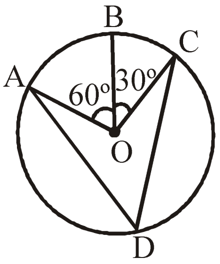
EXERCISE - 10.5

NS. 1

In the given figure, A, B and C are three points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.



Ans. We have a circle with centre O, such that $\angle AOB = 60^\circ$ and $\angle BOC = 30^\circ$

$\because \quad \angle AOB + \angle BOC = \angle AOC$

$\therefore \quad \angle AOC = 60^\circ + 30^\circ = 90^\circ$

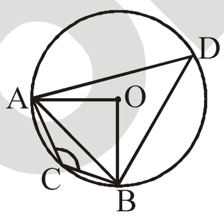
The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle ADC = \frac{1}{2}(\angle AOC) = \frac{1}{2}(90^\circ) = 45^\circ$

NS. 2

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Ans. We have a circle having a chord AB equal to radius of the circle.



$\therefore \quad AO = BO = AB$

$\Rightarrow \quad \Delta AOB$ is an equilateral triangle.

Since, each angle of an equilateral triangle is 60°.

$\Rightarrow \quad \angle AOB = 60^\circ$

Since, the arc ACB makes relfex $\angle AOB = 360^\circ - 60^\circ = 300^\circ$ at the centre of the circle and $\angle ACB$ at a point on the minor arc of the circle.

$\therefore \quad \angle ACB = \frac{1}{2}[\text{reflex } \angle AOB] = \frac{1}{2}[300^\circ] = 150^\circ$

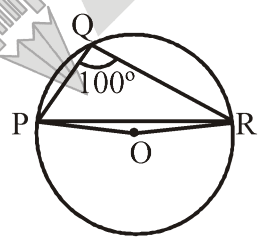
Hence, the angle subtended by the chord on the minor arc = 150°.

Similarly, $\angle ADB = \frac{1}{2}[\angle AOB] = \frac{1}{2} \times 60^\circ = 30^\circ$

Hence, the angle subtended by the chord on the major arc = 30°

NS. 3

In the given figure, $\angle PQR = 100^\circ$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Ans. The angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.

$\therefore \quad \text{reflex } \angle POR = 2\angle PQR$

But $\angle PQR = 100^\circ$

$\therefore \quad \text{reflex } \angle POR = 2 \times 100^\circ = 200^\circ$

Since, $\angle POR + \text{reflex } \angle POR = 360^\circ$

$\Rightarrow \quad \angle POR = 360^\circ - 200^\circ \quad \Rightarrow \quad \angle POR = 160^\circ$

Since, OP = OR [Radii of the same circle]

In ΔPOR, $\angle OPR = \angle ORP$

[Angles opposite to equal sides of a triangle are equal]

Also, $\angle OPR + \angle ORP + \angle POR = 180^\circ$

[Sum of the angles of a triangle is 180°]

$\Rightarrow \quad \angle OPR + \angle ORP + 160^\circ = 180^\circ$

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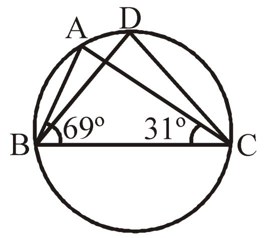
$\Rightarrow \quad 2\angle OPR = 180^\circ - 160^\circ = 20^\circ\ [\angle OPR = \angle ORP]$

$\Rightarrow \quad \angle OPR = \dfrac{20^\circ}{2} = 10^\circ$

NS. 4

In the given figure, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.



Ans. In ΔABC, $\angle ABC + \angle ACB + \angle BAC = 180^\circ$

$\Rightarrow \quad 69^\circ + 31^\circ + \angle BAC = 180^\circ$

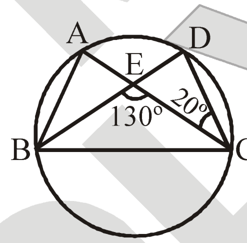
$\Rightarrow \quad \angle BAC = 180^\circ - 100^\circ = 80^\circ$

Since, anlges in the same segment are equal

$\therefore \quad \angle BDC = \angle BAC \ \Rightarrow\ \angle BDC = 80^\circ$

NS. 5

In the figure, A, B, C and D are four points on a circle.AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.



Ans. $\angle BEC = \angle EDC + \angle ECD$

[Sum of interior opposite angles is equal to exterior angle]

$130^\circ = \angle EDC + 20^\circ$

$\Rightarrow \angle EDC = 130^\circ - 20^\circ = 110^\circ \ \Rightarrow \angle BDC = 110^\circ$

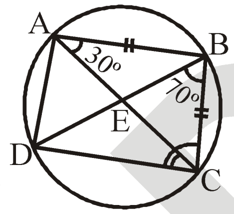
Since, angles in the same segment are equal.

$\therefore \quad \angle BAC = \angle BDC \ \Rightarrow\ \angle BAC = 110^\circ$

NS. 6

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC = 30^\circ$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.

Ans. Since angles in the same segment of a circle are equal.



$\therefore \quad \angle BAC = \angle BDC$

$\Rightarrow \quad \angle BDC = 30^\circ$

Also, $\angle DBC = 70^\circ$ [Given]

In ΔBCD,

$\angle BCD + \angle DBC + \angle CDB = 180^\circ$

$\Rightarrow \quad \angle BCD + 70^\circ + 30^\circ = 180^\circ$

$\Rightarrow \quad \angle BCD = 180^\circ - 100^\circ = 80^\circ$

Now, in ΔABC, AB = BC

$\therefore \quad \angle BCA = \angle BAC$

[Angles opposite to equal sides of a triangle are equal]

$\Rightarrow \quad \angle BCA = 30^\circ$ $[\because \angle BAC = 30^\circ]$

Now, $\angle BCA + \angle ECD = \angle BCD$

$\Rightarrow \quad 30^\circ + \angle ECD = 80^\circ$

$\Rightarrow \quad \angle ECD = 80^\circ - 30^\circ = 50^\circ$

NS. 7

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is rectangle.

Ans. since AC and D are diamteres.

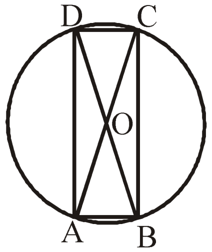
$\Rightarrow \quad AC = BD$ ....(1)

[All diameters of a circle are equal]

Also, $\angle BAD = 90^\circ$

[Angle formed in a semicircle is 90°]

Similarly, ∠ABC = 90°, ∠BCD = 90° and ∠CDA = 90°



Now, in right ΔABC and right ΔBAD, we have

AC = BD [From (1)]

AB = BA [Common]

∠ABC = ∠BAD [Each equal to 90°]

∴ ΔABC ≅ ΔBAD [By RHS congruence criteria]

⇒ BC = AD [C.P.C.T.]

Similarly, AB = DC

Thus, the cyclic quadrilateral ABCD is such that its opposite sides are equal and each of its angle is right angle.

∴ ABCD is a rectangle.

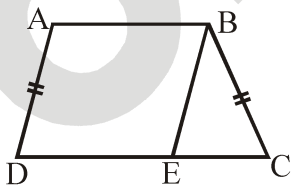
NS. 8

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Ans. We have a trapezium ABCD such that AB || CD and AD = BC.

Let us draw BE || AD such that ABED is a parallelogram

∵ The opposite angles of a parallelogram are equal



∴ ∠BAD = ∠BED ....(1)

and AD = BE ....(2)

[Opposite sides of a parallelogram]

But AD = BC [Given] ....(3)

∴ From (2) and (3), we have

BE = BC ⇒ ∠BEC = ∠BCE ....(4)

[Angle opposite to equal sides of a triangle are equal]

Now, ∠BED + ∠BEC = 180° [Linear pair]

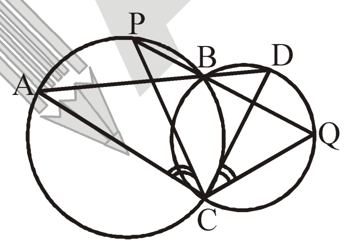
⇒ ∠BAD + ∠BCE = 180° [Using (1) and (4)]

i.e. A pair of opposite angles of quadrilateral ABCD is 180°

⇒ Trapezium ABCD is cyclic.

NS. 9

Two circles intersect at two points B and C. Through B, two line segements ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see the figure). Prove that ∠ACP = ∠QCD.



Ans. Since angles in the same segment of a circle are equal

∴ ∠ACP = ∠ABP ....(1)

Similarly, ∠QCD = ∠QBD ....(2)

Since, ∠ABP = ∠QBD [Vertically opposite angle]

∴ From (1) and (2), we have

∠ACP = ∠QCD

NS. 10

If circles are drawn taking two sides of a tirangle as diameters, prove that the point of intersection of these circles lie on the third side.

Ans. We have ΔABC, and two circles described with diameter as AB and AC respectively. They intersect at a point D, other than A.

Let us join A and D.

∵ AB is a diameter and ∠ADB is an angle formed in a semicircle.

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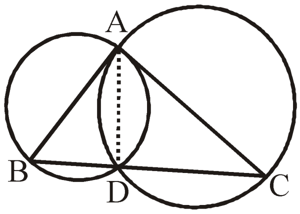


$\Rightarrow \quad \angle ADB = 90^\circ \qquad ....(1)$

Similarly, $\angle ADC = 90^\circ \qquad ....(2)$

Adding (1) and (2), we have

$\angle ADB + \angle ADC = 90^\circ + 90^\circ = 180^\circ$

i.e., B, D and C are collinear points.

$\Rightarrow$ BC is a straight line. Thus, D lies on BC.

NS. 11

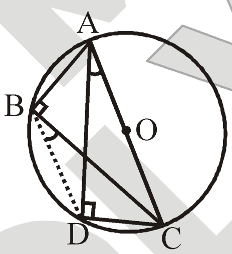
ABC and ADC are two right triangles with common hypotenuse AC. Porve that $\angle CAD = \angle CBD$.

Ans. We have ΔABC and ΔADC such that they are having AC as their common hypotenuse.

$\because$ AC is a hypotenuse and $\angle ADC = 90^\circ = \angle ABC$

$\therefore$ Both the triangles are in semi-cirlce

Case-1 : If both the triangles are in the same semi-circle



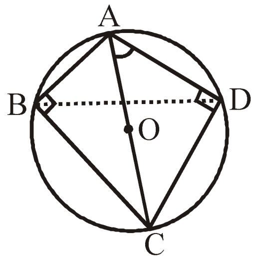
$\Rightarrow$ A, B, C and D are concylic.

Join BD.

Now, DC is a chord and $\angle CAD$ and $\angle CBD$ are formed in the same segment.

$\Rightarrow \quad \angle CAD = \angle CBD$

Case-2 : If both the triangles are not in same semi-circle.



$\Rightarrow$ A, B, C and D are concyclic.

Join BD

Now, DC is a chord and $\angle CAD$ and $\angle CBD$ are formed in the same segment.

$\Rightarrow \quad \angle CAD = \angle CBD$

NS. 12

Prove that a cyclic parallelogram is a rectangle.

Ans. We have a cyclic parallelogram ABCD.

Since, ABCD is a cyclic quadrilateral.

$\therefore$ Sum of its opposite angles is 180°

$\therefore \quad \angle A + \angle C = 180^\circ \qquad ....(1)$

But $\angle A = \angle C \qquad ....(2)$

[Opposite angles of a parallelogram are equal]

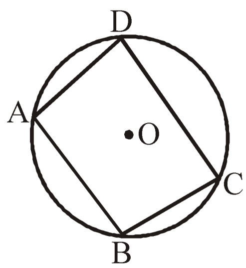
From (1) and (2), we have

$\angle A = \angle C = 90^\circ$

Similarly, $\angle B = \angle D = 90^\circ$

$\Rightarrow$ Each angle of the parallelogram ABCD is 90°.

Thus, ABCD is a rectangle

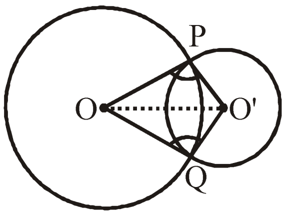


EXERCISE - 10.6

NS. 1

Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Ans. **Given :** Two circles with centres O and O' respectively such that they intersect each other at P and Q.



To Prove : $\angle OPO' = \angle OQO'$.

Construction : Join OP, O'P, OQ, O'Q and OO'.

Proof : In $\Delta OPO'$ and $\Delta OQO'$, we have

OP = OQ [Radii of the same cirlce]

O'P = O'Q [Radii of the same circle]

OO' = OO' [Common]

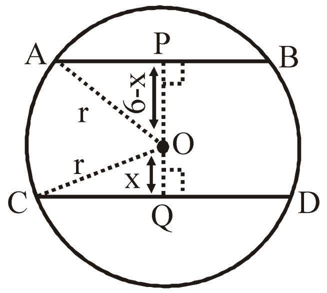
$\Delta OPO' \cong \Delta OQO'$ [By SSS congruence criteria]

$\Rightarrow \angle OPO' = \angle OQO'$ [C.P.C.T.]

NS. 2

Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Ans. We have a circle with centre O. AB || CD and the perpendicular distance between AB and CD is 6 cm and AB = 5 cm, CD = 11 cm. Let 'r' be the radius of the circle.



Let us draw $OP \perp AB$ and $OQ \perp CD$.

Join OA and OC.

Let OQ = x cm

$\therefore$ OP = (6 – x) cm

$\because$ The perpendicular from the centre of a circle to chord bisects the chord.

$\therefore \quad AP = \frac{1}{2}AB = \frac{1}{2}5 = \frac{5}{2}$ cm,

$CQ = \frac{1}{2}CD = \frac{1}{2}\times 11 = \frac{11}{2}$ cm

In ΔCQO, we have $CO^2 = CQ^2 + OQ^2$

$\Rightarrow \quad r^2 = \left(\frac{11}{2}\right)^2 + x^2 \quad \Rightarrow \quad r^2 = \frac{121}{4} + x^2$ ...(1)

In ΔAPO, we have $AO^2 = AP^2 + OP^2$

$\Rightarrow \quad r^2 = \left(\frac{5}{2}\right)^2 + (6-x)^2$

$\Rightarrow \quad r^2 = \frac{25}{4} + [36 - 12x + x^2]$ ...(2)

From (1) and (2), we have

$\frac{25}{4} + 36 - 12x + x^2 = \frac{121}{4} + x^2$

$\Rightarrow \quad -12x = \frac{121}{4} - \frac{25}{4} - 36$

$\Rightarrow \quad 12x = 12 \quad \Rightarrow \quad x = 1$

Substituting the value of x in (1), we get

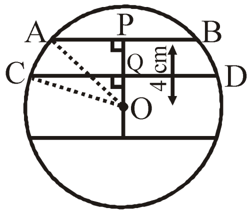
$r^2 = \frac{121}{4} + 1 = \frac{125}{4} \quad \Rightarrow \quad r = \frac{5\sqrt{5}}{2}$ cm

Thus, the radius of the circle is $\frac{5\sqrt{5}}{2}$ cm

NS. 3

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre.

Ans. We have a circle with centre O. Parallel chords AB and CD are such that the smaller chord is 4 cm away from the centre.



Draw OP ⊥ AB and join OA and OC.

Now, P is the mid-point of AB.

$\Rightarrow \quad AP = \frac{1}{2}AB = \frac{1}{2}(6\,cm) = 3\,cm$

Similarly, $CQ = \frac{1}{2}CD = \frac{1}{2}(8\,cm) = 4\,cm$

Now, in ΔOPA, we have $OA^2 = OP^2 + AP^2$

$\Rightarrow \quad r^2 = 4^2 + 3^2$

$\Rightarrow \quad r^2 = 16 + 9 = 25$

$\Rightarrow \quad r = \sqrt{25} = 5\,cm$

[$r \neq -5$, because distance cannot be negative]

Again, in ΔCQO, we have $OC^2 = OQ^2 + CQ^2$

$\Rightarrow \quad r^2 = OQ^2 + 4^2 \quad \Rightarrow \quad OQ^2 = r^2 - 4^2$

$\Rightarrow \quad OQ^2 = 5^2 - 4^2 = 25 - 16 = 6 \qquad [\because\ r = 5\text{ cm}]$

$\Rightarrow \quad OQ = \sqrt{9} = 3\,cm$

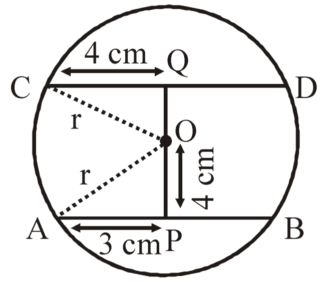
The distance of the other chord (CD) from the centre is 3 cm.

Note : In case we take the two parallel chords on either side of the centre, then

In ΔPOA, $OA^2 = OP^2 + PA^2$

$\Rightarrow \quad r^2 = 4^2 + 3^2 = 5^2$

$\Rightarrow \quad r = 5$ cm



In ΔOQC, $OC^2 = CQ^2 + OQ^2$

$\Rightarrow \quad r^2 = 4^2 + OQ^2$

$\Rightarrow \quad OQ^2 = 5^2 - 4^2 = 9$

$\Rightarrow \quad OQ = 3$ cm

NS. 4

Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Ans. **Given :** ∠ABC such that when we produce arms BA and BC, they make two equal chords AD and CE. Join AC, DE and AE

Proof : An exterior angle of a triangle is equal to the sum of interior opposite angles.

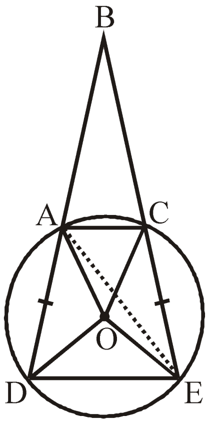
∴ In ΔBAE, we have

$\angle DAE = \angle ABC + \angle AEC \qquad ....(1)$

The chord DE subtends ∠DOE at the centre and ∠DAE in the remaining part of the circle.

$\therefore \quad \angle DAE = \frac{1}{2}\angle DOE \qquad ....(2)$

Similarly, $\angle AEC = \frac{1}{2}\angle AOC \qquad ....(3)$



From (1), (2) and (3), we have

$\frac{1}{2}\angle DOE = \angle ABC + \frac{1}{2}\angle AOC$

$\Rightarrow \quad \angle ABC = \frac{1}{2}\angle DOE - \frac{1}{2}\angle AOC$

$\Rightarrow \quad \angle ABC = \frac{1}{2}[\angle DOE - \angle AOC]$

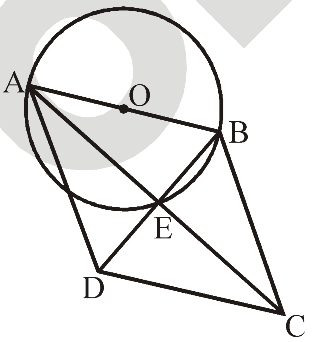
$\Rightarrow \quad \angle ABC = \frac{1}{2}$ [(Angle subtended by the chord DE at the centre) – (Angle subtended by the chord AC at the centre)]

$\Rightarrow \quad \angle ABC = \frac{1}{2}$ [Difference of the angles subtended by the chords DE and AC at the centre]

NS. 5

Prove that the circle drawn with any side of a rhombus as a diameter, passes through the point of intersection of its diagonals.

Ans. Let ABCD be a rhombus whose diagonals AC and BD intersect at E. Let O be centre of the circle with diameter AB. We know that the diagonals of a rhombus intersect each other at a right angle.



$\Rightarrow \quad \angle AEB = 90^\circ$

$\Rightarrow$ Circle with AB as diameter passes through E *i.e.*, the point of intersection of its diagonals.

NS. 6

ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

Ans. **Given :** A circle passing through A, B and C is drawn such that it intersects CD at E.

To prove : AE = AD

Proof : ABCE is a cyclic quadrilateral

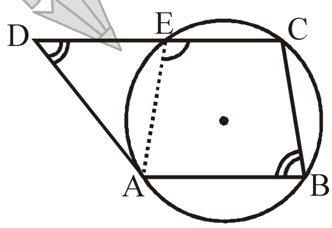
$\therefore \quad \angle AEC + \angle B = 180^\circ$ ...(1)

[Opposite angles of a cyclic quadrilateral are supplementary]

But ABCD is a parallelogram. [Given]

$\therefore \quad \angle D = \angle B$ ...(2)

[Opposite angles of a parallelogram are equal]



From (1) and (2), we have

$\angle AEC + \angle D = 180^\circ$ ...(3)

But $\angle AEC + \angle AED = 180^\circ$ ...(4)

From (3) and (4), we have

$\angle D = \angle AED$

i.e., The base angles of ΔADE are equal.

$\therefore$ Opposite sides must be equal.

$\Rightarrow \quad AD = AE$

NS. 7

AC and BD are chords of a circle which bisect each other. Prove that

(i) AC and BD are diameters,

(ii) ABCD is a rectangle

Ans. **Given :** A circle with centre at O. Two chords AC and BD are such that they bisect each other. Let their point of intersection be O.

To Prove : (i) AC and BD are diameters

(ii) ABCD is a rectangle

Construction : Join AB, BC, CD and DA.

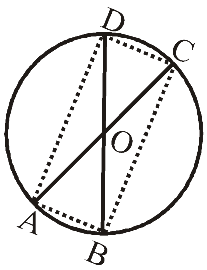
Proof : (i) In ΔAOB and ΔCOD, we have

AO = CO [O is the mid-point of AC]

BO = DO [O is the mid-point of BD]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\therefore$ Using the SAS criterion of congruence,



$\Delta AOB \cong \Delta COD$

$\Rightarrow$ AB = CD [C.P.C.T.]

$\Rightarrow$ arc AB = arc CD ...(1)

Similarly, arc AD = arc BC ...(2)

[$\because$ If two chords are qual, then corresponding arcs are equal (congruent)]

Adding (1) and (2), we get

arc AB + arc AD = arc CD + arc BC

$\Rightarrow$ $\overset{\frown}{BAD} = \overset{\frown}{BCD}$

$\Rightarrow$ BD divides the circle into two equal parts

$\therefore$ BD is a diameter

Similarly, AC is a diameter

(ii) We know that $\Delta AOB \cong \Delta COD$

$\Rightarrow$ $\angle OAB = \angle OCD$ [C.P.C.T.]

$\Rightarrow$ $\angle CAB = \angle ACD$ $\Rightarrow$ AB || DC

Similarly, AD || BC

$\therefore$ ABCD is a parallelogram

Since, opposite angles of a parallelogram are equal

$\therefore$ $\angle DAB = \angle DCB$

But $\angle DAB + \angle DCB = 180°$

[Sum of the opposite angles of a cyclic quadrilateral is 180°]

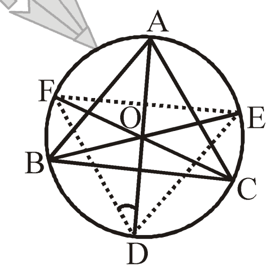
$\Rightarrow$ $\angle DAB = 90° = \angle DCB$

Thus, ABCD is a rectangle.

NS. 8

Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle D, E and F respectively. Prove that the angles of the triangle DEF are $90° - \frac{1}{2}\angle A$, $90° - \frac{1}{2}\angle B$, $90° - \frac{1}{2}\angle C$.

Ans. **Given :** A triangle ABC inscribed in a circle, such that bisectors of $\angle A$, $\angle B$ and $\angle C$ intersect the circumcircle at D, E and F respectively.



Construction : Join DE, EF and FD.

Proof : Since, angles in the same segment are equal.

$\therefore$ $\angle FDA = \angle FCA$ ....(1)

$\angle EDA = \angle EBA$ ....(2)

Adding (1) and (2), we have

$\angle FDA + \angle EDA = \angle FCA + \angle EBA$

$\Rightarrow$ $\angle FDE + \angle FCA = \angle EBA$

$$= \frac{1}{2}\angle C + \frac{1}{2}\angle B = \frac{1}{2}[\angle C + \angle B]$$

$$= \frac{1}{2}[180° - \angle A] = \left(90° - \frac{\angle A}{2}\right)$$

Similarly, $\angle FED = \left(90° - \frac{\angle B}{2}\right)$

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and $\angle EFD = \left(90^\circ - \frac{\angle C}{2}\right)$

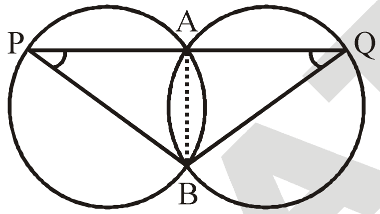
Thus, the angles of ΔDEF are

$\left(90^\circ - \frac{\angle A}{2}\right), \left(90^\circ - \frac{\angle B}{2}\right)$ and $\left(90^\circ - \frac{\angle C}{2}\right)$

NS. 9

Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Ans. We have two congruent circles such that they intersect each other at A and B. A line passing through A, meets the circles at P and Q. Let us draw the common chord AB.



Since angles subtended by equal chords in the congruent circles are equal.

$\Rightarrow \quad \angle APB = \angle AQB$

Now, in ΔPBQ, we have

$\angle APB = \angle AQB$

So, their opposite sides must be equal.

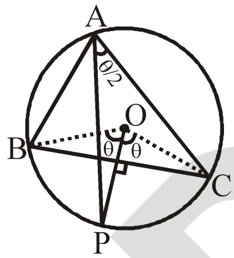
$\Rightarrow \quad PB = BQ.$

NS. 10

In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Ans. **Given :** ΔABC with O as centre of its circumcircle. The perpendicular bisector of BC passes through O. Suppose it cut circumcircle at P.

Proof : In order to prove that the perpendicular bisector of BC and bisector of angle A of ΔABC intersect at P, it is sufficient to show that AP is bisector of $\angle A$ of ΔABC.



Arc BC makes angle θ on the circumference

$\therefore \quad \angle BOC = 2\theta$

[Angle at centre is double the angle made by an arc at circumference]

Also, in ΔBOC, OB = OC and OP is perpendicular bisector of BC.

So, $\angle BOP = \angle COP = \theta$

Arc CP makes angle θ at O, so it will make angle $\frac{\theta}{2}$ at circumference.

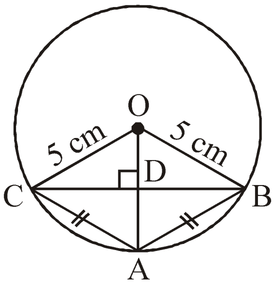
So, $\angle CAP = \frac{\theta}{2}$

Hence, AP is angle bisector of $\angle A$ of ΔABC.

EXERCISE – I

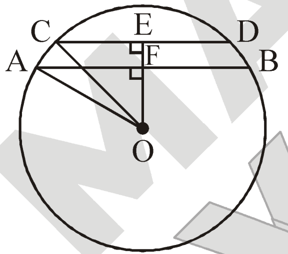
ONLY ONE CORRECT TYPE

1. In the given figure, O is the centre of the circle and chords AB = AC = 6 cm. The length of BC, if radius is 5 cm, is :



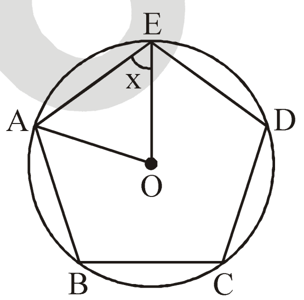
(A) 9.6 cm (B) 4.8 cm
(C) 19.2 cm (D) 8.0 cm

2. In the given figure, O is the centre of the circle and OE ⊥ CD, OF ⊥ AB, AB || CD, AB = 48 cm, CD = 20 cm, radius OA = 26 cm. The length of EF is :



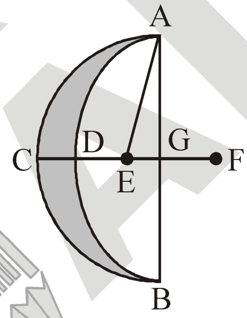
(A) 6 cm (B) 8 cm
(C) 14 cm (D) 16 cm

3. In the given pentagon ABCDE, AB = BC = CD = DE = AE. The value of x is :



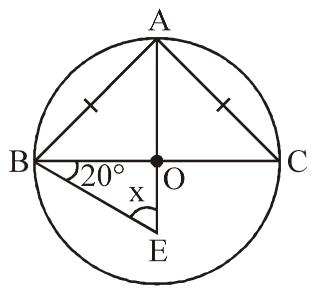
(A) 36° (B) 54°
(C) 72° (D) 108°

4. A cresecent formed of two circular arcs ACB, ADB of equal radius with respective, centres E and F as shown in the given figure. The perpendicular bisector of AB cuts the crescent at C and D, where CD = 12 cm, AB = 16 cm. The radius of arc ACB is :



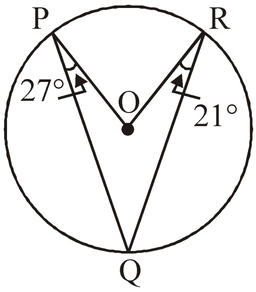
(A) 18 cm (B) 16 cm
(C) 12 cm (D) 10 cm

5. In the given figure, E is any point in the interior of the circle with centre O. Chord AB = AC. If ∠OBE = 20°, the value of x is :



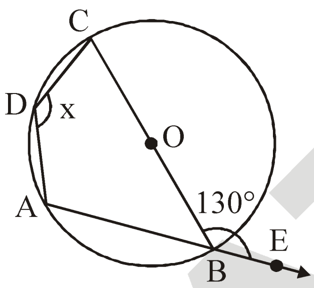
(A) 40° (B) 45°
(C) 50° (D) 70°

6. In the given figure, O is the centre of circle. ∠OPQ = 27° and ∠ORQ = 21°. The values of ∠POR and ∠PQR respectively are :



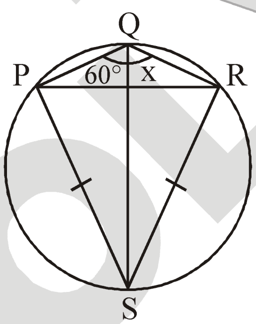
(A) 84°, 42° (B) 96°, 48°
(C) 54°, 42° (D) 108°, 54°

7. ABCD is a cyclic quadrilateral with centre O in the given figure. Chord AB is produced to E where ∠CBE = 130°, the value of x is equal to :



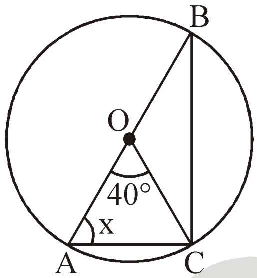
(A) 130° (B) 260°
(C) 140° (D) 280°

8. In the given figure, PQRS is a cyclic quadrilateral in which PS = RS and ∠PQS = 60°. The value of x is :



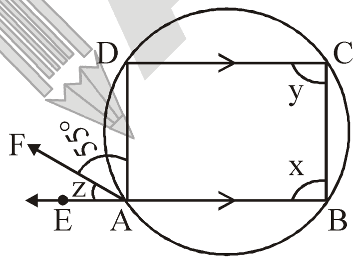
(A) 30° (B) 60°
(C) 75° (D) 80°

9. In the given figure, AB is diameter, ∠AOC = 40°. The value of x is :



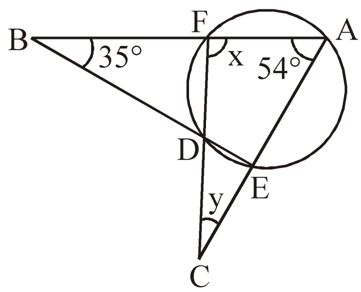
(A) 50° (B) 60°
(C) 70° (D) 80°

10. In the given figure, ABCD is a cyclic quadrilateral. BA is produced to E and DC || AB. If y : x is equal to 4 : 5, then value of z is :



(A) 15° (B) 20°
(C) 25° (D) 30°

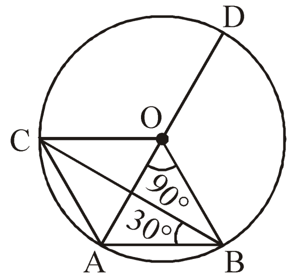
11. In the given figure, AEDF is a cyclic quadrilateral. If we extend the lines AF & ED to meet at B and lines FD & AE to meet at C. The values of x and y respectively are :



(A) 79°, 47° (B) 89°, 37°
(C) 89°, 47° (D) 79°, 37°

MATRIX OLYMPIAD

12. In the given figure, O is the centre of the circle and $\angle AOB = 90°$ and $\angle ABC = 30°$. Then, $\angle CAO$ is :



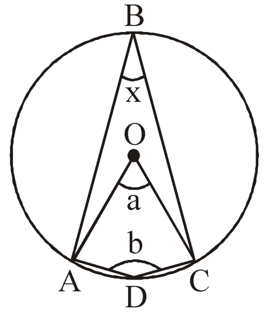
(A) 30° (B) 45°
(C) 60° (D) 90°

13. In a cyclic quadrilateral, the difference between two opposite angles is 58°, the measures of opposite angles are :
(A) 158°, 22° (B) 129°, 51°
(C) 109°, 71° (D) 119°, 61°

14. Which of the following statements is true for a regular pentagon ?
(A) All vertices are concyclic
(B) All vertices are not concyclic
(C) Only four vertices are concyclic
(D) Cannot say anything about regular pentagon

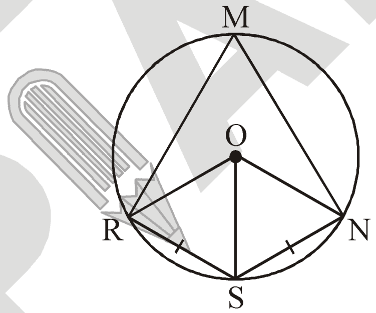
15. In a cyclic quadrilateral ABCD, if two sides are parallel, which of the following statements is definitely false ?
(A) Remaining two sides are equal
(B) Diagonals are not equal
(C) Diagonals intersect at the centre of circle
(D) Both (A) and (C)

16. In the given figure, ABCD is a cyclic quadrilateral, O is the centre of the circle and a : b = 2 : 5. The value of x is :



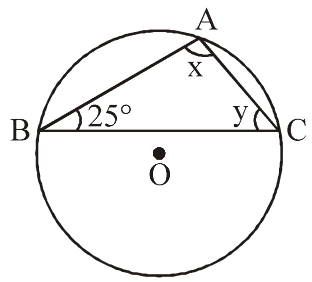
(A) 20° (B) 25°
(C) 30° (D) 35°

17. In the given figure, chord RS = chord NS. How $\overset{\frown}{RS}$ is related with $\overset{\frown}{NS}$?



(A) $\overset{\frown}{RS}$ is smaller than $\overset{\frown}{NS}$
(B) Both are equal
(C) $\overset{\frown}{RS}$ is greater than $\overset{\frown}{NS}$
(D) None of these

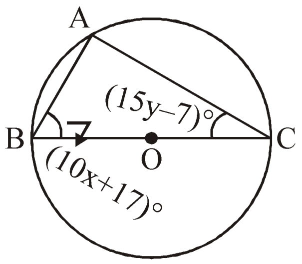
18. In the given figure, O is the centre of the circle. For what values of x and y, if chord BC passes through the centre of circle where points A, B and C are on the circle ?



(A) x = 90°, y = 60° (B) x = 75°, y = 30°
(C) x = 65°, y = 90° (D) x = 90°, y = 65°

MATRIX OLYMPIAD

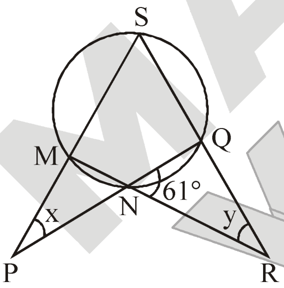
19. In the given figure, chord BC passes through the centre of a circle where points A, B and C are concylic and ∠B is 44° more than ∠C. The values of x and y respectively are :



(A) x = 4; y = 3 (B) x = 3; y = 5

(C) x = 7; y = 2 (D) x = 5; y = 2

20. In the given figure, MNQS is a cyclic quadrilateral in which ∠QNR = 61° and x : y is 2 : 1. If SQ & MN are extended to meet at R and SM & QN are extended they meet at P. Then the values of x and y respectively are :



(A) $18\frac{1°}{4}, 37\frac{3°}{4}$ (B) $38\frac{2°}{3}, 19\frac{1°}{3}$

(C) $21\frac{1°}{3}, 33\frac{2°}{3}$ (D) $19\frac{1°}{4}, 38\frac{1°}{4}$

21. Equal chords of a circle subtend equal angles at

(A) Centre

(B) Circumference

(C) Both (A) and (B)

(D) None of these

22. The line joining the centre of a circle to the midpoint of a chord is always

(A) Parallel to the chord

(B) Perpendicular to the chord

(C) Equal to the chord

(D) Tangent to the chord

23. There is one and only one circle passing through three given ________ points

(A) collinear (B) non-collinear

(C) far-off (D) nearest

24. Which of the following statements is true for the longest chord of a circle ?

(A) It is equal to radius

(B) It is two times of radius

(C) It is never equal to diameter

(D) It is two times of diameter

25. When two chords of a circle bisect each other, then which of the following statemetns is true ?

(A) Both chords are perpendicular to each other

(B) Both chords are parallel to each other

(C) Both chords are unequal

(D) Both are diameters of the circle

PARAGRAPH TYPE

Passage – I : The perpendicular from the centre of a circle to a chord bisects the chord.

26. The radius of a circle is 13 cm and length of one of its chord is 10 cm. The distance of the chord from the centre is :

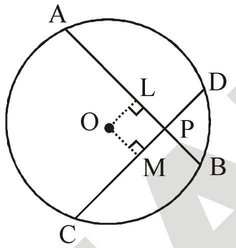
(A) 11 cm (B) 12 cm

(C) 13 cm (D) 14 cm

27. LM and NP are two parallel chords of a circle such that LM = 10 cm and NP = 24 cm. If the chords are on the opposite sides of the centre and the distance between them is 17 cm, the radius of the circle is :
(A) 11 cm (B) 12 cm
(C) 13 cm (D) 14 cm

28. The radius of a circle is 10 cm and the distance of the chord from the centre is 6 cm. Then the length of the chord is :
(A) 12 cm (B) 13 cm
(C) 14 cm (D) 16 cm

Passage – II : Two equal chords AB and CD of a circle C(O, r) intersect at a point P within a circle then :



29. AP =
(A) DP (B) OL
(C) CP (D) AB

30. DP =
(A) BP (B) MP
(C) PL (D) OP

31. ∠OPL =
(A) ∠OMP (B) ∠OMC
(C) ∠OLP (D) ∠OPM

MATCH THE COLUMN TYPE

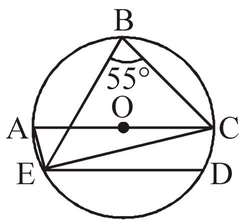
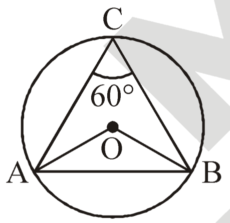
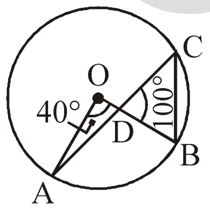
In this section, each question has two matching lists. Choices for the correct combination of elements from Column I and Column II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the following :

Column – I	**Column – II**
(P) The radius of a circle is 8 cm and the length of one of its chords is 12 cm. The distance of the chord from the centre is	(i) 23 cm
(Q) Two parallel chords of length 30 cm and 16 cm are drawn on the opposite sides of the centre of a circle of radius 17 cm. The distance between the chords is	(ii) 5.196 cm
(R) The length of a chord which is at a distance of 4 cm from the centre of the circle of radius 6 cm is	(iii) 5.291 cm
(S) An equilateral triangle of side 9 cm is incribed in a circle. The radius of the circle is	(iv) 8.94 cm

(A) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)
(B) (P) → (iii), (Q) → (iv), (R) → (i), (S) → (ii)
(C) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)
(D) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)

33. Match the following :

Column – I	**Column – II**
(P) C is a point on the minor arc AB of the circle with centre O. If ∠ACB = x calculate x, if ACBO is a parallelogram	(i) 60°
(Q) Chord ED is parallel to the diameter AC of the circle. If ∠CBE = 55°, then ∠DEC is 	(ii) 120°
(R) In the given figure, O is the centre of the circle. If ∠ACB = 60°, find ∠OAB. 	(iii) 35°
(S) In the given figure, O is the centre of a circle, ∠AOB = 40° and ∠BDC = 100°. Find ∠OBC. 	(iv) 30°

(A) (P) → (ii), (Q) → (iv), (R) → (iii), (S) → (i)

(B) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)

(C) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(D) (P) → (i), (Q) → (iii), (R) → (ii), (S) → (iv)

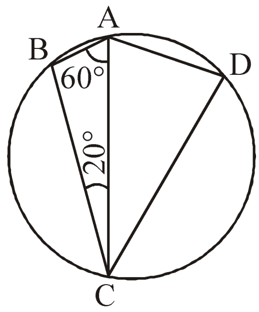
MATRIX OLYMPIAD

EXERCISE – II

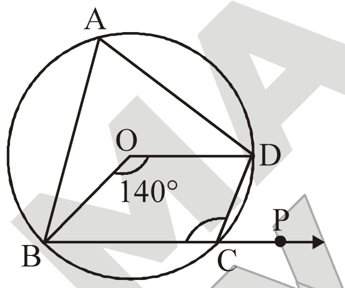
VERY SHORT ANSWER TYPE

1. Two chords AB and CD of a circle are parallel and a line *l* is the perpendicular bisector of AB. Show that *l* bisects CD.

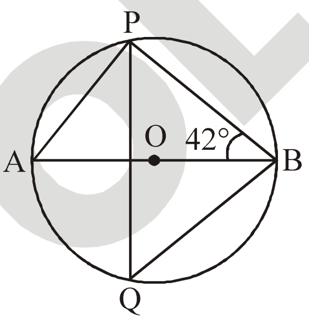
2. In given figure, if ∠BAC = 60° and ∠BCA = 20°, find ∠ADC.



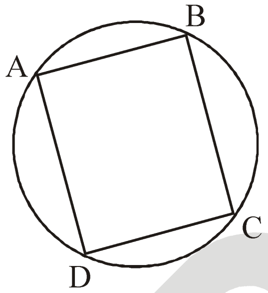
3. In the given figure, O is the centre of the circle. The angle subtended by the arc BCD at the centre is 140°. BC is produced to P. Determine ∠BAD.



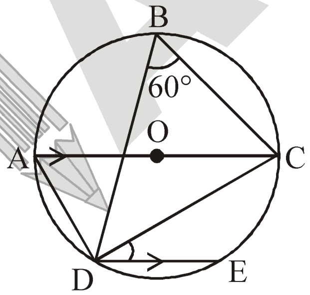
4. In given figure, find m∠PQB where O is the centre of the circle.



5. In a cyclic quadrilateral ABCD, if ∠A = 3∠C. Find ∠A.

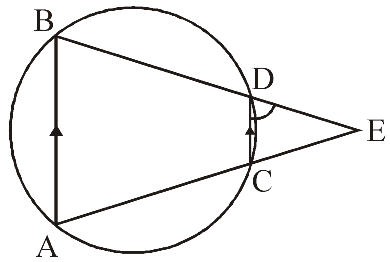


6. In the given figure, DE is a chord parallel to diameter AC of the circle with centre O. If ∠CBD = 60°, then find ∠CDE.

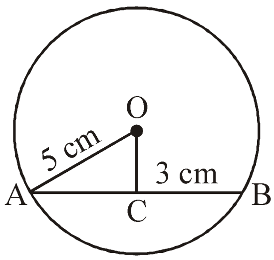


7. Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

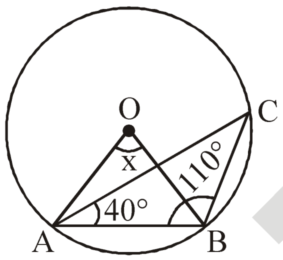
8. In the given figure, AB and CD are two parallel chords of a circle. If BDE and ACE are straight lines, intersecting at E, prove that ΔAEB is isosceles.



9. In the given figure, find the length of AB, if OA = 5 cm and OC = 3 cm.



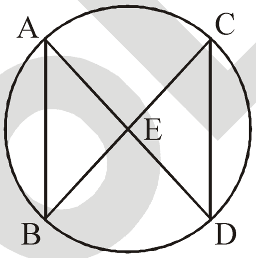
10. In the given figure, O is the centre of the circle. $\angle CAB = 40°$, $\angle CBA = 110°$, then find value of x.



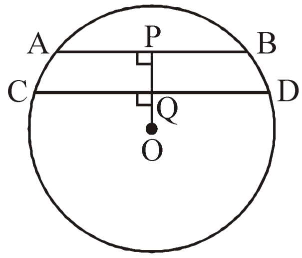
SHORT ANSWER TYPE

1. If O is the circumcentre of a ΔABC and $OD \perp BC$, prove that $\angle BOD = \angle A$.

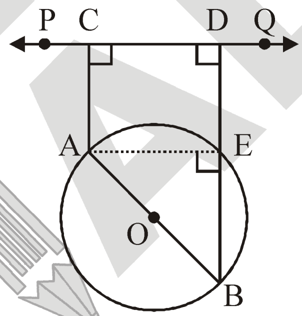
2. In the given figure, AB = CD. Prove that BE = DE and AE = CE, where E is the point of intersection of AD and BC.



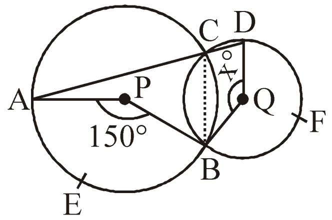
3. In the given figure, AB and CD are two parallel chords of a circle with centre O and radius 13 cm such that AB = 10 cm and CD = 24 cm. If $OP \perp AB$ and $OQ \perp CD$, find the length of PQ.



4. In the given figure, AB is a diameter of the circle with centre O. If AC and BD are perpendicular on a line PQ, and BD meets the circle at E, prove that AC = ED.



5. In the given figure, P and Q are centres of two circles, intersecting at B and C, and ACD is a straight line. If $\angle APB = 150°$ and $\angle BQD = x°$, find the value of x.



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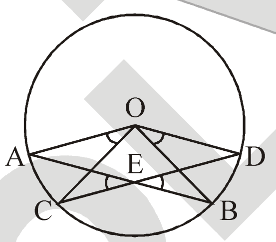
LONG ANSWER TYPE

1. The bisectors of opposite angles ∠P and ∠R of a cyclic quadrilateral PQRS intersect the corresponding circle at A and B respectively. Prove that AB is a diameter of the circle.
2. D and E are points on equal sides AB and AC of an isosceles triangle ABC such that AD = AE. Prove that B, C, D, E are concyclic.
3. AB and CD are two chords of a circle such that AB = 6 cm, CD = 12 cm and AB || CD. If the distance between AB and CD is 3 cm, find the radius of the circle.
4. D and E are the points on equal sides AB and AC respectively on an isosceles, ΔABC such that B, C, E and D are concyclic. If O is the point of intersection of CD and BE, prove that AO is the bisector of line segment DE.
5. In the given figure, O is the centre of the given circle and chords AB and CD intersect at a point E inside the circle.

   Prove that ∠AOC + ∠BOD = 2∠AEC.



TRUE/FALSE

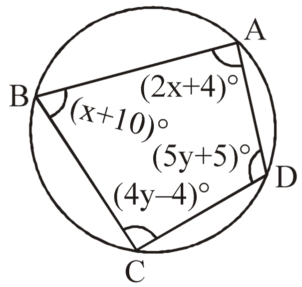
1. The circumference of a circle is π times the diameter.
2. More than one circle exists through three non collinear points.
3. At a point on a circle, only one tangent can be drawn.
4. From any point inside a circle two tangent can be drawn to the circle.
5. Two circle may touch each other internally.

FILL IN THE BLANKS

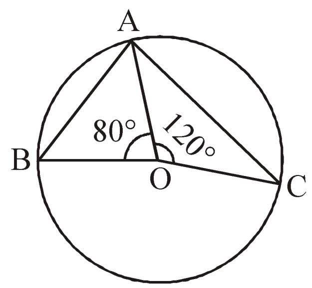
1. The opposite angles of a cyclic quadrilateral are __________ .
2. The perimeter of the circle is _______.
3. Two equal chords of a circle are _______ from the centre of the circle.
4. The angles in the same segment of a circle are _____.
5. The longest chord of a circle is _______ times of radius.

NUMERICAL PROBLEMS

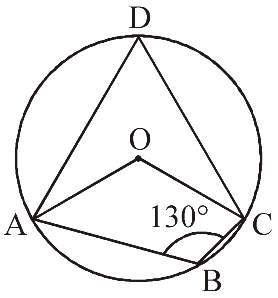
1. In the given figure, find out the value of (x – y) when ∠A = (2x + 4)°, ∠B = (x + 10)°, ∠C = (4y – 4)° and ∠D = (5y + 5)°



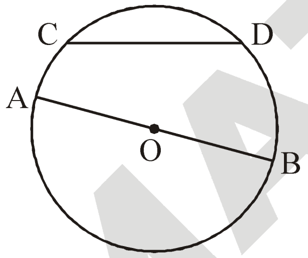
2. In the given figure, A, B, C are three points on a circle such that the angles subtended by the chords AB and AC at the centre O are 80° and 120° respectively. If ∠BAC is k°, then value of k is :



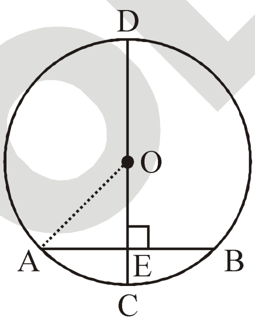
3. In the given figure, O is the centre of the circle and the measure of $\angle ABC$ is 130°. The value of $\dfrac{(\angle AOC - \angle ADC)}{25^\circ}$ is :



4. In the given figure, AOB is a diameter of a circle with centre O, such that AB = 34 cm and CD is a chord of length 30 cm. Then the distance (in cm) of CD from AB is :

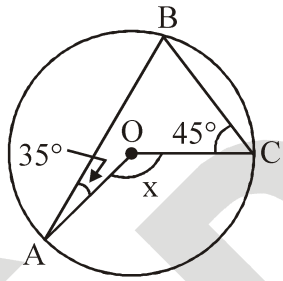


5. In the given figure, CD is the diameter of a circle with centre O and CD is perpendicular to chord AB. If AB = 12 cm and CE = 3 cm, then radius of the circle is r cm. The value of 2r is :



ANALYTICAL PROBLEMS & BRAIN TEASER

1. In the given figure O is the centre of circle. If $\angle BAO = 35^\circ$ and $\angle BCO = 45^\circ$, then the value of x will be :



(A) 160° (B) 170°
(C) 80° (D) 140°

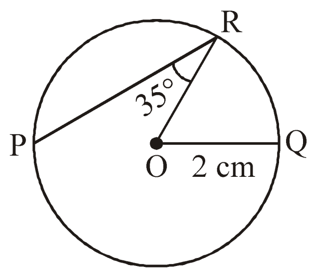
2. ABCD is a cyclic quadrilateral, if $\angle BAC = 60^\circ$, $\angle BCA = 20^\circ$ then find the value of $\angle ADC$.

(A) 15° (B) 50°
(C) 80° (D) 40°

3. Angles A, B, C and D of a cyclic quadrilateral ABCD are in the ratio 3 : 3 : 2 : 2 respectively. If AB = 5 cm, BC = 3.5 cm and CD = 8 cm, then the length of AD is :

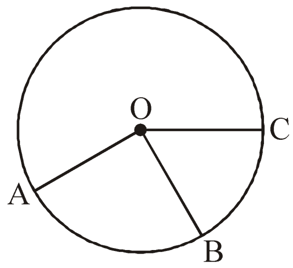
(A) 5 cm (B) 3.5 cm
(C) 8 cm (D) 4 cm

4. In the given figure, O is the centre of the circle. The distance between P and Q is 4 cm. Find the $\angle ROQ$.



(A) 50° (B) 60°
(C) 70° (D) 35°

5. In the given figure, what would be $\angle COB$, if the ratio of arc AB and BC is 3 : 2 and $\angle AOB = 96°$?



(A) 96° (B) 32°
(C) 64° (D) 16°

Space for Notes :

MATRIX OLYMPIAD

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	C	B	D	D	B	A	B	C	C	B	C	D	A	B
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	B	D	D	B	C	B	B	B	D	B	C	D	C	A
31	32	33												
D	A	B												

EXERCISE II

VERY SHORT ANSWER TYPE

2. 80° **3.** 70° **4.** 48° **5.** 135° **6.** 30° **9.** 8 cm **10.** 60°

SHORT ANSWER TYPE

3. 7 cm **5.** 150°

LONG ANSWER TYPE

3. 6.7cm

TRUE FALSE

1. T 2. F 3. T 4. F 5. T

FILL IN THE BLANKS

1. supplementary 2. 2πr 3. equidistant 4. equal 5. two

ANALYTICAL PROBLEMS & BRAIN TEASER

1. A **2.** C **3.** B **4.** C **5.** C

NUMERICAL PROBLEMS

1. 15 **2.** 80 **3.** 2 **4.** 8 **5.** 15

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : CIRCLES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Exercises			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

MATRIX OLYMPIAD

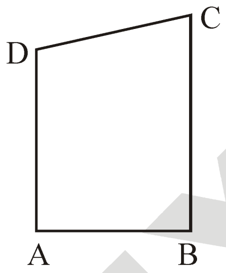
Space for Notes :

QUADRILATERALS 8

Concepts

INTRODUCTION

A closed figure formed by four intersecting lines is called a quadrilateral. It has four sides, four angles and four vertices. In quadrilateral ABCD; AB, BC, CD and DA are the four sides; A, B, C and D are the four vertices and ∠A, ∠B, ∠C and ∠D are the four angles formed at the vertices. The part of the plane enclosed by a simple closed figure is called a planar region corresponding to that figure.



1. ANGLE SUM PROPERTY OF A QUADRILATERAL

The sum of the four angles of a quadrilateral is 360°.

Given : We have a quadrilateral ABCD. AC is one of its diagonals.

Proof : In ΔABC, we have

$\angle 1 + \angle 3 + \angle B = 180°$ ....(1) [Angle sum property of a triangle]

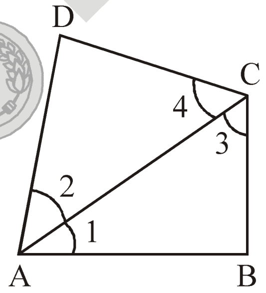
In ΔADC, we have

$\angle 2 + \angle 4 + \angle D = 180°$ ....(2) [Angle sum property of a triangle]

Adding (1) and (2), we have, $(\angle 1 + \angle 2) + (\angle 3 + \angle 4) + \angle B + \angle D = 180° + 180°$

$\Rightarrow \angle BAD + \angle BCD + \angle B + \angle D = 360° \Rightarrow \angle A + \angle C + \angle B + \angle D = 360°$

∴ Sum of the angles of a quadrilateral is 360°.

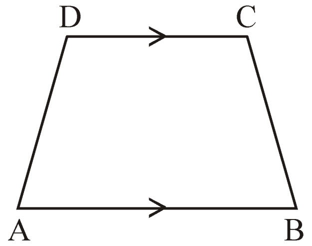


2. TYPES OF QUADRILATERALS

We shall define various types of quadrilaterals :

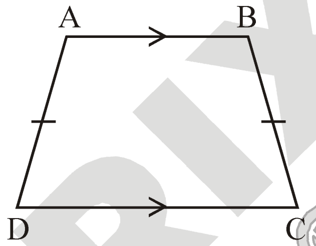
2.1 TRAPEZIUM

A quadrilateral having exactly one pair of parallel sides, is called a trapezium. In the given figure, ABCD is a trapezium in which AB || DC.



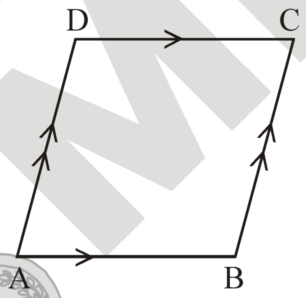
2.2 ISOSCELES TRAPEZIUM

A trapezium is said to be a an isosceles trapezium, if its non-parallel sides are equal. In the given figure, quadrilateral ABCD is an isosceles trapezium, in which AB || DC and AD = BC.



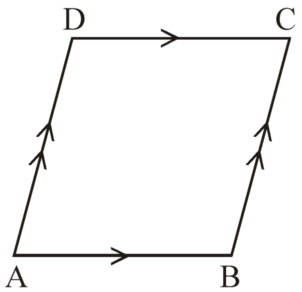
2.3 PARALLELOGRAM

A quadrilateral is a parallelogram if its both pairs of opposite sides are parallel. In the given figure, quadrilateral ABCD is a parallelogram, in which AB || DC and AD || BC.



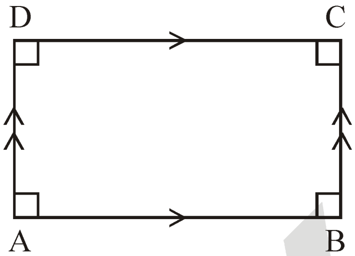
2.4 RHOMBUS

A parallelogram having all sides equal is called a rhombus. In the given figure, a parallelogram ABCD is a rhombus, if AB = BC = CD = AD. In other words, a quadrilateral ABCD is a rhombus, if AB || DC, AD || BC and AB = BC = CD = DA.



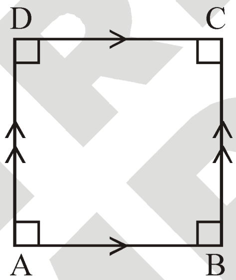
2.5 RECTANGLE

A parallelogram whose each angle is a right angle, is called a rectangle. In the given figure, ABCD is a rectangle in which AB || DC, AD || BC and ∠A = ∠B = ∠C = ∠D = 90°.



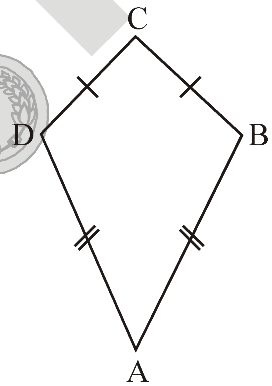
2.6 SQUARE

A parallelogram having all sides equal and each angle equal to a right angle, is called a square. In the given figure, ABCD is a square in which AB || DC, AD || BC, AB = BC = CD = DA and ∠A = ∠B = ∠C = ∠D = 90°.



2.7 KITE

A quadrilateral is a kite if it has two pairs of equal adjacent sides and unequal opposite sides. In the given figure, a quadrilateral ABCD is a kite, if AB = AD, BC = DC but AD ≠ BC and AB ≠ DC.



It follows from the above definitions that :

(i) Square, rectangle and rhombus are parallelograms.

(ii) A parallelogram is a trapezium but a trapezium is not a parallelogram.

(iii) A rectangle or a rhombus is not necessarily a square.

(iv) A kite is not a parallelogram.

Focus Point

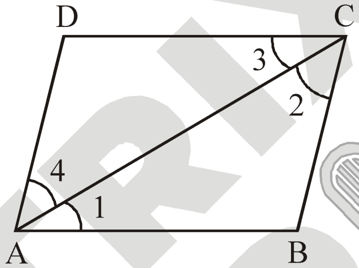
- In a parallelogram, bisectors of any two consecutive angles intersect at right angle.

3. PROPERTIES OF PARALLELOGRAM

Theorem -1

Statement : A diagonal of a parallelogram divides it into two congruent triangles.

Given : AC is a diagonal of the parallelogram ABCD as shown in the figure.



To Prove : $\Delta ABC \cong \Delta CDA$

Proof : AB || DC [Pair of opposite sides of parallelogram ABCD]

$\Rightarrow \angle 1 = \angle 3$ .....(1) [Pair of alternate angles with transversal AC]

Now, BC || AD [Pair of opposite sides of parallelogram ABCD]

$\Rightarrow \angle 2 = \angle 4$ .....(2) [Pair of alternate angles with transversal AC]

In ΔABC and ΔCDA, we have

$\angle 1 = \angle 3$ [By equation (1)]

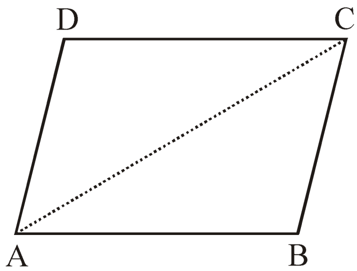
$\angle 2 = \angle 4$ [By equation (2)]

AC = CA [Common side]

$\therefore \Delta ABC \cong \Delta CDA$ [By ASA congruency]

Theorem -2

Statement : In a parallelogram, opposite sides are equal.



Given : ABCD is a parallelogram.

To Prove : AB = CD and DA = BC.

Construction : Join AC.

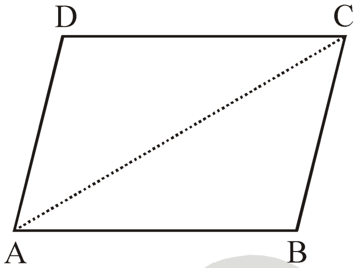
Proof : By Theorem 1, we have proved that $\Delta ABC \cong \Delta CDA$.

$\Rightarrow$ AD = CB and DC = BA [By C.P.C.T.]

Theorem -3

Statement (Converse of Theorem 2) : A quadrilateral is a parallelogram if its opposite sides are equal.

Given : A quadrilateral ABCD in which AB = CD and BC = DA.



To Prove : ABCD is a parallelogram.

Construction : Join AC.

Proof : In ΔACB and ΔCAD,

AC = CA [Common side]

CB = AD, AB = CD [Given]

∴ ΔACB ≅ ΔCAD [By SSS congruency]

⇒ ∠CAB = ∠ACD and ∠ACB = ∠CAD [By C.P.C.T.] .....(1)

Now, line AC intersects AB and DC at A and C respectively, such that ∠CAB = ∠ACD [From (1)]

i.e., alternate interior angles are equal. ∴ AB || DC .....(2)

Similarly, line AC intersects BC and AD at C and A respectively, such that ∠ACB = ∠CAD [From (1)]

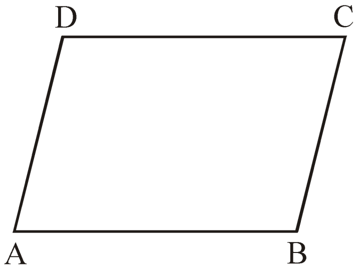
i.e., alternate interior angles are equal. ∴ BC || AD ......(3)

From (2) and (3), we have AB || DC and BC || AD.

Hence, ABCD is a parallelogram.

Theorem -4

Statement : The opposite angles of a parallelogram are equal.



Given : ABCD is a parallelogram.

To Prove : ∠A = ∠C and ∠B = ∠D

Proof : Since, ABCD is a parallelogram.

∴ AB || DC and AD || BC.

∵ AB || DC and transversal AD intersects them at A and D respectively.

∴ ∠A + ∠D = 180° .....(1) [Co-interior angles]

∵ AD || BC and transversal DC intersects them at D and C respectively.

∴ ∠D + ∠C = 180° .....(2) [Co-interior angles]

From (1) and (2), we get ∠A + ∠D = ∠D + ∠C ⇒ ∠A = ∠C.

Similarly, ∠B = ∠D. Hence, ∠A = ∠C and ∠B = ∠D.

Theorem -5

Statement (Converse of Theorem 4) : A quadrilateral is a parallelogram if its opposite angles are equal.

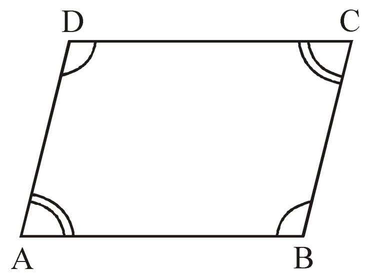
Given : A quadrilateral ABCD in which ∠A = ∠C and ∠B = ∠D.

To Prove : ABCD is a parallelogram.

Proof : In quadrilateral ABCD, we have

$\angle A = \angle C$ .....(1) [Given]

$\angle B = \angle D$ .....(2) [Given]



$\because$ Sum of the angles of a quadrilateral is 360°

$\therefore \angle A + \angle B + \angle C + \angle D = 360°$ .....(3)

$\Rightarrow (\angle A + \angle B) + (\angle A + \angle B) = 360°$ [Using equations (1) and (2)]

$\Rightarrow 2(\angle A + \angle B) = 360° \Rightarrow \angle A + \angle B = 180°$

$\Rightarrow \angle A + \angle B = \angle C + \angle D = 180°$ .....(4)

Transversal AB intersects AD and BC at A and B respectively, such that

$\angle A + \angle B = 180°$, which forms co-interior angles.

$\therefore$ AD || BC .....(5)

$\because \angle A + \angle B = 180°$

$\Rightarrow \angle C + \angle B = 180°$ [By equations (1) and (2)]

Now, transversal BC intersects AB and DC at B and C respectively, such that

$\angle B + \angle C = 180°$, which forms co-interior angles.

$\therefore$ AB || DC .....(6)

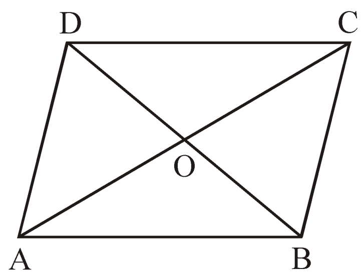
From (5) and (6), we get AD || BC and AB || DC. Hence, ABCD is a parallelogram.

Theorem -6

Statement : The diagonals of a parallelogram bisect each other.

Given : A parallelogram ABCD such that its diagonals AC and BD intersects at O.

To Prove : OA = OC and OB = OD.



Proof : Since, ABCD is a parallelogram. Therefore,

AB || DC and AD || BC.

As AB || DC and AC is a transversal line.

$\therefore \angle BAC = \angle DCA$ [$\because$ Alternate interior angles are equal]

$\Rightarrow \angle BAO = \angle DCO$ .....(1)

Again, AB || DC and BD is a transversal line.

$\therefore \angle ABD = \angle CDB$ [$\because$ Alternate interior angles are equal]

$\Rightarrow \angle ABO = \angle CDO$ .....(2)

In ΔAOB and ΔCOD, we have

$\angle BAO = \angle DCO$ [From equation =(1)]

AB = CD [Opposite sides of a parallelogram]

and, $\angle ABO = \angle CDO$ [From (2)]

$\therefore \Delta AOB \cong \Delta COD$ [By ASA congruency]

⇒ OA = OC and OB = OD [By C.P.C.T.]

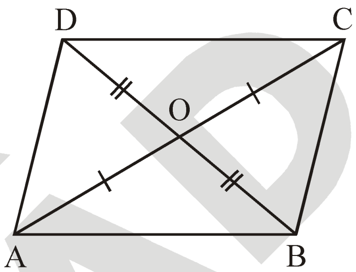
Hence, OA = OC and OB = OD.

Theorem -7

Statement (Converse of Theorem 6) : If the diagonals of a quadrilateral bisects each other, then the quadrilateral is a parallelogram.

Given : A quadrilateral ABCD in which the diagonals AC and BD intersect at O such that AO = OC and BO = OD.

To Prove : Quadrilateral ABCD is a parallelogram.



Proof : In ΔAOD and ΔCOB, we have

OA = OC, OD = OB [Given]

∠AOD = ∠COB [Vertically opposite angles]

∴ ΔAOD ≅ ΔCOB [By SAS congruency]

⇒ ∠OAD = ∠OCB .....(1) [By C.P.C.T.]

Now, line AC intersects AD and BC at A and C respectively, such that ∠OAD = ∠OCB [From (1)]

i.e., alternate interior angles are equal. ∴ AD || BC

Similarly, AB || DC

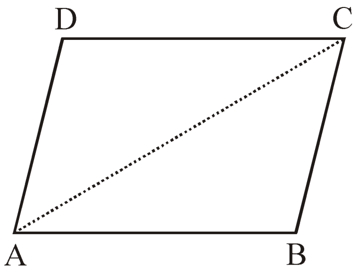
Hence, ABCD is a parallelogram.

Theorem -8

Statement : A quadrilateral is a parallelogram, if its one pair of opposite sides is equal and parallel.

Given : A quadrilateral ABCD, in which AB = DC and AB || DC.

To Prove : ABCD is a parallelogram.



Construction : Join AC.

Proof : In ΔABC and ΔCDA, we have

AB = CD [Given]

AC = CA [Common side]

∠BAC = ∠DCA [As AB || DC ∴ alternate interior angles are equal]

∴ ΔABC ≅ ΔCDA [By SAS congruency]

⇒ ∠BCA = ∠DAC [By C.P.C.T.]

Thus, line AC intersects AD and BC at A and C respectively such that ∠DAC = ∠BCA.

i.e., alternate interior angles are equal.

∴ AD || BC .....(1)

∵ AB || DC [Given] .....(2)

Hence, by equations (1) and (2), quadrilateral ABCD is a parallelogram.

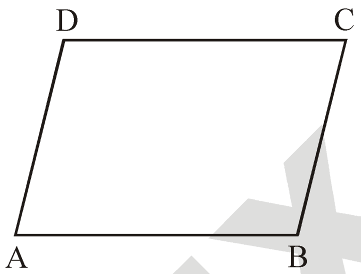
Example 1

In a parallelogram ABCD, prove that sum of any two consecutive angles is 180°.

Solution :

Since, ABCD is a parallelogram,

$\Rightarrow$ AD || BC.



Now, AD || BC and AB is a transversal line.

$\therefore\ \angle A + \angle B = 180°$ [Co-interior angles]

Similarly, $\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$ and $\angle D + \angle A = 180°$.

$\therefore$ The sum of any two consecutive angles is 180°.

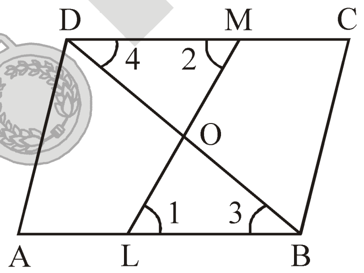
Example 2

ABCD is a parallelogram. L and M are the points on the sides AB and DC respectively and AL = CM. Prove that LM and BD bisect each other.

Solution :

AL = CM [Given]

$\Rightarrow$ BL = DM .....(1) [ABCD is a parallelogram $\Rightarrow$ AB = DC]



Now, AB || DC and transversal BD and LM intersect them.

$\therefore\ \angle 3 = \angle 4$ and $\angle 1 = \angle 2$ .....(2) [$\because$ Alternate angles are equal]

In ΔOBL and ΔODM, we have

$\angle 1 = \angle 2$ [From equation (2)]

BL = MD [From equation (1)]

$\angle 3 = \angle 4$ [From equation (2)]

$\therefore\ \Delta OBL \cong \Delta ODM$ [By ASA congruency]

⇒ OB = OD and OL = OM [By C.P.C.T.]

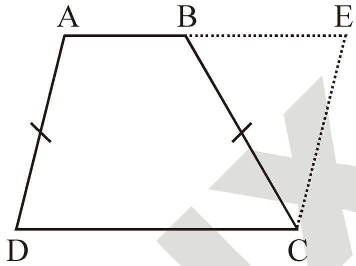
⇒ BD and LM bisect each other.

Example 3

If ABCD is a quadrilateral in which AB || DC and AD = BC, prove that ∠A = ∠B.

Solution :

Produce AB to E and draw CE || DA.



∵ AB || DC [Given]

⇒ AE || DC and DA || CE [By construction]

⇒ AECD is a parallelogram.

⇒ DA = CE [Opposite sides of a parallelogram]

⇒ CB = CE .....(1) [AD = BC (given)]

∴ In ΔBCE, CB = CE [From equation(1)]

⇒ ∠CEB = ∠CBE .....(2) [Angles opposite to equal sides of a Δ are equal]

Now, ∠CBE + ∠ABC = 180° .....(3) [Linear pair]

and ∠BAD + ∠CEB = 180° [Co-interior angles]

⇒ ∠BAD + ∠CBE = 180° .....(4) [From equation (2)]

∴ ∠CBE + ∠ABC = ∠BAD + ∠CBE [From equations (3) and (4)]

⇒ ∠ABC = ∠BAD ⇒ ∠B = ∠A.

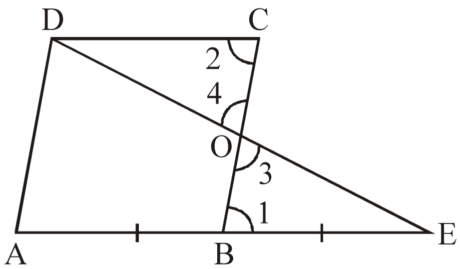
Example 4

ABCD is a parallelogram. AB is produced to E, so that BE = AB. Prove that ED bisects BC.

Solution :

Since, ABCD is a parallelogram.

⇒ AB || DC.



$\Rightarrow$ AE || DC and transversal BC intersects them,

$\therefore \angle 1 = \angle 2$ [alternate interior angles are equal] .....(1)

Also, AB = BE [Given]

and AB = DC [Opposite sides of a parallelogram]

$\therefore$ BE = DC .....(2)

In ΔBOE and ΔCOD, we have

$\angle 1 = \angle 2$ [From equation (1)]

$\angle 3 = \angle 4$ [Vertically opposite angles]

BE = CD [From equation (2)]

$\therefore \Delta BOE \cong \Delta COD$ [By AAS congruency]

$\Rightarrow$ BO = CO [By C.P.C.T.]

$\Rightarrow$ O is the mid point of BC $\Rightarrow$ ED bisects BC.

4. MIDPOINT THEOREM

Theorem -9

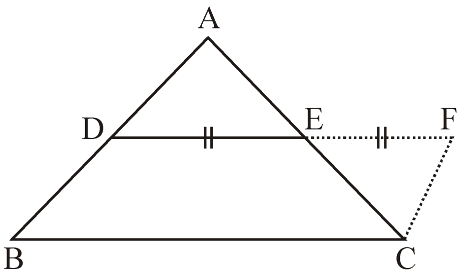
Statement : The line segment joining the midpoint of any two sides of a triangle, is parallel to the third side and equal to half of it.

Given : In a triangle ABC, D is the midpoint of side AB and E is the midpoint of side AC. Join DE.

To Prove : (i) DE || BC (ii) DE = $\frac{1}{2}$ BC.

Construction : Produce line segment DE to F such that DE = EF. Join FC.

Proof : In ΔAED and ΔCEF, we have



AE = EC [E is the midpoint of AC]

$\angle AED = \angle CEF$ [Vertically opposite angles]

DE = EF [By construction]

$\Delta AED \cong \Delta CEF$ [By SAS congruency]

Hence, AD = CF .....(1)

and $\angle ADE = \angle CFE$ .....(2) [By C.P.C.T.]

Also, DB = FC .....(3) [As AD = CF and AD = BD]

$\because \angle ADE = \angle CFE$ (By (2)) and DF is a transveral line.

$\therefore$ AD || FC $\Rightarrow$ DB || FC .....(4)

From (3) and (4), BCFD is a parallelogram.

Hence DF || BC and DF = BC .....(5) [Opposite sides of parallelogram]

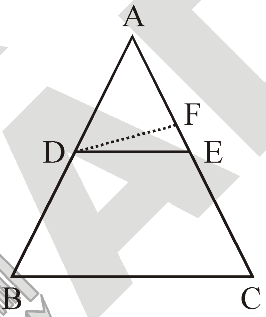
But DE = EF [By construction]

$\Rightarrow$ DE = $\frac{1}{2}$ DF $\Rightarrow$ DE = $\frac{1}{2}$ BC [By equation (5)].

Hence, DE || BC and DE = $\frac{1}{2}$ BC.

Theorem -10

Statement (Converse of Midpoint Theorem) : The line drawn through the midpoint of one side of a triangle, parallel to another side, intersects the third side at its midpoint.



Given : A ΔABC in which D is the midpoint of AB and DE || BC.

To Prove : E is the midpoint of AC.

Proof : Let, if possible E is not the midpoint of AC.

Draw a line DF from a point D intersecting AC at F such that,

F is the midpoint of AC.

In ΔABC, D is the midpoint of AB. [Given]

and F is the midpoint of AC. Therefore, by mid point theorem,

DF || BC .....(1)

Also, DE || BC .....(2) [Given]

From (1) and (2), two intersecting lines DE and DF are both parallel to the line BC. This contradicts the parallel line axiom. So, our supposition is wrong.

Hence, E is the midpoint of AC.

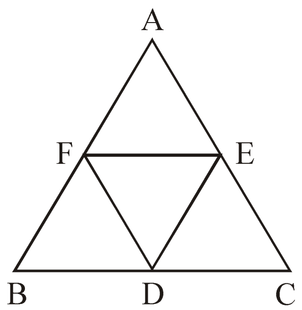
Example 5

If D, E and F are respectively the midpoint of the sides BC, CA and AB of an equilateral triangle ABC, prove that ΔDEF is also an equilateral triangle.

Solution :

Since, D and E are the midpoint of sides BC and CA respectively, we have

DE = $\frac{1}{2}$ AB [By midpoint theorem]



Similarly, FE = $\frac{1}{2}$ BC and DF = $\frac{1}{2}$ CA.

Also, AB = BC = CA [ABC is an equilateral Δ]

$\Rightarrow$ DE = EF = FD

So, ΔDEF is an equilateral triangle.

Example 6

Let ABC be a triangle right-angled at B and D be the midpoint of AC. Show that DA = DB = DC.

Solution :

Through D, draw DE || BC, meeting AB at E.

Now, $\angle AED = \angle ABC = 90°$ [Corresponding angles of parallel sides DE and BC]

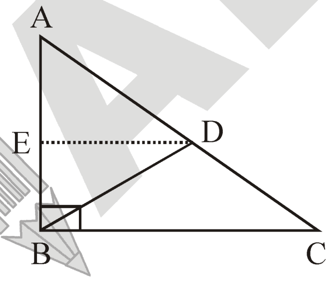
$\therefore \angle BED = \angle AED = 90°$ [$\because \angle AED + \angle BED = 180°$]

Now, in ΔABC, it is given that D is the midpoint of AC and DE || BC [By construction]

∴ E must be the midpoint of AB. [By converse of mid point theorem]

∴ AE = BE .....(1)

Now, in ΔAED and ΔBED, we have



AE = BE [From (1)],

ED = DE [Common side]

$\angle AED = \angle BED$ [Each 90°]

$\therefore \Delta AED \cong \Delta BED$ [By SAS congruency]

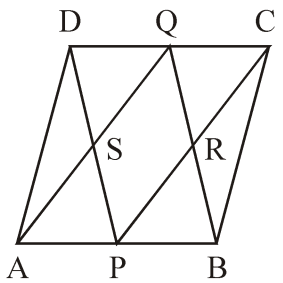
⇒ DA = DB. .....(2) [By C.P.C.T.]

Given, DA = DC .....(3) [As D is the midpoint of AC].

Hence, DA = DB = DC. [From equations (2) and (3)]

Example 7

ABCD is a parallelogram in which P and Q are midpoint of opposite sides AB and CD respectively. If AQ intersects DP at S and BQ intersects CP at R, show that :



(i) APCQ is a parallelogram.

(ii) DPBQ is a parallelogram.

(iii) PSQR is a parallelogram.

Solution :

(i) In quadrilateral APCQ,

AP || QC .....(1) [Since AB || CD]

$\Rightarrow AP = \frac{1}{2} AB$ and $CQ = \frac{1}{2} CD$ [Since P and Q are the midpoint of the sides AB and CD]

Given, AB = CD [Opposite sides of a parallelogram]

So, AP = CQ ......(2)

$\Rightarrow$ APCQ is a parallelogram [From equations (1) and (2)]

(ii) Similarly, quadrilateral DPBQ is a parallelogram, as DQ || PB and DQ = PB.

(iii) In quadrilateral PSQR, SP || QR (SP is a part of DP and QR is a part of QB)

Similarly, SQ || PR.

So, PSQR is a parallelogram.

MATRIX OLYMPIAD

SOLVED EXAMPLES

SE. 1

In a quadrilateral ABCD, bisectors of $\angle B$ and $\angle D$ meets CD and AB produced at P and Q respectively.

Prove that $\angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$.

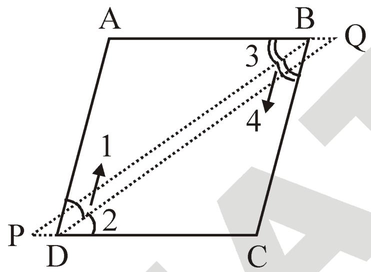
Ans. In ΔPBC, we have $\angle P + \angle 4 + \angle C = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle P + \frac{1}{2}\angle B + \angle C = 180°$ .....(1)

In ΔQAD, we have $\angle A + \angle Q + \angle 1 = 180°$

[Angle sum property of a triangle]



$\Rightarrow \angle Q + \angle A + \frac{1}{2}\angle D = 180°$ .....(2)

Adding equations (1) and (2), we get

$\angle P + \angle Q + \angle A + \angle C + \frac{1}{2}\angle B + \frac{1}{2}\angle D = 360°$

But, $\angle A + \angle B + \angle C + \angle D = 360°$

[Angle sum property of a quadrilateral ABCD]

$\therefore \angle P + \angle Q + \angle A + \angle C + \frac{1}{2}(\angle B + \angle D)$

$= \angle A + \angle B + \angle C + \angle D$

$\Rightarrow \angle P + \angle Q = \frac{1}{2}(\angle B + \angle D)$

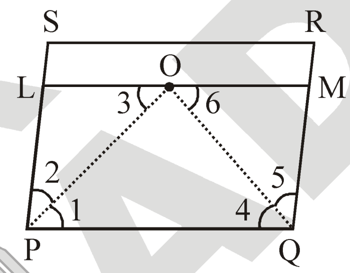
$\therefore \angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$

SE. 2

PQRS is a parallelogram. If PO and QO are, respectively, the angle bisectors of $\angle P$ and $\angle Q$ and line LOM is drawn parallel to PQ inside PQRS. Then, prove that :

(i) PL = QM (ii) LO = OM

Ans. Since PQRS is a parallelogram.



$\therefore$ PS || QR $\Rightarrow$ PL || QM

Thus, we have

PL || QM and LM || PQ

$\Rightarrow$ PQML is a parallelogram

$\Rightarrow$ PL = QM [$\because$ Opposite sides of a $||^{gm}$ are equal]

This proves (i).

Now, OP is the bisector of $\angle P$

$\therefore \angle 1 = \angle 2$ .....(1)

Now, PQ || LM and transversal OP intersects them

$\therefore \angle 1 = \angle 3$ .....(ii)

From equations (i) and (ii), we get $\angle 2 = \angle 3$

Thus, in ΔOPL, we have $\angle 2 = \angle 3$

$\therefore$ OL = PL .....(iii) [Opposite sides of equal angles in a triangle are equal]

Since OQ is the bisector of $\angle Q$

$\therefore \angle 4 = \angle 5$ .....(iv)

Also, PQ || LM and tranversal OQ intersects them

$\therefore \angle 4 = \angle 6$ .....(v)

From (iv) and (v), we get $\angle 5 = \angle 6$

$\Rightarrow$ OM = QM [$\because$ Opposite sides of equal angles

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are equal] .....(vi)

But, PL = QM [PLMQ is a $||^{gm}$] .....(vii)

From (iii), (vi) and (vii), we get OL = OM

SE. 3

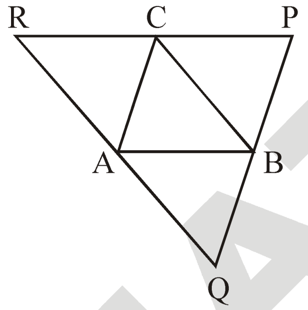
Given ΔABC, lines are drawn through A, B and C parallel to the sides BC, CA and AB respectively, forming ΔPQR. Show that $BC = \frac{1}{2}QR$.

Ans. We have, AQ || CB and AC || QB

⇒ AQBC is a parallelogram.

⇒ BC = AQ .....(1)

[Opposite sides of a parallelogram]



Again, AR || BC and AB || RC

⇒ ARCB is a parallelogram.

⇒ BC = AR .....(2)

[Oppsite sides of a parallelogram]

From equations (1) and (2), we get AQ = RA

$\Rightarrow AQ = RA = \frac{1}{2}QR \Rightarrow BC = \frac{1}{2}QR.$

SE. 4

ABCD is a parallelogram in which ∠DAB = 60°. If the bisectors AP and BP of angles A and B respectively, meet at P on CD, prove that P is the midpoint of CD.

Ans. ∠DAB = 60° [Given]

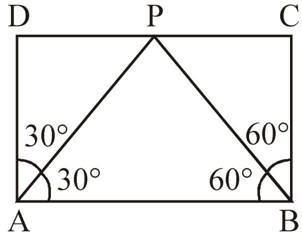
Since, ∠A + ∠B = 180° [Co-interior angles]

∴ 60° + ∠B = 180° ⇒ ∠B = 120°

Now, AB || DC and transversal AP intersects them.

∴ ∠PAB = ∠APD

⇒ ∠APD = 30° [∠PAB = 30°]



In ΔAPD, ∠PAD = ∠APD [Each equal to 30°]

⇒ AD = PD .....(1)

[Sides opposite to equal angles of a triangle]

Since, BP is the bisector of ∠B.

∴ ∠ABP = ∠PBC = 60°

Now, AB || DC and transversal BP intersects them.

∴ ∠CPB = ∠ABP ⇒ ∠CPB = 60° [∠ABP = 60°]

In ΔCBP, ∠CBP = ∠CPB [Each equal to 60°]

⇒ CP = BC ..... (2)

[Sides opposite to equal angles of a triangle]

Also, AD = BC [∵ ABCD is a parallelogram]

⇒ AD = CP .....(3) [From equatoins (2)]

From equations (1) and (3), we get PD = CP

⇒ P is the midpoint of CD.

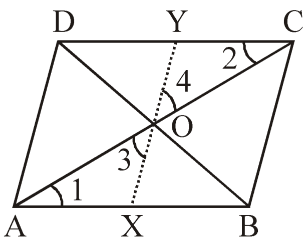
SE. 5

The diagonals of a parallelogram ABCD intersects at O. A line through O intersects AB at X and DC at Y. Prove that OX = OY.

Ans. Since, ABCD is a parallelogram

⇒ AB || DC.

Also, AC is a transversal line.



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$\therefore \angle 1 = \angle 2$ [Alternate angles are equal] ...(1)

Since, the diagonals of a parallelogram bisect each other.

$\therefore$ OA = OC .....(2)

Also, $\angle 3 = \angle 4$ .....(3)

[Vertically opposite angles are equal]

Thus, in ΔOAX and ΔOCY, we have

$\angle 1 = \angle 2$ [From equations (1)]

OA = OC [From equations (2)]

$\angle 3 = \angle 4$ [From equations (3)]

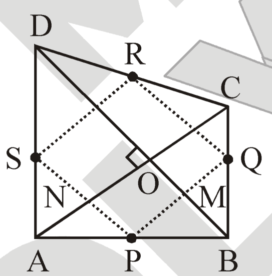
$\therefore$ ΔOAX ≅ ΔOCY [By ASA congruency]

⇒ OX = OY [By C.P.C.T.]

SE. 6

The diagonals of a quadrilateral ABCD are perpendicular. Show that the quadrilateral, formed by joining the midpoints of its sides, is a rectangle.

Ans. **Given :** A quadrilateral whose diagonals AC and BD are perpendicular to each other. P, Q, R and S are the midpoints of sides AB, BC, CD and DA respectively. Join PQ, QR, RS and SP.



To Prove : PQRS is a rectangle.

Proof : In ΔABC, P and Q are the midpoint of AB and BC respectively.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$ AC .....(1)

[By midpoint theorem]

In ΔADC, R and S are the midpoints of CD and AD respectively.

$\therefore$ RS || AC and RS = $\frac{1}{2}$ AC .....(2)

[By midpoint theorem]

PQ || RS and PQ = RS [From equations (1) and (2)] Thus, in quadrilateral PQRS, a pair of opposite sides are equal and parallel. So, PQRS is a parallelogram.

Suppose the diagonals AC and BD of quadrilateral ABCD intersect at O. Now in ΔABD, P is the midpoint of AB and S is the midpoint of AD.

$\therefore$ PS || BD ⇒ PN || MO

Also, from equations (1), PQ || AC ⇒ PM || NO

Thus, in quadrilateral PMON, PN || MO and PM || NO. ⇒ PMON is a parallelogram.

⇒ $\angle$MPN = $\angle$MON

[Opposite angles of a parallelogram]

⇒ $\angle$MPN = $\angle$BOA

⇒ $\angle$MPN = 90° [AC ⊥ BD $\therefore$ $\angle$BOA = 90°]

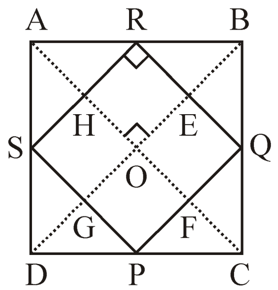
⇒ $\angle$QPS = 90° [$\angle$MPN = $\angle$QPS]

Thus, PQRS is a parallelogram whose one angle $\angle$QPS = 90°. Hence, PQRS is a rectangle.

SE. 7

Show that the quadrilateral formed by joining the midpoints of the sides of a square is also a square.

Ans. **Given :** ABCD is a square. P, Q, R and S are the midpoint of the sides DC, CB, BA and AD respectively. Join PQ, QR, RS and SP.



To Prove : PQRS is a square.

Construction : Join AC and BD.

Proof : Since in a ΔABC, R and Q are the mid points of the sides AB and BC.

$\therefore$ RQ || AC and RQ = $\frac{1}{2}$ AC .....(1)

Similarly SP || AC and SP = $\frac{1}{2}$ AC .....(2)

[By midpoint theorem]

$\therefore$ RQ = SP and RQ || SP [By (1) and (2)]

$\therefore$ PQRS is a parallelogram.

As RQ || AC $\Rightarrow$ RE || HO

and SR || BD $\Rightarrow$ HR || OE

$\Rightarrow$ OERH is a parallelogram.

Now, $\angle HRE = \angle HOE$

[Opposite angles of a parallelogram]

$\Rightarrow \angle HRE = 90°$

[$\angle HOE = 90°$ as diagonals of a square are perpendicular to each other]

$\Rightarrow \angle SRQ = 90°$ [$\angle SRQ = \angle HRE$]

So, quadrilateral PQRS is a rectangle. .....(3)

As AC = BD, RQ = $\frac{1}{2}$ AC = $\frac{1}{2}$ BD

and PQ = $\frac{1}{2}$ BD

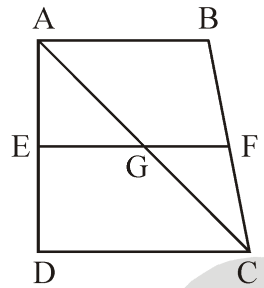
$\Rightarrow$ RQ = PQ .....(4)

$\therefore$ RQ = QP = RS = SP [By equations (3) and (4)]

So, quadrilateral PQRS is a square.

SE. 8

In the given figure, ABCD is a trapezium in which side AB is parallel to side DC and E is the midpoint of side AD. If F is a point on the side BC such that the segment EF is parallel to side DC. prove that EF = $\frac{1}{2}$(AB + DC).



Ans. **Given :** A trapezium ABCD in which AB || DC, E is the midpoint of AD and F is a point on BC such that EF || DC.

To Prove : EF = $\frac{1}{2}$ (AB + DC)

Proof : In ΔADC, E is the midpoint of AD and EG || DC [Given]

$\therefore$ G is the midpoint of AC

[By converse of mid point theorem]

$\Rightarrow$ EG = $\frac{1}{2}$ DC .....(1)

ABCD is a trapezium in which AB || DC. Also, EF || DC.

$\therefore$ EF || AB $\Rightarrow$ GF || AB.

In ΔABC, G is the midpoint of AC and GF || AB.

$\therefore$ F is the midpoint of BC.

[By converse of mid point theorem]

$\Rightarrow$ GF = $\frac{1}{2}$ AB .....(2)

From equations (1) and (2), we have

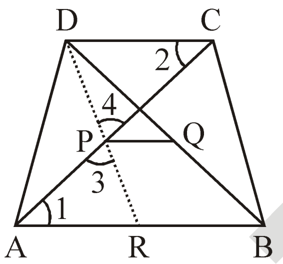
EG + GF = $\frac{1}{2}$ DC + $\frac{1}{2}$ AB

$\Rightarrow EF = \frac{1}{2}(AB + DC)$

SE. 9

Prove that the line segment joining the midpoint of the diagonals of a trapezium are parallel to each of the parallel sides and is equal to half the difference of these sides.

Ans. **Given :** A trapezium ABCD in which AB || DC and P, Q are the midpoints of its diagonals AC, BD respectively.



To Prove : (i) PQ || AB or DC

(ii) $PQ = \frac{1}{2}(AB - DC)$

Construction : Join DP and produce DP to meet AB at R.

Proof : Since, AB || DC and AC is a transversal line.

$\therefore \angle 1 = \angle 2$ [∵ Alternate angles are equal] ...(1)

In ΔAPR and ΔCPD,

$\angle 1 = \angle 2$ [From equation (1)]

AP = CP [P is the midpoint of AC]

$\angle 3 = \angle 4$ [Vertically opposite angles]

$\therefore$ ΔAPR ≅ ΔCPD [By ASA congruency]

⇒ AR = CD and PR = PD .....(2) [By C.P.C.T.]

In ΔDRB, P and Q are the midpoints of sides DR and DB respectively.

$\therefore$ PQ || RB [By mid point theorem]

⇒ PQ || AB [RB is a part of AB]

⇒ PQ || AB or DC [AB || DC]

This proves (i).

Again, P and Q are the midpoints of sides DR and DB respectively in ΔDRB.

$\therefore PQ = \frac{1}{2} RB$ [By midpoint theorem]

$\Rightarrow PQ = \frac{1}{2}(AB - AR)$

$\Rightarrow PQ = \frac{1}{2}(AB - DC)$ [From equation (2), AR = DC]

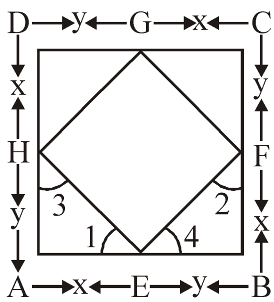
This proves (ii).

SE. 10

ABCD is a square. E, F, G and H are the points on the sides AB, BC, CD and DA respectively, such that AE = BF = CG = DH. Prove that EFGH is a square.

Ans. We have, AE = BF = CG = DH = x (say)

$\therefore$ BE = CF = DG = AH = y (say)



In ΔAEH and ΔBFE, we have

AE = BF [Given]

$\angle A = \angle B$ [each angle is 90°]

AH = BE [AD = BC ⇒ $\frac{1}{2}$ AD = $\frac{1}{2}$ BC]

$\therefore$ ΔAEH ≅ ΔBFE [By SAS congruency]

Also, HE = EF, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [By C.P.C.T.]

But, $\angle 1 + \angle 3 + \angle 2 + \angle 4 = 90° + 90°$

$\Rightarrow \angle 1 + \angle 4 + \angle 1 + \angle 4 = 180°$

$\Rightarrow 2(\angle 1 + \angle 4) = 180° \Rightarrow \angle 1 + \angle 4 = 90°$

$\Rightarrow \angle HEF = 90°$

Similarly, we have

$\angle EFG = \angle HGF = \angle GHE = 90°$ and $HE = EF = FG = GH = \sqrt{x^2 + y^2}$

Hence, EFGH is a square.

SE. 11

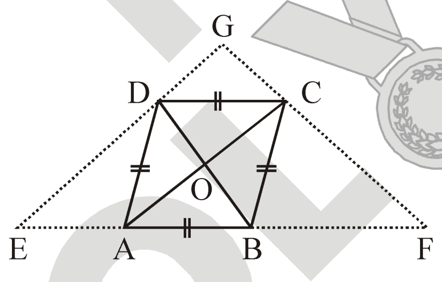
ABCD is a rhombus, EABF is a straight line such that EA = AB = BF. Prove that ED and FC when produced meet at right angle.

Ans. We know that the diagonals of a rhombus are perpendicular bisector of each other.

$\therefore$ OA = OC, OB = OD .....(1)

and $\angle AOB = \angle COB = 90° = \angle AOD = \angle COD$ .....(2)

In ΔBDE, A and O are midpoints of BE and BD respectively.



$\therefore$ OA || DE

$\Rightarrow$ OC || DG [By equation (1)]

In ΔCFA, B and O are midpoints of AF and AC respectively.

$\therefore$ OB || CF $\Rightarrow$ OD || GC [By equation (1)]

Thus, in quadrilateral DOCG, we have OC || DG and OD || GC.

$\Rightarrow$ DOCG is a parallelogram. [Opposite angles of a parallelogram]

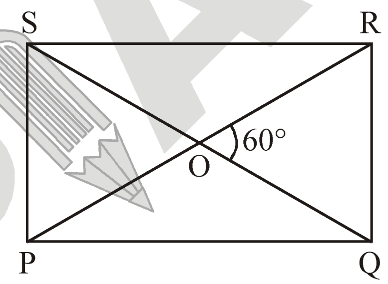
$\therefore \angle DGC = \angle DOC = 90°$ [By (2)]

SE. 12

The diagonals of a rectangle PQRS intersect at O. If $\angle ROQ = 60°$, then find $\angle OSP$.

Ans. $\angle ROQ = \angle SOP = 60°$ .....(1)

[Vertically opposite angles]



Also, PR = SQ $\Rightarrow$ PO = SO

[Diagonals of a rectangle are equal]

$\Rightarrow \angle OPS = \angle OSP$ .....(2)

[$\because$ In a triangle, angles opposite to equal sides are equal]

In ΔPOS, we have

$\angle OSP + \angle OPS + \angle SOP = 180°$

[Angle sum property of a Δ]

$\Rightarrow 2\angle OSP = 180° - 60°$ [Using (1) and (2)]

$\Rightarrow \angle OSP = 60°$

EXERCISE – 8.1

NS. 1

The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of a quadrilateral.

Ans. Let the angles of a quadrilateral be 3x, 5x, 9x and 13x.

$\therefore$ 3x + 5x + 9x + 13x = 360°

[Angle sum property of a quadrilateral]

$\Rightarrow 30x = 360° \Rightarrow x = \dfrac{360°}{30} = 12°$

$\therefore$ 3x = 3 × 12° = 36°

5x = 5 × 12° = 60°

9x = 9 × 12° = 108°

13x = 13 × 12° = 156°

$\Rightarrow$ The required angles of a quadrilateral are 36°, 60°, 108° and 156°.

NS. 2

If the diagonals of a parallelogram are equal, then show that it is a rectangle.

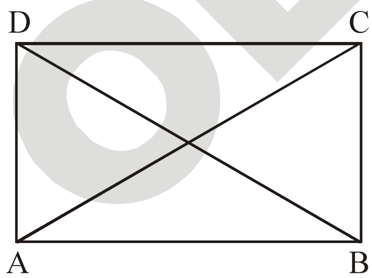
Ans. ABCD is a parallelogram such that AC = BD.

In ΔABC and ΔDCB,

AC = DB [Given]

AB = DC [Opposite sides of a parallelogram]

BC = CB [Common]



$\therefore$ ΔABC ≅ ΔDCB [By SSS congruency]

$\Rightarrow$ ∠ABC = ∠DCB [By C.P.C.T.] ...(1)

Now, AB || DC and BC is a transversal.

[∵ ABCD is a parallelogram]

$\therefore$ ∠ABC + ∠DCB = 180° .....(2)

[Co-interior angles]

From (1) and (2), we have ∠ABC = ∠DCB = 90°

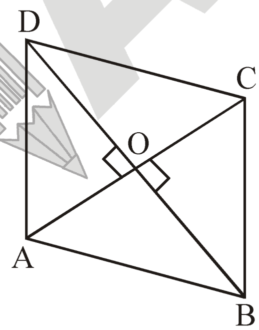
i.e., ABCD is a parallelogram having an angle equal to 90°.

$\therefore$ ABCD is a rectangle.

NS. 3

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Ans. We have a quadrilateral ABCD such that the diagonals AC and BD bisect each other at right angles at O.



$\therefore$ In ΔAOB and ΔAOD, we have

AO = OA [Common]

OB = OD [O is the midpoint of BD]

∠AOB = ∠AOD [Each 90°]

$\therefore$ ΔAOB ≅ ΔAOD [By SAS congruency]

$\therefore$ AB = AD [By C.P.C.T.] .....(1)

Similarly, AB = BC .....(2)

BC = CD .....(3)

CD = DA .....(4)

$\therefore$ From (1), (2), (3) and (4),

we have AB = BC = CD = DA

Thus, a quadrilateral ABCD is a rhombus.

Alternatively : ABCD can be proved first a parallelogram then proving one pair of adjacent sides equal will result in rhombus.

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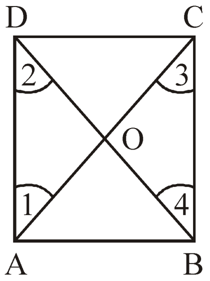
NS. 4

Show that the diagonals of a square are equal and bisect each other at right angles.

Ans. We have a square ABCD such that its diagonals AC and BD intersect at O.



(i) To prove that the diagonals are equal, i.e., AC = BD. In ΔABC and ΔBAD, we have

AB = BA [Common]

BC = AD [Sides of a square ABCD]

∠ABC = ∠BAD [∵ each angle is 90°]

∴ ΔABC ≅ ΔBAD [By SAS congruency]

⇒ AC = BD [By C.P.C.T.] .....(1)

(ii) ∵ AD || BC and AC is a transversal.

[∵ A square is a parallelogram]

∴ ∠1 = ∠3

[Alternate interior angles are equal]

Similarly, ∠2 = ∠4

Now, in ΔOAD and ΔOCB, we have

AD = CB [Sides of a square ABCD]

∠1 = ∠3 [Proved]

∠2 = ∠4 [Proved]

∴ ΔOAD ≅ ΔOCB [By ASA congruency]

⇒ OA = OC and OD = OB [By C.P.C.T.]

i.e., the diagonals AC and BD bisect each other at O. .....(2)

(iii) In ΔOBA and ΔODA, we have

OB = OD [Proved]

BA = DA [Sides of a square]

OA = AO [Common]

∴ ΔOBA ≅ ΔODA [By SSS congruency]

⇒ ∠AOB = ∠AOD .....(3)

[By C.P.C.T.]

∵ ∠AOB and ∠AOD form a linear pair.

∴ ∠AOB + ∠AOD = 180°

∴ ∠AOB = ∠AOD = 90° [By (3)]

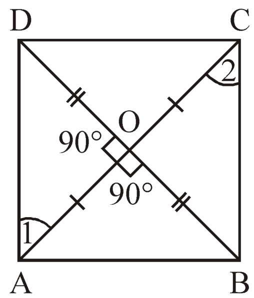
⇒ AC ⊥ BD .....(4)

From (1), (2) and (4), we get AC and BD are equal and bisect each other at right angles.

NS. 5

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Ans. We have a quadrilateral ABCD such that O is the midpoint of AC and BD.



Also, AC ⊥ BD.

Now, in ΔAOD and ΔAOB, we have

∠AOD = ∠AOB [Each 90°]

AO = OA [Common]

OD = OB [∵ O is the midpoint of BD]

∴ ΔAOD ≅ ΔAOB [By SAS congruency]

⇒ AD = AB [By C.P.C.T.] .....(1)

Similarly, we have AB = BC .....(2)

BC = CD .....(3)

CD = DA .....(4)

From (1), (2), (3) and (4), we have

AB = BC = CD = DA

∴ Quadrilateral ABCD have all sides equal.

In ΔAOD and ΔCOB, we have

AO = CO [Given]

OD = OB [Given]

∠AOD = ∠COB [Vertically opposite angles]

So, ΔAOD ≅ ΔCOB [By SAS congruency]

∴ ∠1 = ∠2 [By C.P.C.T.]

But, they form a pair of alternate interior angles.

∴ AD || BC

Similarly, AB || DC

∴ ABCD is a parallelogram

∴ Parallelogram having all its sides equal is a rhombus.

∴ ABCD is a rhombus.

Now, in ΔABC and ΔBAD, we have

AC = BD [Given]

BC = AD [Proved]

AB = BA [Common]

∴ ΔABC ≅ ΔBAD [By SSS congruency]

∴ ∠ABC = ∠BAD [By C.P.C.T.] .....(5)

Since, AD || BC and AB is a transversal.

∴ ∠ABC + ∠BAD = 180° .....(6)

[Adjacent angles are supplementary]

⇒ ∠ABC = ∠BAD = 90° [By (5) and (6)]

So, rhombus ABCD is having one angle equal to 90°.

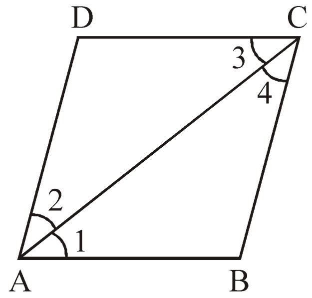
Thus, ABCD is a square.

NS. 6

Diagonal AC of a parallelogram ABCD bisects ∠A. Show that :

(i) it bisects ∠C also.

(ii) ABCD is a rhombus.



Ans. We have a parallelogram ABCD in which diagonal AC bisects ∠A ⇒ ∠DAC = ∠BAC

(i) Since, ABCD is a parallelogram.

∴ AB || DC and AC is a transversal.

∴ ∠1 = ∠3 [Alternate interior angles]....(1)

Also, BC || AD and AC is a transversal.

∴ ∠2 = ∠4 [Alternate interior angles]....(2)

Also, ∠1 = ∠2 [∵ AC bisects ∠A] ....(3)

From (1), (2) and (3), we have ∠3 = ∠4

⇒ AC, bisects ∠C.

(ii) In ΔABC, we have

∠1 = ∠4 [From (2) and (3)]

⇒ BC = AB .....(4)

[∵ Sides opposite to equal angles of a Δ are equal]

Similarly, AD = DC .....(5)

But, ABCD is a parallelogram [Given]

∴ AB = DC .....(6)

From (4), (5) and (6), we have

AB = BC = CD = DA

Thus, ABCD is a rhombus.

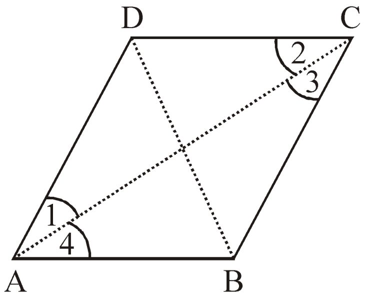
NS. 7

ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Ans. Since ABCD is a rhombus

⇒ AB = BC = CD = DA

Also, AB || CD and AD || BC



Now, AD = CD ⇒ ∠1 = ∠2 .....(1)

[∵ angles opposite to equal sides of a triangle are equal]

Also, AD || BC and AC is the transversal.

[∵ every rhombus is a parallelogram]

∠1 = ∠3 .....(2)

[∵ Alternate interior angles are equal]

From (1) and (2), we have

∠2 = ∠3 .....(3)

Since, AB || DC and AC is transversal.

∴ ∠2 = ∠4 .....(4)

[Alternate interior angles are equal]

From (1) and (4), we have

∠1 = ∠4

⇒ AC bisects ∠C as well as ∠A.

Similarly, we can prove that BD bisects ∠B as well as ∠D.

NS. 8

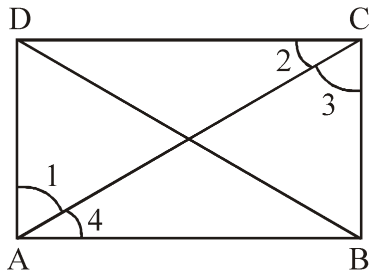
ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that :

(i) ABCD is a square.

(ii) Diagonal BD bisects ∠B as well as ∠D.

Ans. We have a rectangle ABCD such that AC bisects ∠A as well as ∠C.

i.e., ∠1 = ∠4 and ∠2 = ∠3 .....(1)



(i) Since, every rectangle is a parallelogram.

∴ ABCD is a parallelogram.

⇒ AB || CD and AC is a transversal.

∴ ∠2 = ∠4 [Alternate interior angles] ....(2)

From (1) and (2), we have

∠3 = ∠4

In ΔABC, ∠3 = ∠4

⇒ AB = BC

[∵ sides opposite to equal angles of a Δ are equal]

⇒ ABCD is a rectangle having adjacent sides equal.

⇒ ABCD is a square.

(ii) Since, ABCD is a square and diagonals of a square bisect the opposite angles.

So, BD bisects ∠B as well as ∠D.

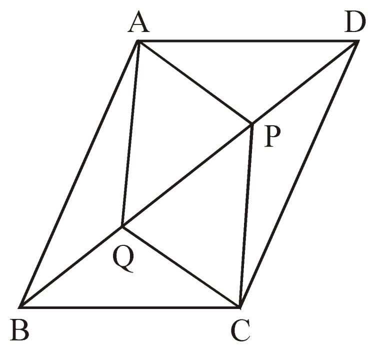
NS. 9

In a parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ. Show that:

(i) ΔAPD ≅ ΔCQB (ii) AP = CQ

(iii) ΔAQB ≅ ΔCPD (iv) AQ = CP

(v) APCQ is a parallelogram



Ans. We have parallelogram ABCD, BD is the diagonal and points P and Q are such that

PD = QB [Given]

(i) Since, AD || BC and BD is a transversal.

$\therefore \angle ADB = \angle CBD$

[$\because$ Alternate interior angles are equal]

$\Rightarrow \angle ADP = \angle CBQ$

Now, in ΔAPD and ΔCQB, we have

AD = CB [Opposite sides of a parallelogram ABCD are equal]

PD = QB [Given]

$\angle ADP = \angle CBQ$ [Proved]

$\therefore \Delta APD \cong \Delta CQB$ [By SAS congruency]

(ii) Since, $\Delta APD \cong \Delta CQB$ [Proved]

$\Rightarrow$ AP = CQ [By C.P.C.T.]

(iii) Since, AB || CD and BD is a transversal.

$\therefore \angle ABD = \angle CDB$

$\Rightarrow \angle ABQ = \angle CDP$

Now, in $\angle AQB$ and $\angle CPD$, we have

QB = PD [Given]

$\angle ABQ = \angle CDP$ [Proved]

AB = CD [$\because$ Opposite sides of a parallelogram ABCD are equal]

$\therefore \Delta AQB \cong \Delta CPD$ [By SAS congruency]

(iv) Since, $\Delta AQB \cong \Delta CPD$ [Proved]

$\Rightarrow$ AQ = CP [By C.P.C.T.]

(v) $\because$ In a quadrilateral APCQ

Opposite sides are equal. [Proved]

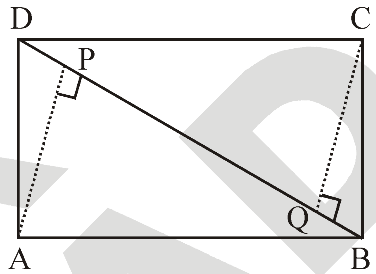
$\therefore$ APCQ is a parallelogram.

NS. 10

ABCD is a parallelogram and AP and CQ are perpendicular from vertices A and C on diagonal BD. Show that :

(i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ



Ans. (i) In ΔAPB and ΔCQD, we have

$\angle APB = \angle CQD$ [Each 90°]

AB = CD [$\because$ Opposite sides of a parallelogram ABCD are equal]

$\angle ABP = \angle CDQ$ [$\because$ Alternate angles are equal as AB || CD and BD is a transversal]

$\therefore \Delta APB \cong \Delta CQD$ [By AAS congruency]

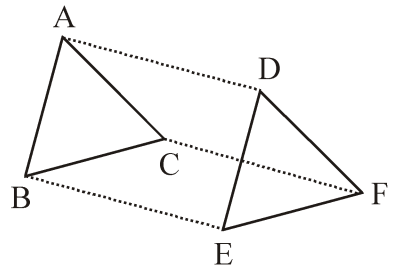
(ii) Since,

$\Delta APB \cong \Delta CQD$ [Proved]

$\Rightarrow$ AP = CQ [By C.P.C.T.]

NS. 11

In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively. Show that :



(i) quadrilateral ABED is a parallelogarm.

(ii) quadrilateral BEFC is a parallelogram.

(iii) AD || CF and AD = CF.

(iv) quadrilateral ACFD is a parallelogram.

(v) AC = DF

(vi) ΔABC ≅ ΔDEF

Ans. (i) We have

AB = DE [Given]

AB || DE [Given]

i.e., ABED is a quadrilateral in which a pair of opposite sides (AB and DE) are parallel and of equal length.

∴ ABED is a parallelogram.

(ii) BC = EF [Given]

and BC || EF [Given]

i.e., BEFC is a quadrilateral in which a pair of opposite sides (BC and EF) are parallel and of equal length.

∴ BEFC is a parallelogram.

(iii) ABED is a parallelogram [Proved]

∴ AD || BE and AD = BE [∵ Opposite sides of a parallelogram are equal and parallel] .....(1)

Also, BEFC is a parallelogram. [Proved]

∴ BE || CF and BE = CF [∵ Opposite sides of a parallelogram are equal and parallel] .....(2)

From (1) and (2), we have

AD || CF and AD = CF

(iv) Since, AD || CF and AD = CF [Proved]

i.e., In quadrilateral ACFD, one pair of opposite sides (AD and CF) are parallel and equal in length.

∴ Quadrilateral ACFD is a parallelogram.

(v) Since, ACFD is a parallelogram. [Proved]

So, AC = DF [∵ opposite sides of a parallelogram are equal]

(vi) In ΔABC and ΔDEF, we have

AB = DE [∵ opposite sides of a parallelogram ABED are equal]

BC = EF [∵ opposite sides of a parallelogram BEFC are equal]

AC = DF [Proved in (v)]

∴ ΔABC ≅ ΔDEF [By SSS congruency]

NS. 12

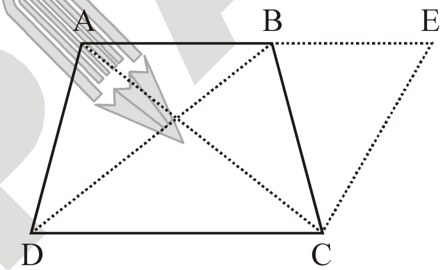
ABCD is a trapezium in which AB || CD and AD = BC. Show that :

(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) ΔABC ≅ ΔBAD

(iv) Diagonal AC = Diagonal BD



Hint : Extend AB and draw a line through C parallel to DA intersecting AB produced at E.

Ans. (i) Produce AB to E and draw CE || AD.

∵ AB || DC ⇒ AE || DC

Also AD || CE [By construction]

∴ AECD is a parallelogram.

⇒ AD = CE .....(1)

[∵ opposite sides of the parallelogram AECD are equal]

But AD = BC .....(2) [Given]

By (1) and (2), we have, BC = CE

Now, in ΔBCE, we have BC = CE

⇒ ∠CEB = ∠CBE .....(1)

[∵ angles opposite to equal sides of a triangle are equal]

Also, $\angle ABC + \angle CBE = 180°$ .....(2) [Linear pair]

and $\angle A + \angle CEB = 180°$ .....(3)

[co-interior angles of a parallelogram ADCE]

From (2) and (3), we get

$\angle ABC + \angle CBE = \angle A + \angle CEB$

$\Rightarrow \angle ABC = \angle A$ [From (1)]

$\Rightarrow \angle B = \angle A$ .....(4)

(ii) AB || CD and AD is a transversal.

$\therefore \angle A + \angle D = 180°$ .....(5)

[Co-interior angles of a parallelogram]

Similarly, $\angle B + \angle C = 180°$ .....(6)

From (5) and (6), we get

$\angle A + \angle D = \angle B + \angle C$

$\Rightarrow \angle C = \angle D$ [From (4)]

(iii) In ΔABC and ΔBAD, we have

AB = BA [Common]

BC = AD [Given]

$\angle ABC = \angle BAD$ [Proved]

$\therefore \Delta ABC \cong \Delta BAD$ [By SAS congruency]

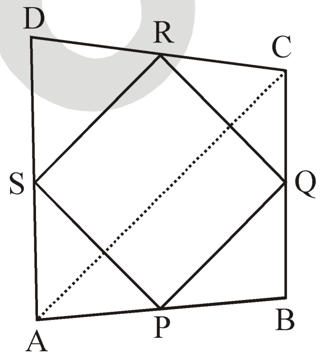
(iv) Since, $\Delta ABC \cong \Delta BAD$ [Proved]

$\Rightarrow$ AC = BD [By C.P.C.T.]

EXERCISE – 8.2

NS. 1

ABCD is a quadrilateral in which P, Q, R and S are midpoints of the sides AB, BC, CD and DA. AC is a diagonal. Show that :



(i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram

Ans. (i) In ΔACD, we have

S is the midpoint of AD and R is the midpoint of CD.

$\therefore$ SR = $\frac{1}{2}$ AC and SR || AC .....(1)

[By mid-point theorem]

(ii) In ΔABC,

P is the midpoint of AB and Q is the midpoint of BC.

$\therefore$ PQ = $\frac{1}{2}$ AC and PQ || AC .....(2)

[By mid-point theorem]

From (1) and (2), we get

PQ = $\frac{1}{2}$ AC = SR and PQ || AC || SR

$\Rightarrow$ PQ = SR and PQ || SR

(iii) In a quadrilateral PQRS.

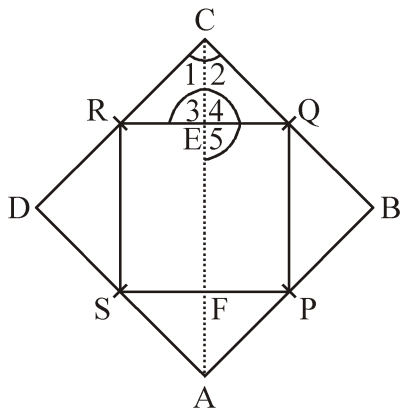
PQ = SR and PQ || SR [Proved]

$\therefore$ PQRS is a parallelogram.

NS. 2

ABCD is a rhombus and P, Q, R, and S are the midpoint of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Ans. Join AC. In ΔABC, P and Q are the midpoints of AB and BC respectively.



$\therefore$ PQ = $\frac{1}{2}$ AC and PQ || AC .....(1)

[By mid-point theorem]

In ΔADC, R and S are the midpoints of CD and DA respectively.

$\therefore$ SR = $\frac{1}{2}$ AC and SR || AC .....(2)

[By mid-point theorem]

From (1) and (2), we get

PQ = $\frac{1}{2}$ AC = SR and PQ || AC || SR

$\Rightarrow$ PQ = SR and PQ || SR

$\therefore$ PQRS is a parallelogram. .....(3)

Now, in ΔERC and ΔEQC,

$\angle 1 = \angle 2$ [$\because$ The diagonals of a rhombus bisects the opposite angles]

CR = CQ $\left[\because \frac{CD}{2} = \frac{BC}{2}\right]$

CE = EC [Common]

$\therefore$ ΔERC ≅ ΔEQC [By SAS congruency]

$\Rightarrow \angle 3 = \angle 4$ .....(4) [By C.P.C.T.]

But $\angle 3 + \angle 4 = 180°$ .....(5) [Linear pair]

From (4) and (5), we get

$\Rightarrow \angle 3 = \angle 4 = 90°$

Now, $\angle RQP = 180° - \angle 5$ [$\because$ Co-interior angles for PQ || AC and EQ is transversal]

But $\angle 5 = \angle 3$ [$\because$ Vertically opposite angles are equal]

$\therefore \angle 5 = 90°$

So, $\angle RQP = 180° - \angle 5 = 90°$

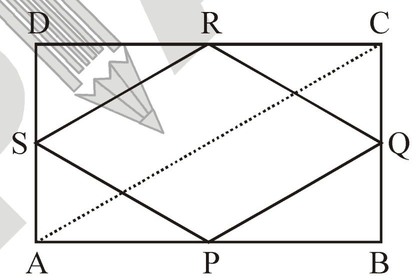
$\therefore$ One angle of parallelogram PQRS is 90°.

Thus, PQRS is a rectangle.

NS. 3

ABCD is a rectangle and P, Q, R and S are midpoints of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Ans.



Now, in ΔABC, we have

PQ = $\frac{1}{2}$ AC and PQ || AC .....(1)

[By mid-point theorem]

Similarly, in ΔADC, we have

SR = $\frac{1}{2}$ AC and SR || AC .....(2)

From (1) and (2), we get

PQ = SR and PQ || SR

$\therefore$ PQRS is a parallelogram.

Now, in ΔPAS and ΔPBQ, we have

$\angle A = \angle B$ [Each 90°]

AP = BP [$\because$ P is the midpoint of AB]

AS = BQ $\left[\because \frac{1}{2}AD = \frac{1}{2}BC\right]$

$\therefore \Delta PAS \cong \Delta PBQ$ [By SAS congruency]

$\Rightarrow PS = PQ$ [By C.P.C.T.]

Also, PS = QR and PQ = SR.

[∵ opposite sides of parallelogram PQRS are equal]

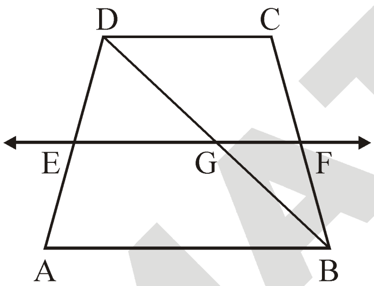
So, PQ = QR = RS = SP

i.e., PQRS is a parallelogram having all of its sides equal.

Hence, PQRS is a rhombus.

NS. 4

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the midpoint of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is the mid point of BC.



Ans. In ΔDAB, we know that E is the midpoint of AD and EG || AB [∵ EF || AB]

∴ Using the converse of midpoint theorem, we get, G is the midpoint of BD.

Again in ΔBDC, we have

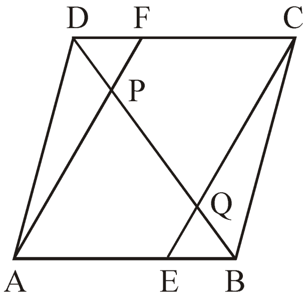
G is the midpoint of BD and GF || DC.

[∵ AB || DC and EF || AB and GF is a part of EF]

Using the converse of the midpoint theorem, we get, F is the midpoint of BC.

NS. 5

In a parallelogram ABCD, E and F are the midpoint of sides AB and CD respectively. Show that the line segment AF and EC trisect the diagonal BD.



Ans. Since, the opposite sides of a parallelogram are parallel and equal.

∴ AB || DC ⇒ AE || FC .....(1)

and AB = DC

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}DC \Rightarrow AE = FC$ .....(2)

From (1) and (2), we have

AE || FC and AE = FC

∴ AECF is a parallelogram.

Now, in ΔDQC, we have

F is the midpoint of DC and FP || CQ [∵ AF || CE]

⇒ DP = PQ .....(3)

[By converse of midpoint theorem]

Similarly, in ΔBAP, E is the midpoint of AB and EQ || AP [∵ AF || CE]

⇒ BQ = PQ .....(4)

[By converse of midpoint theorem]

∴ From (3) and (4), we have

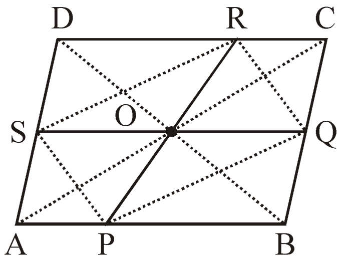
DP = PQ = BQ

So, the line segment AF and EC trisect the diagonal BD.

NS. 6

Show that the line segments joining the midpoints of the opposite sides of a quadrilateral bisect each other.

Ans. Join PQ, QR, RS and SP. Let us also join PR and SQ.



Now, in ΔABC, we have P and Q are the midpoints of its sides AB and BC respectively.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$ AC .....(1)

[By mid-point theorem]

Similarly, RS || AC and RS = $\frac{1}{2}$ AC .....(2)

$\therefore$ By (1) and (2), we get

PQ || RS, PQ = RS

$\therefore$ PQRS is a parallelogram.

And the diagonals of a parallelogram bisect each other, i.e., PR and SQ bisect each other.

Thus, the line segments joining the mid-point of opposite sides of a quadrilateral ABCD bisect each other.

NS. 7

ABC is a triangle right angled at C. A line through the midpoint M of hypotenuse AB and parallel to BC intersects AC at D. Show that :

(i) D is mid point

(ii) MD $\perp$ AC

(iii) CM = MA = $\frac{1}{2}$ AB

Ans. (i) In ΔACB, we have

M is the midpoint of AB. [Given]

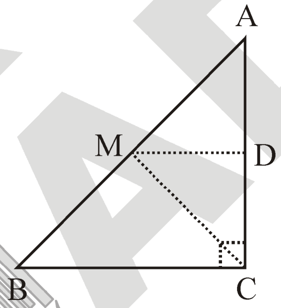
MD || BC [Given]

$\therefore$ Using the converse of mid-point theorem, D is the midpoint of AC.

(ii) Since, MD || BC and AC is a transversal.

$\therefore \angle MDA = \angle BCA$

[$\because$ Corresponding angles are equal]



As $\angle BCA = 90°$ [Given]

$\therefore \angle MDA = 90°$

$\Rightarrow$ MD $\perp$ AC.

(iii) In ΔADM and ΔCDM, we have

$\angle ADM = \angle CDM$ [Each equal to 90°]

MD = DM [Common]

AD = CD [$\because$ D is the midpoint of AC]

$\therefore \Delta ADM \cong \Delta CDM$ [By SAS congruency]

$\Rightarrow$ MA = MC [By C.P.C.T.] .....(1)

$\because$ M is the midpoint of AB [Given]

$\therefore$ MA = $\frac{1}{2}$ AB .....(2)

From (1) and (2), we have

CM = MA = $\frac{1}{2}$ AB.

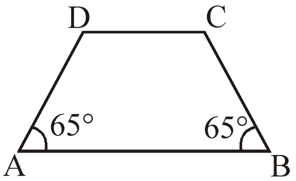
EXERCISE – I

ONLY ONE CORRECT TYPE

1. A quadrilateral having only one pair opposite sides parallel is called a
(A) Square (B) Rhombus
(C) Trapezium (D) Parallelogram

2. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4. The largest angle is :
(A) 36° (B) 72°
(C) 108° (D) 144°

3. In the given figure AB || CD, then measure of ∠C is :

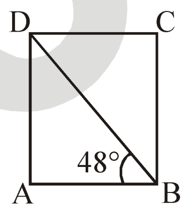


(A) 65° (B) 115°
(C) 135° (D) 125°

4. A quadrilateral has three acute angles each measuring 70°. The measure of fourth angle is :
(A) 140° (B) 150°
(C) 105° (D) 120°

5. In a parallelogram ABCD, ∠A = 115°. The measure of ∠D is equal to :
(A) 115° (B) 65°
(C) 135° (D) 165°

6. In the given figure, ABCD is a rectangle. The measure of ∠DBC is equal to :



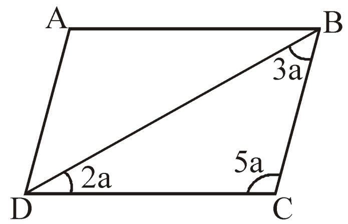
(A) 48° (B) 38°
(C) 42° (D) 52°

7. If in a quadrialteral, two adjacent sides are equal and the opposite sides are unequal, then it is called a
(A) Parallelogram (B) Square
(C) Rectangle (D) Kite

8. The angles of a quadrilateral are x°, (x – 10)°, (x + 30)° and (2x)°, the smallest angle is equal to
(A) 68° (B) 52°
(C) 58° (D) 47°

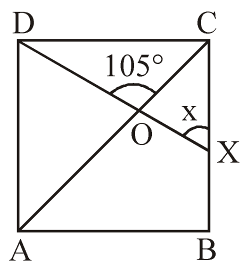
9. Which of the following statements is true ?
(A) In a parallelogram, the diagonals are equal.
(B) In a parallelogram, the diagonals bisect each other.
(C) In a parallelogram, the diagonals intersect each other at right angles.
(D) In any quadrilateral, if a pair of opposite sides are equal, it is parallelogram.

10. In the given figure, the measure of ∠C is equal to



(A) 90° (B) 80°
(C) 75° (D) 95°

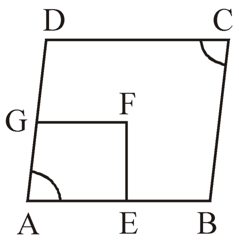
11. In the given figure, if ABCD is a square, the value of x is :



(A) 45° (B) 60°
(C) 70° (D) 36°

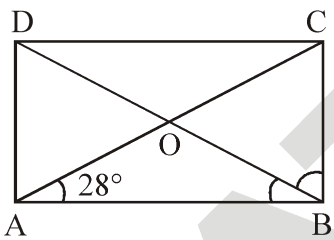
MATRIX OLYMPIAD

12. In the following figure, ABCD and AEFG are two parallelograms. If ∠C = 55°, find ∠F.



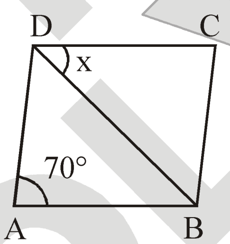
(A) 65° (B) 75°
(C) 85° (D) 55°

13. In the given figure, ABCD is a rectangle whose diagonals AC and BD intersect at O. If ∠OAB = 28°, then ∠OBC is equal to



(A) 72° (B) 50°
(C) 62° (D) 75°

14. In the given figure, ABCD is a rhombus. If ∠A = 70°, then ∠CDB is equal to



(A) 65° (B) 55°
(C) 75° (D) 80°

15. Which is not correct about rectangle EFGH ?
(A) ∠E = ∠F = ∠G = ∠H = 90°
(B) EG = FH
(C) EF = GH and HE = FG
(D) EG and FH are ⊥ bisectors

16. In a parallelogram ABCD, if ∠A = 80° then ∠B is equal to
(A) 80° (B) 180°
(C) 100° (D) 120°

17. Two adjacent angles of parallelogram are in the ratio 2 : 3. The angle are
(A) 90°, 180° (B) 36°, 144°
(C) 72°, 108° (D) 52°, 104°

18. PQRS is a square, PR and SQ intersect at O. The measure of ∠POQ is :
(A) 45° (B) 90°
(C) 180° (D) None of these

19. In a quadrilateral ABCD, ∠A + ∠C = 180°, then ∠B + ∠D is equal to
(A) 360° (B) 100°
(C) 180° (D) 80°

20. In a parallelogram ABCD, diagonals AC and BD intersect at O and AC = 12.8 cm and BD = 7.6 cm. The measure of OC and OD respectively are
(A) 6.4 cm, 3.8 cm (B) 2.4 cm, 3.8 cm
(C) 4.5 cm, 6.4 cm (D) 3.8 cm, 6.4 cm

21. Two adjacent angles of a parallelogram are (2x + 25)° and (3x – 5)°. The value of x is :
(A) 28° (B) 32°
(C) 36° (D) 42°

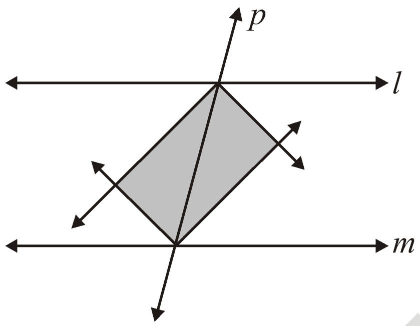
22. The length and breadth of a rectangle are in the ratio 4 : 3. If the diagonal measures 25 cm, then the perimeter of the rectangle is :
(A) 58 cm (B) 60 cm
(C) 70 cm (D) 80 cm

23. In a square ABCD, AB = (2x + 3) cm and BC = (3x – 5) cm. Then, the value of x is :

(A) 5 (B) 7

(C) 8 (D) 10

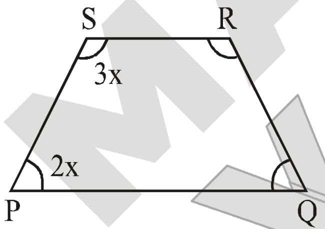
24. Two parallel lines *l* and m are intersected by a transversal p. The quadrilateral formed by the bisectors of interior angles is a



(A) Trapezium (B) Kite

(C) Parallelogram (D) Square

25. In figure, PQRS is an isosceles trapezium. Find x.



(A) 30° (B) 40°

(C) 36° (D) 35°

PARAGRAPH TYPE

Passage – I : The sum of the four angles of a quadrilateral is 360°.

26. The angles of a quadrilateral are 100°, 98°, 92° respectively. Find the fourth angle.

(A) 70° (B) 80°

(C) 40° (D) 90°

27. In a quadrilateral ABCD, the angles A, B, C and D are in the ratio 1 : 2 : 4 : 5, then the measure of each angle of a quadrilateral is :

(A) 36°, 60°, 108°, 156°

(B) 30°, 60°, 120°, 150°

(C) 42°, 54°, 110°, 154°

(D) 72°, 108°, 36°, 144°

28. Three angles of a quadrilateral are respectively equal to 110°, 50° and 40°. Find its fourth angle.

(A) 160° (B) 120°

(C) 80° (D) 140°

Passage – II : In a parallelogram ABCD, the sum of any two consecutive angles is 180° and opposite angles are equal.

29. In a parallelogram ABCD, ∠D = 115°, determine the measure of ∠A and ∠B.

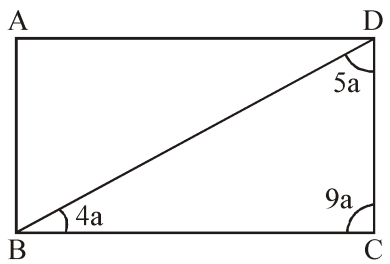
(A) ∠A = 85°, ∠B = 115°

(B) ∠A = 65°, ∠B = 65°

(C) ∠A = 65°, ∠B = 115°

(D) ∠A = 75°, ∠B = 105°

30. In the given figure, find ∠A in the parallelogram ABCD.



(A) 90° (B) 60°

(C) 30° (D) 110°

MATRIX OLYMPIAD

31. Find the value of ∠Q and ∠P, if ∠P = 10a and ∠R = 50° in a parallelogram PQRS.

(A) ∠Q = 50°, ∠P = 130°

(B) ∠Q = 130°, ∠P = 50°

(C) ∠Q = 100°, ∠P = 120°

(D) ∠Q = 50°, ∠P = 100°

MATCH THE COLUMN TYPE

In this section, each question has two matching lists. Choices for the correct combination of elements from Column I and Column II are given as options (A), (B), (C) and (D) out of which one is correct.

32. Match the following :

Column – I	**Column – II**
(P) Trapezium	(i) Each angle is 90°.
(Q) Rectangle	(ii) Equal adjacent sides but unequal opposite sides.
(R) Rhombus	(iii) Unequal sides.
(S) Kite	(iv) All sides are equal.

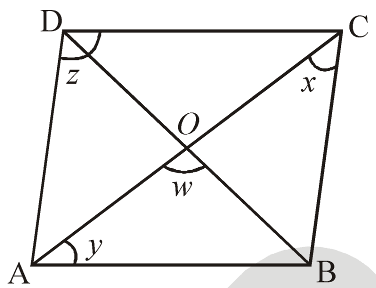
(A) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(B) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)

(C) (P) → (iv), (Q) → (iii), (R) → (ii), (S) → (i)

(D) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)

33. By using a given figure of quadrilateral ABCD, match Column – I with Column – II.



Column – I	**Column – II**
(P) If ABCD is a parallelogram, then sum of the angles x, y and z is	(i) 25°
(Q) If ABCD is a rhombus, where ∠D = 130°, then the value of x is	(ii) 180°
(R) If ABCD is a rhombus, the value of w is	(iii) 50°
(S) If ABCD is a parallelogram, where x + y = 130°, the value of z is	(iv) 90°

(A) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

(B) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)

(C) (P) → (ii), (Q) → (i), (R) → (iv), (S) → (iii)

(D) (P) → (ii), (Q) → (iv), (R) → (iii), (S) → (i)

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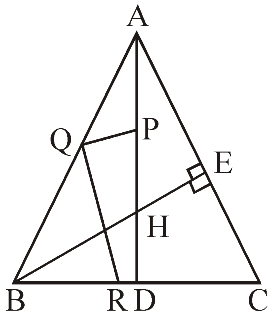
EXERCISE – II

VERY SHORT ANSWER TYPE

1. Find the measure of all the angles of a parallelogram, if one angle of the adjacent angles is 20° less than thrice the smallest angle.

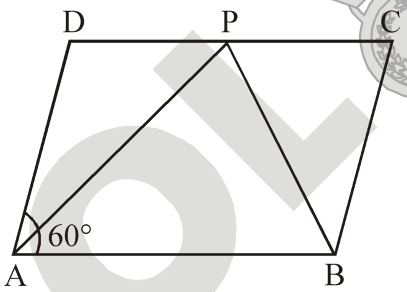
2. In the given figure, BE ⊥ AC. AD is any line from A to BC intersecting BE at H. P, Q and R are the midpoints of AH, AB and BC respectively. Prove that ∠PQR = 90°.



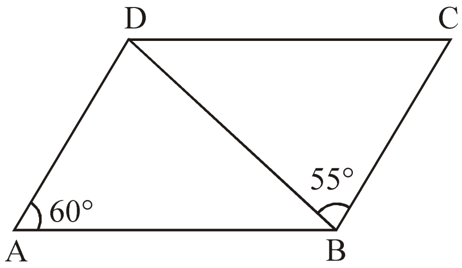
3. In a parallelogram ABCD, ∠D = 135°, determine the measures of ∠A and ∠B.

4. ABCD is a parallelogram in which ∠A = 78°. Compute ∠B, ∠C and ∠D.

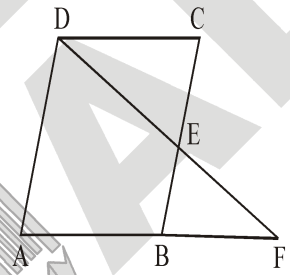
5. In the given figure, ABCD is a parallelogram in which ∠A = 60°. If the bisectors of ∠A and ∠B meet at P, prove that AD = DP.



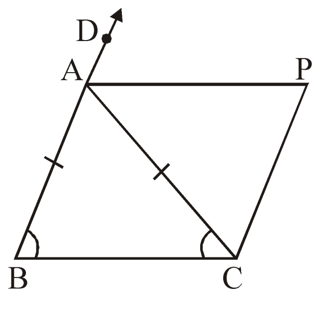
6. In the given figure, ABCD is a parallelogram in which ∠DAB = 60° and ∠DBC = 55°. Compute ∠CDB and ∠ADB.



7. In the given figure, ABCD is a parallelogram and E is the midpoint of side BC. If DE and AB when produced meet at F, prove that AF = 2AB.



8. In the given figure, AB = AC and CP || BA and AP is the bisector of exterior ∠CAD of ΔABC. Prove that ∠PAC = ∠BCA and ABCP is a parallelogram.



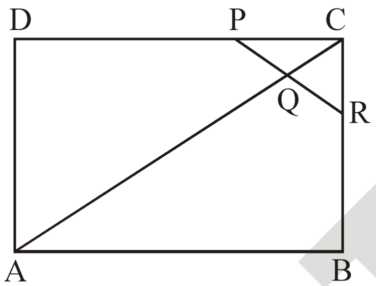
9. ABCD is a rectangle with ∠ABD = 50°. Determine ∠DBC.

10. In ΔABC, ∠A = 50°, ∠B = 30° and ∠C = 100°. What are the angles of the triangle formed by joining the midpoints of the sides of this triangle ?

MATRIX OLYMPIAD

SHORT ANSWER TYPE

1. Let ABC be an isosceles triangle in which AB = AC. If D, E, F be the midpoints of the sides BC, CA and AB respectively, show that AD and EF bisect each other at right angles.
2. In given figure, ABCD is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$. If PQ produced meet BC at R, prove that R is a midpoint of BC.



3. ABCD is parallelogram. P is a point on AD such that $AP = \frac{1}{3}AD$ and Q is a point on BC such that $CQ = \frac{1}{3}BC$. Prove that AQCP is a parallelogram.
4. In a parallelogram ABCD, prove that it is a rhombus, if diagonals bisect each other at 90°.
5. ABC is a triangle. D is a point on AB such that $AD = \frac{1}{4}AB$ and E is a point on AC such that $AE = \frac{1}{4}AC$. Prove that $DE = \frac{1}{4}BC$.

LONG ANSWER TYPE

1. In a parallelogram ABCD, the bisector of ∠A also bisects BC at X. Prove that AD = 2AB.
2. ABCD is a parallelogram, AD is produced to E so that DE = DC and EC produced meets AB produced in F. Prove that BF = BC.
3. ABCD and APCR are the two parallelograms and AC is the common diagonal. Prove that PBRD is a parallelogram.
4. ABCD is a parallelogram. AB and AD are produced to P and Q respectively such that BP = AB and DQ = AD. Prove that P, C, Q lie on a straight line.
5. ABCD is a parallelogram. BT bisects ∠ABC and meets AD at T. A straight line through C and parallel to BT meets AB produced at P and AD produced at R. Prove that ΔRAP is isosceles and the sum of two equal sides of ΔRAP is equal to the perimeter of the parallelogram ABCD.

TRUE / FALSE

1. All rectangles are squares .
2. All rhombuses are parallelograms .
3. All squaers are rhombuses & also rectangles .
4. All kites are rhombuses.
5. All parallelograms are trapeziums .

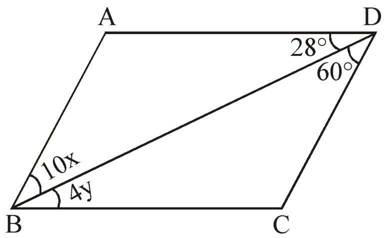
FILL IN THE BLANKS

1. Opposite angles of a parallelogram are______.
2. Diagonals of a square ______ each other at right angles.
3. In a square ABCD, AB= 7+3y & $BC = 28 + \frac{2}{3}y$ then y is ______.
4. The diagonls of a rhombus are 16 cm & 12 cm . Then the side of the rhombus is ______.
5. ______ is made by the bisectors of angles of a parallelogram .

MATRIX OLYMPIAD

NUMERICAL PROBLEMS

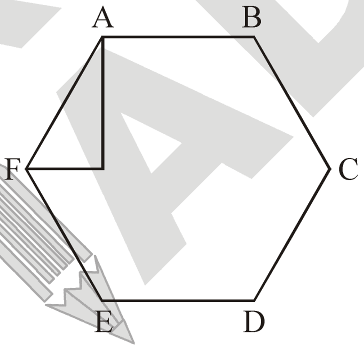
1. The perimeter of a parallelogram is 20 cm. If the longer side measures 6 cm, then measure of the shorter side is equal to.
2. In a rhombus ABCD, $\angle A = 60°$ and AB = 6 cm. The length of the diagonal BD is equal to.
3. In the given figure, if ABCD is a parallelogram, then the value of 2x + y is equal to _____ degress.



4. The lengths of the diagonal of a rhombus are 16 cm and 12 cm. The length of each side of the rhombus is k cm. The value of 3k is.
5. In a square ABCD, AB = (4x + 3) cm and BC = (5x – 6) cm. Then, the value of x is.

ANALYTICAL PROBLEMS & BRAIN TEASER

1. The length of the parallel sides of a trapezium are 14 cm and 7 cm. If the length of third side is 8 cm and of fourth side is x cm, then the number of possible integral value of x is.
   (A) 12 (B) 13
   (C) 14 (D) 17
2. If the diagonals of a rhombus are 30 cm and 40 cm, then the length of side of rhombus is
   (A) 20 cm (B) 22 cm
   (C) 25 cm (D) 45 cm
3. In the given figure, ABCDEF is a regular hexagon and $\angle AOF = 90°$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF ?
   (A) $\frac{1}{12}$ (B) $\frac{1}{6}$
   (C) $\frac{1}{24}$ (D) $\frac{1}{8}$



4. The diagonals of rectangle ABCD intersect each other at O. If $\angle BOC = 44°$, then the value of $\angle OAD$ will be :
   (A) 120° (B) 68°
   (C) 90° (D) 44°

Answer Key

EXERCISE–I														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	D	B	B	B	C	D	C	B	A	B	D	C	B	D
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	C	B	C	A	B	C	C	C	C	A	B	A	C	A
31	32	33												
B	D	C												

EXERCISE II

VERY SHORT ANSWER TYPE

1. 50°, 130°, 50°, 130° **3.** 45°, 135° **4.** 102°, 78°, 102°

6. 65°, 55° **9.** 40° **10.** 100°, 30°, 50°

TRUE / FLASE

1. F 2. T 3. T 4. F 5. T

FILL IN THE BLANKS.

1. equal 2. bisect 3. 9 4. 10 5. rectangle

ANALYTICAL PROBLEMS & BRAIN TEASER

1. B **2.** C **3.** A **4.** B

NUMERICAL PROBLEMS

1. 4 **2.** 6 **3.** 19 **4.** 30 **5.** 9

MATRIX OLYMPIAD

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : QUADRILATERALS)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Exercises			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

MATRIX OLYMPIAD

Space for Notes :

AREAS OF PARALLELOGRAM AND TRIANGLES

9

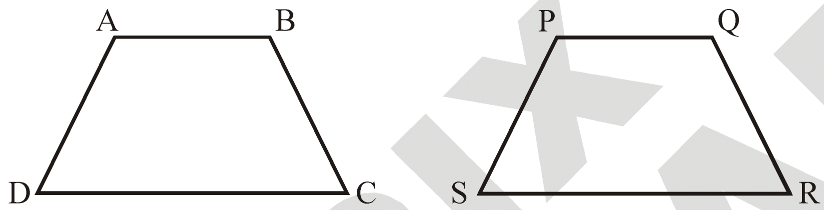
Concepts

INTRODUCTION

The magnitude or measure of the planar region is called its Area. The part of the plane enclosed by a simple closed figure is called a planar region corresponding to that figure. The magnitude or measure is always expressed with the help of a number such as 5 cm^2, 10 cm^2, etc.

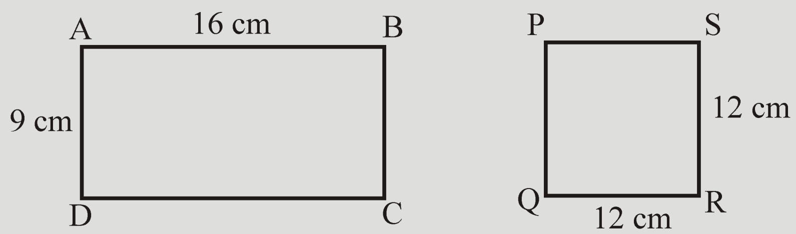
Let us look at the two figures given below.

If two quadrilateral ABCD and PQRS, overlap each other completely, then both the quadrilaterals are congruent i.e., they have same shape and size.

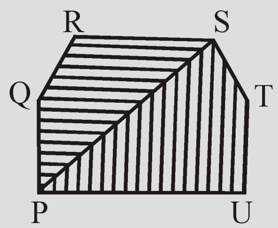


Focus Point

- Two figures are congruent, then they have equal areas but the converse of this statement is not true. For below example, rectangles ABCD and PQRS have equal areas (16×9 cm^2 and 12×12 cm^2) but clearly they are not congruent.



Now let us look at the figure PQRSTU which is made up of quadrilaterals PQRS and PSTU.



So, Area of figure PQRSTU = Area of figure PQRS + Area of figure PSTU.

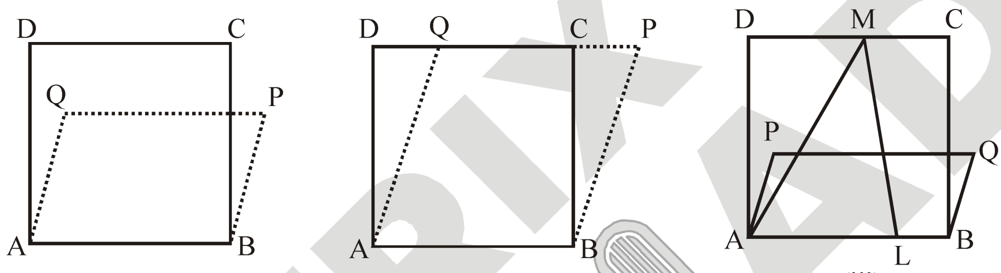
- The area of figure X is denoted as ar(X).

1. AREA AXIOMS

(i) If R_1 and R_2 be two congruent figures, i.e., $R_1 \cong R_2$ then $ar(R_1) = ar(R_2)$.

(ii) If a planar region formed by a figure R is made up of two non–overlapping regions formed by figures P and Q then ar(R) = ar(P) + ar(Q).

2. FIGURES ON THE SAME BASE AND BETWEEN THE SAME PARALLELS



(i) (ii) (iii)

Look at the following figures

(i) parallelogram ABCD and ABPQ are on the same base AB. If two geometric figures have a common side, we say that they are on the same base.

In fig. (ii) parallelograms ABCD and ABPQ are on the same base AB and between the same parallels AB and DP as the vertices C & D of parallelogram ABCD and P & Q of parallelogram ABPQ lie on a line DP parallel to base AB. Thus, two geometric figures are said to be on the same base and between the same parallel lines, if they have a common side (base) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

In fig. (iii) we find the parallelograms ABCD and ABQP are on the same base AB but they are not between the same parallel lines as the vertices P, Q of parallelogram ABQP and C, D of parallelogram ABCD do not lie on the same line. Also, ΔMAL and parallelogram ABCD are between the same parallel lines but they are not on the common base.

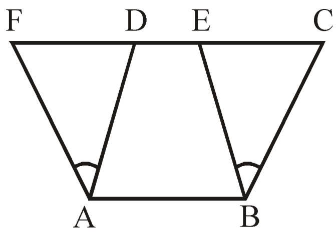
Theorem - 1

Statement : Parallelograms on the same base and between the same parallels are equal in area.

Given : Two $||^{gms}$ ABCD and ABEF are on the same base AB and between the same parallel lines AB and FC.

To Prove : ar($||^{gm}$ ABCD) = ar($||^{gm}$ ABEF)

MATRIX OLYMPIAD



Proof : In ΔADF and ΔBCE, we have :

AD = BC [Opposite sides of a $||^{gm}$ ABCD]

AF = BE [Opposite sides of a $||^{gm}$ ABEF]

∠DAF = ∠CBE

[AD || BC and AF || BE so angle between sides AD and AF = angle between sides BC and BE]

∴ ΔADF ≅ ΔBCE [By SAS Congruency]

So, ar(ΔADF) = (ΔBCE) [Area axiom]

Now, ar($||^{gm}$ ABCD) = ar(ABED) + ar(ΔBCE)

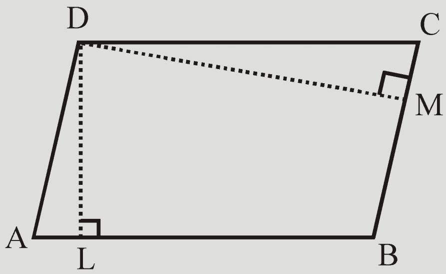
= ar(ABED) + ar(ΔADF)

= ar($||^{gm}$ ABEF)

Hence, ar($||^{gm}$ ABCD) = ar ($||^{gm}$ ABEF)

Focus Point

- **BASE :-** Any side of parallelogram can be called its base.
- **ALTITUDE :-** The length of the line segment which is perpendicular to the base from the opposite vertex is called the altitude or height of the parallelogram corresponding to the given base.



In the adjoining figure

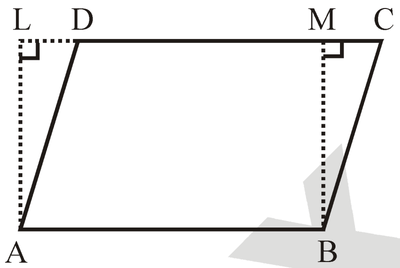
(i) DL is the altitude of $||^{gm}$ ABCD, corresponding to the base AB.

(ii) DM is the altitude of $||^{gm}$ ABCD, corresponding to the base BC.

Corollary - 1

A parallelogram and a rectangle on the same base and between the same parallels are equal in area. Since a rectangle is also a parallelogram.

Corollary - 2



Area of a parallelogram = base × height.

Given : A | |^{gm} ABCD in which AB is the base and AL is the corresponding height.

Construction : Draw BM ⊥ DC so that rectangle ABML is formed.

Proof : Parallelogram ABCD and rectangle ABML are on the same base AB and between the same parallel lines AB and LC.

∴ ar (| |^{gm} ABCD) =ar(rect. ABML) = AB × AL

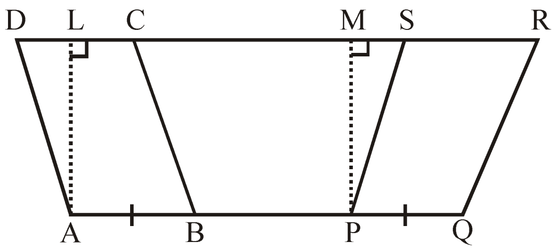
∴ area of a | |^{gm} = base × height.

Corollary - 3

Parallelograms on equal bases and between the same parallels are equal in area.

Given : Two | |^{gms} ABCD and PQRS with equal bases AB and PQ and between the same parallels, AQ and DR.

To Prove : ar(| |^{gm} ABCD) = ar(| |^{gm} PQRS).



Construction : Draw AL ⊥ DR and PM ⊥ DR

Proof : AB | | DR, AL ⊥ DR and PM ⊥ DR

∴ AL = PM [∵ The perpendicualr distance between two same parallel lines is same]

∴ ar(| |^{gm} ABCD) = AB × AL = PQ × PM [AB = PQ and AL = PM]

= ar(| |^{gm} PQRS).

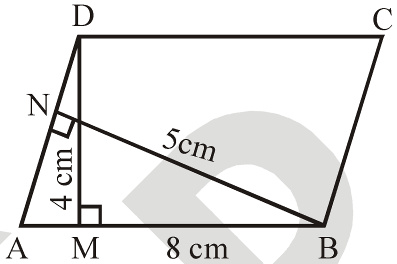
Hence, ar(| |^{gm} ABCD) = ar (| |^{gm} PQRS).

Example 1

In a parallelogram ABCD, AB = 8 cm. The altitudes corresponding to sides AB and AD are 4 cm and 5 cm respectively. Find AD.

Solution :

We know that, area of a parallelogram = base × Corresponding altitude.



$\therefore$ Area of parallelogram ABCD

= AD × BN

= AB × DM

$\Rightarrow$ AD × 5cm = 8 × 4 cm^2

$\Rightarrow AD = \frac{8 \times 4}{5}$ cm = 6.4 cm.

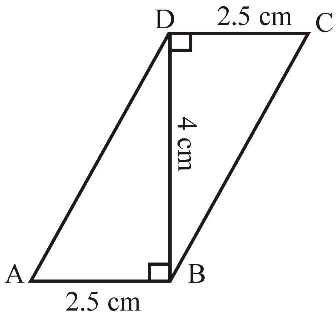
Example 2

ABCD is a quadrilateral and BD is one of its diagonals; as shown in the figure. Prove that quadrilateral ABCD is a parallelogram, also find its area.

Solution :

From figure, the transversal DB is intersecting a pair of lines DC and AB such that $\angle CDB = \angle ABD = 90^\circ$.

Hence these angles form a pair of alternate equal angles.



$\therefore$ DC | | AB and we have given AB = DC

$\therefore$ Quadrilateral ABCD is a parallelogram.

Now, area of parallelogram ABCD

= Base × Corresponding altitude

= 2.5 × 4cm^2 = 10cm^2

Example 3

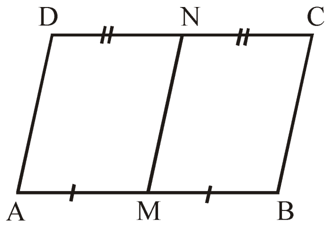
Show that the line segment joining the mid–point of a pair of opposite sides of a parallelogram divides it into two equal parallelograms.

Solution :

Given : Let ABCD be a parallelogram in which M and N are the mid–points of a pair of its opposite sides AB and DC respectively. MN is joined.

To Prove : ar(| |^{gm} AMND) = ar(| |^{gm} MBCN)

Proof : ABCD is a parallelogram.



$\therefore$ AB = DC and AB | | DC

$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ DC and AM | | DN

$\therefore$ AM = DN and AM | | DN

$\Rightarrow$ Quadrilateral AMND is a parallelogram.

Similarly, quad. MBCN is a parallelogram.

Now since the parallelogram AMND and MDCN are on the equal bases AM and MB

(M is the mid–point of AB) and between the same parallels AB and DC.

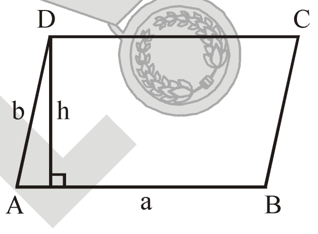
$\therefore$ ar(| |^{gm} AMND) = ar(| |^{gm} MBCN).

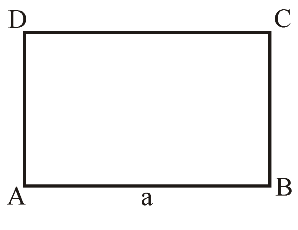
Example 4

Prove that of all parallelograms of which the sides are given, the parallelograms which is rectangle has the greatest area.

Solution :

Let ABCD be a parallelogram in which AB = a and AD = b. Let h be the altitude corresponding to the base AB. Then,





ar(| |^{gm} ABCD) = AB × h = ah .....(1)

We can construct infinitely many parallelograms with different h and b but with same base a.

Also from equation (1)

ar(| |^{gm} ABCD) is maximum or greatest when h is maximum.

Since the fig. (i) shows that ΔADM is a right angled triangle.

$\therefore$ b > h

The maximum value which h can attain is AD = b and this is possible when AD is perpendicular to AB i.e., the | |gm ABCD becomes a rectangle.

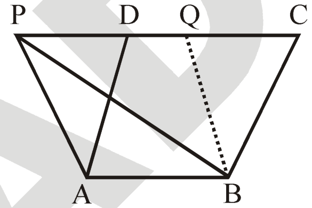
Thus, ar(| |^{gm} ABCD) is greatest when AD ⊥ AB i.e., when | |^{gm} ABCD is a rectangle.

3. TRIANGLES ON THE SAME BASE AND BETWEEN THE SAME PARALLELS

Theorem - 2

Statement : If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

Given : Let ΔABP and parallelogram ABCD are on the same base AB and between the same parallels AB and PC (see fig).



To Prove : ar(ΔPAB) = $\frac{1}{2}$ ar(| |^{gm} ABCD)

Construction : Draw BQ | | AP to obtain another parallelogram ABQP.

Proof : Parallelograms ABQP and ABCD are on the same base AB and between the same parallels AB and PC.

Therefore, ar(| |^{gm} ABQP) = ar(| |^{gm} ABCD) [By Theorem 1] ...(i)

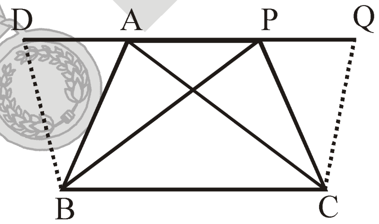
Also, ΔPAB ≅ ΔBQP [Diagonal PB divides parallelogram ABQP into two congruent triangles]

So, ar(ΔPAB) = ar(ΔBQP) ...(ii)

Therefore, ar(ΔPAB) = $\frac{1}{2}$ ar(| |^{gm} ABQP) [From (ii)] ...(iii)

This gives ar(ΔPAB) = $\frac{1}{2}$ ar(| |^{gm} ABCD) [From (i) and (iii)]

Theorem - 3



Statement : Triangles on the same base and between the same parallels are equal in area.

Given : Let two triangles ABC and PBC are on the same base BC and between the same parallel lines BC and AP.

To Prove : ar(ΔABC) = ar(ΔPBC)

Construction : Through B, draw BD | | CP, intersecting line AP at D and through C, draw CQ | | BA, intersecting line AP at Q.

Proof : Quadrilaterals BCQA & BCPD are parallelograms. [By construction]

Parallelograms BCQA and BCPD are on the same base BC and between the same parallels BC and DQ.

$\therefore$ ar(| |^{gm} BCQA) = ar(| |^{gm} BCPD) ...(i)

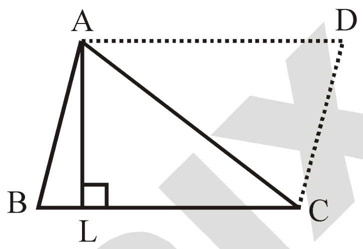
Now, CA is a diagonal of parallelogram BCQA,

$\therefore$ ar(ΔABC) = ar(ΔAQC) = $\frac{1}{2}$ ar(| |^{gm} BCQA) ...(ii)

Similarly, ar(ΔPBC) = $\frac{1}{2}$ ar(| |^{gm} BCPD) ...(iii)

From (i), (ii) and (iii), we get ar(ΔPBC) = ar(ΔABC)

Theorem - 4



Statement : The area of a triangle is half the product of any of its sides and the corresponding altitude.

Given : Let a ΔABC in which AL is the altitude to the side BC.

To Prove : ar(ΔABC) = $\frac{1}{2}$ (BC) × (AL)

Construction : Through C and A, draw lines parallel to BA and BC, respectively intersecting each other at D.

Proof : Quadrilateral ABCD is a parallelogram. [By construction]

$\because$ AC is a diagonal of parallelogram ABCD.

$\therefore$ ar(ΔABC) = $\frac{1}{2}$ ar(| |^{gm} ABCD) [By Theorem 2] ...(i)

BC is a side of parallelogram BCDA and AL is the corresponding altitude.

$\therefore$ ar(| |gm ABCD) = BC × AL. ...(iii)

From (i) and (ii), we get ar(ΔABC) = $\frac{1}{2}$ BC × AL.

Example 5

Show that a median of a triangle divides it into two triangles of equal areas.

Solution :

Given : Let ABC be a triangle and AD be the median.

To Prove : ar(ΔABD) = ar(ΔACD).

Construction : Draw AN $\perp$ BC.

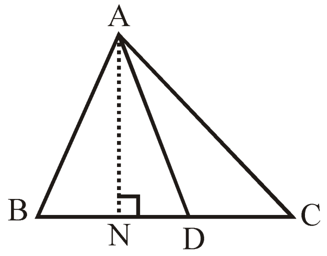
Proof : ar(ΔABD) = $\frac{1}{2}$ × base × altitude

$= \frac{1}{2} \times BD \times AN$

$= \frac{1}{2} \times CD \times AN$ (As BD = CD)

= ar(ΔACD)

∴ ar(ΔABD) = ar(ΔACD)



Example 6

In the given fig., ABCD is a quadrilateral and BE | | AC and also BE meets DC produced at E. Show that area of ΔADE is equal to the area of the quadrilateral ABCD.

Solution :

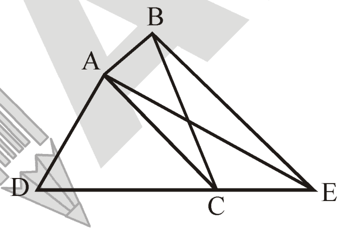
ΔBAC and ΔEAC lie on the same base AC and between the same parallels AC and BE.

Therefore, ar(ΔBAC) = ar(ΔEAC) [By theorem 3]

So, ar(ΔBAC) + ar(ΔADC) = ar(ΔEAC) + ar(ΔADC)

[Adding same areas on both sides]

⇒ ar(quad. ABCD) = ar(ΔADE).



Example 7

The diagonals of quadrilateral ABCD, AC and BD intersects at O. Prove that if BO = OD then the triangles ABC and ADC are equal in area.

Solution :

Given : A quadrilateral ABCD in which its diagonals AC and BD intersects at O such that BO = OD.

To Prove : ar(ΔABC) = ar(ΔADC)

Proof : In ΔABD, we have BO = OD

⇒ O is the mid–point of BD

⇒ AO is the median

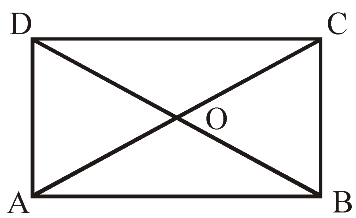
Since median of a triangle divides it into two triangles of equal area.

∴ ar(ΔAOB) = ar(ΔAOD) ...(i)

In ΔCBD, O is the mid–point of BD.

∴ CO is the median

⇒ ar(ΔCOB) = ar(ΔCOD) ...(ii)



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Adding (i) and (ii), we get

ar(ΔAOB) + ar(ΔCOB) = ar(ΔAOD) + ar(ΔCOD)

⇒ ar(ΔABC) = ar(ΔADC)

BUILD THE CONCEPT

- The area of a trapezium is half the product of the sum of its parallel sides and the perpendicular distance between them.

  Given : Trapezium ABCD in which

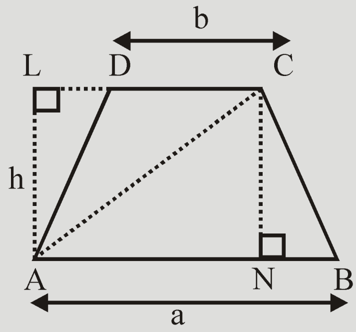
  AB | | DC, AL ⊥ DC, CN ⊥ AB, AL = CN = h (say,)

  AB = a, DC = b.

  To Prove : ar(trap. ABCD) = $\frac{1}{2}$ h × (a + b)

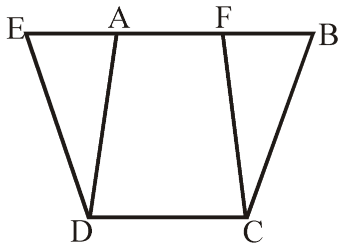
  Proof : ar(trap. ABCD) = ar (ΔABC) + ar(ΔACD)

  = $\frac{1}{2}$ h × a + $\frac{1}{2}$ h × b = $\frac{1}{2}$ h (a + b).

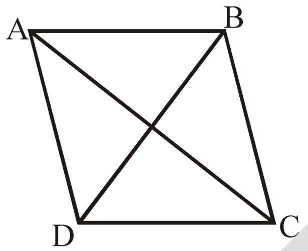


SE. 1

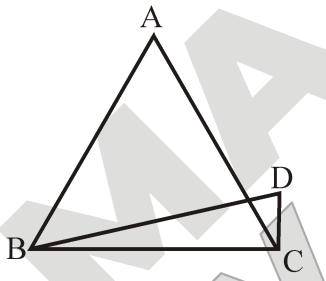
Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



(i)



(ii)



(iii)

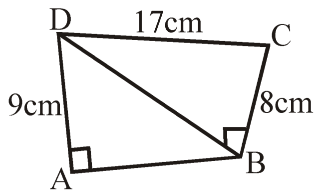
Ans. Fig. (i) and (ii) lies on the same base and between the same parallels.

Fig. (i) Common base = DC and two parallels are DC and EB.

Fig. (ii) Common base = DC and two parallels are AB and DC.

SE. 2

Compute the area of quadrilateral ABCD.



Ans. In ΔBCD, we have

$CD^2 = BD^2 + BC^2$

$\Rightarrow (17)^2 = BD^2 + (8)^2$

$\Rightarrow BD^2 = 289 - 64 = 225 \Rightarrow BD = 15$

In ΔABD, we have

$BD^2 = AB^2 + AD^2$

$\Rightarrow (15)^2 = AB^2 + (9)^2$

$\Rightarrow AB^2 = 225 - 81 = 144 \Rightarrow AB = 12$ cm

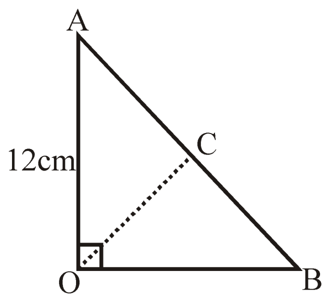
So, ar(ABCD) = ar(ΔABD) + ar(ΔBCD)

$= \frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 15) = 54 + 60 = 144$

$\therefore$ ar(ABCD) = 144 cm^2

SE. 3

In the given figure $\angle AOB = 90^\circ$, AC = BC, OA = 12 cm and OC = 6.5 cm. Find the area of ΔAOB.



Ans. Since the mid–point of the hypotenuse of a right triangle is equidistant from the vertices.

$\therefore$ CA = CB = OC

$\Rightarrow$ CA = CB = 6.5 cm

$\Rightarrow$ AB = 13 cm

In right triangle OAB, we have

$AB^2 = OB^2 + OA^2 \Rightarrow 13^2 = OB^2 + 12^2$

$\Rightarrow$ OB = 5 cm

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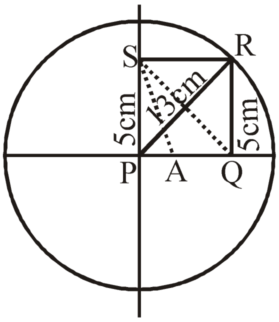
$\therefore$ ar(ΔAOB) = $\frac{1}{2}$ (OA × OB) = $\frac{1}{2}$ (12 × 5) cm^2

= 30 cm^2

SE. 4

PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5cm, then show that ar(ΔPAS) = 30 cm^2 is false.



Ans. In ΔPQR,

$PR^2 = PQ^2 + RQ^2$ (by Pythagoras theorem)

$\Rightarrow PQ^2 = PR^2 - RQ^2 = (13)^2 - (5)^2$

$\Rightarrow PQ = \sqrt{169-25} = \sqrt{144} = 12cm$

Now, area of rectangle PQRS = 12 × 5 = 60 cm^2

Area of ΔPSQ = $\frac{1}{2} \times 60 = 30\ cm^2$

(since, diagonals of rectangle bisect it into two triangles of equal areas.)

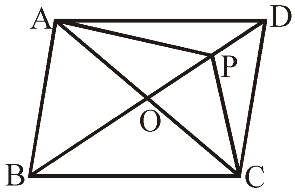
Now, A is any point on PQ. So, area of ΔAPS depends on the position of A. If A is on Q point, then area will be 30 cm^2.

$\therefore$ ar(ΔAPS) < 30 cm^2

Hence, the given statement is false.

SE. 5

P is any point on the diagonal BD of the parallelogram ABCD. Prove that ar(ΔAPD) = ar(ΔCPD).



Ans. The diagonals BD and AC intersect each other at O. Since, the diagonals of a parallelogram bisect each other.

$\therefore$ O is the mid – point of AC

i.e., DO is the median of ΔDAC

Hence, ar(ΔDAO) = ar(ΔDCO) ...(i)

Again, PO is the median of ΔPAC

$\therefore$ ar(ΔPAO) = ar(ΔPCO) ...(ii)

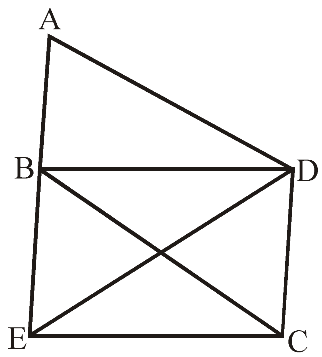
Subtracting equation(ii) frome quation (i), we get

ar(ΔDAO) – ar(ΔPAO) = ar(ΔDCO) – ar(ΔPCO)

$\Rightarrow$ ar(ΔAPD) = ar(ΔCPD)

SE. 6

ABCD is a quadrilateral. The straight line through C parallel to the diagonal DB intersects AB produced at E. Prove that the ar(quad. ABCD) = ar(ΔADE).



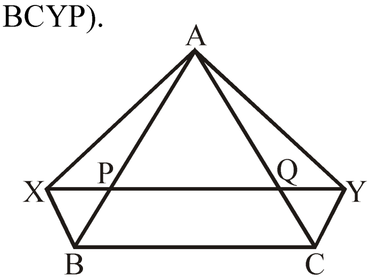
Ans. ar(ΔBCD) = ar(ΔBED) [Since, they are on the same base BD and between same lines BD and EC]

$\therefore$ ar(ΔBCD) + ar(ΔABD) = ar(ΔBED) + ar(ΔABD) [Adding ar (ΔABD) to both sides]

$\therefore$ ar(ABCD) = ar(ΔADE)

SE. 7

In the adjoining figure, PQ is a line parallel to side BC of ΔABC. If BX || CA and CY || AB meet the line PQ produced to X and Y respectively, show that $ar(\Delta ABX) = \frac{1}{2} ar(||^{gm} XBCQ)$ and $ar(\Delta ACY) = \frac{1}{2} ar(||^{gm} BCYP)$.

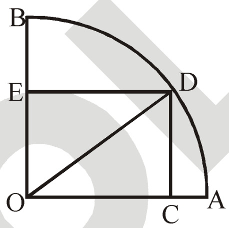


Ans. $||^{gm}$ XBCQ and ΔABX being on the same base XB and between the same parallels XB and CA, we have $ar(\Delta ABX) = \frac{1}{2} ar(||^{gm} XBCQ)$

Again, $||^{gm}$ BCYP and ΔACY being on the same base CY and between the same parallels CY and BA, we have $ar(\Delta ACY) = \frac{1}{2} ar(||^{gm} BCYP)$

SE. 8

In fig. OCDE is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If $OE = 2\sqrt{5}$ cm, then find the area of the rectangle OCDE.



Ans. We have, OD = 10 cm and $OE = 2\sqrt{5}$ cm

$\therefore$ In ΔODE, $OD^2 = OE^2 + DE^2$

$\Rightarrow DE = \sqrt{OD^2 - OE^2} = \sqrt{(10)^2 - (2\sqrt{5})^2} = 4\sqrt{5}$cm

$\therefore$ ar(rect. OCDE) = OE × DE = $2\sqrt{5} \times 4\sqrt{5}$

$= 40\ cm^2$

SE. 9

If the diagonals AC, BD of a quadrilateral ABCD, intersect at O and separate the quadrilateral into four triangles of equal areas, show that quadrilateral ABCD is a parallelogram.

Ans. Given : A quadrilateral ABCD such that its diagonals AC and BD intersect at O and separate it into four parts such that

$ar(\Delta AOB) = ar(\Delta BOC) = ar(\Delta COD) = ar(\Delta AOD)$

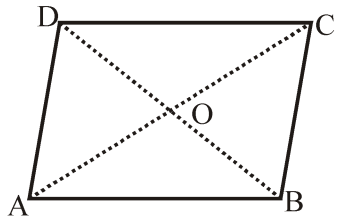
To Prove : Quadrilateral ABCD is a parallelogram.

Proof : We have, ar (ΔAOD) = ar(ΔBOC)

$\therefore$ ar(ΔAOD) + ar(ΔAOB) = ar(ΔBOC) + ar(ΔAOB) [Adding ar(ΔAOB) to both sides.]

$\Rightarrow$ ar(ΔABD) = ar(ΔABC)

Thus, triangles Δ ABD and Δ ABC have the same base AB and have equal areas. So, their corresponding altitudes must be equal.



$\therefore$ Altitude from D of ΔABD = Altitude from C of ΔABC

$\Rightarrow$ DC || AB.

Similarly, we have, AD || BC. Hence, quadrilateral ABCD is a parallelogram.

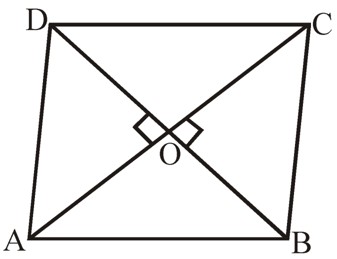
SE. 10

Prove that area of a rhombus is half the product of the lengths of its diagonals.

Ans. **Given** : A rhombus ABCD whose diagonals AC and BD intersect at a point O.

To Prove : $ar(ABCD) = \frac{1}{2} \times AC \times BD$.

Proof : Since the diagonals of a rhombus intersect each other at right angles, we have BO ⊥ AC and DO ⊥ AC.



$\therefore$ ar(ABCD) = ar(ΔABC) + ar(ΔACD)

$$= \left(\frac{1}{2}\times AC\times BO\right)+\left(\frac{1}{2}\times AC\times DO\right)$$

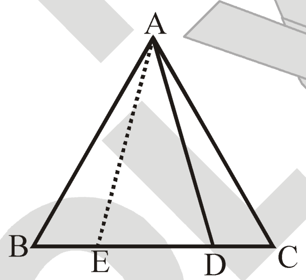
$$= \frac{1}{2}\times AC\times (BO + DO) = \frac{1}{2}\times AC\times BD$$

Hence, Area of a rhombus

$$= \frac{1}{2}\times \text{(product of diagonals)}$$

SE. 11

A point D is taken on the side BC of a ΔABC such that BD = 2DC. Prove that ar(ΔABD) = 2ar(ΔADC).



Ans. In ΔABC, we have BD = 2DC

Let E be the mid–point of BD. Then,

BE = ED = DC

Since AE and AD are the medians of ΔABD and ΔAEC respectively.

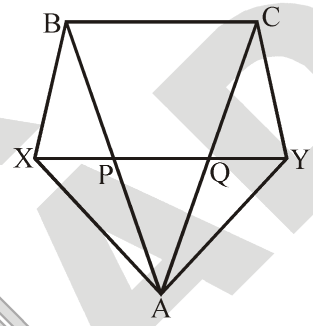
$\therefore$ ar(ΔABD) = 2ar(ΔAED) ...(i)

and ar(ΔADC) = ar(ΔAED) ...(ii)

From (i) and (ii), we get ar(ΔABD) = 2ar(ΔADC)

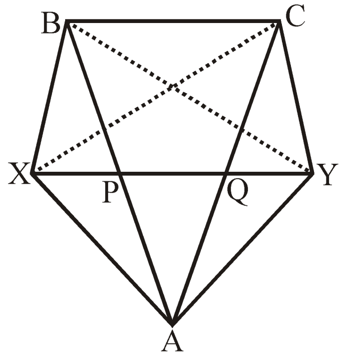
SE. 12

In the given figure, BC || XY, BX || CA and AB || YC. Prove that ar(ΔABX) = ar(ΔACY)



Ans. Join XC and BY.

Since triangles BXC and BCY are on the same base BC and between the same parallels BC and XY.



$\therefore$ ar(ΔBXC) = ar(ΔBCY) ...(i)

Also, triangle BXC and ABX are on the same base BX and between the same parallels BX and AC

$\therefore$ ar(ΔBXC) = ar(ΔABX) ...(ii)

Clearly, triangles BCY and ACY are on the same base CY and between the same parallels AB and CY.

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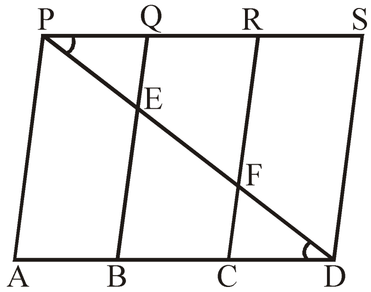
$\therefore$ ar(ΔBCY) = ar(ΔACY) ...(iii)

From (i), (ii) and (iii), we get ar(ΔABX) = ar(ΔACY)

SE. 13

In the given figure PSDA is a parallelogram in which PQ = QR = RS and AP || BQ || CR || DS. Prove that ar(ΔPQE) = ar(ΔDCF).



Ans. Since AP || BQ || CR || DS.

$\therefore$ PQ = CD ...(i)

In ΔBED, C is the mid–point of BD and CF || BE

$\therefore$ F is the mid – point of ED

[By converse of Mid Point Theorem]

$\Rightarrow$ EF = FD

Similarly, EF = PE $\Rightarrow$ PE = FD ...(ii)

In ΔPQE and ΔDCF, we have PE = FD

[Proved above]

Also, ∠EPQ = ∠FDC [Alternate interior angles]

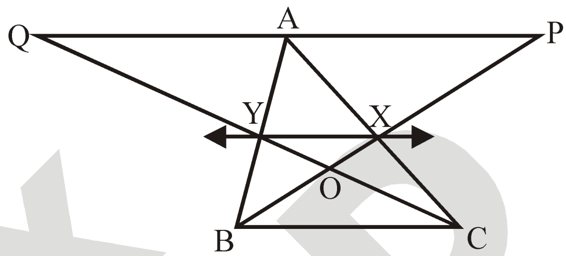
and PQ = CD [from (i)]

So, by SAS congruency, we have ΔPQE ≅ ΔDCF

$\Rightarrow$ ar(ΔPQE) = ar(ΔDCF)

SE. 14

In the given figure X and Y are the midpoints of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar(ΔABP) = ar(ΔACQ).



Ans. Since X and Y are the mid–points of AC and AB respectively.

$\therefore$ XY || CB

Clearly, triangles ΔBXY and ΔCXY are on the same base XY and between the same parallels XY and BC

$\Rightarrow$ ar(ΔBXY) = ar(ΔCXY) ...(i)

We observe that the quadrilaterals XYAP and XYQA are on the same base XY and between the same parallels XY and PQ.

$\because$ ar (quad XYAP) = ar(quad XYQA) ...(ii)

Adding (i) and (ii), we get

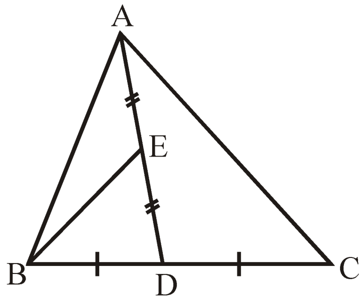
ar(ΔBXY) + ar(quad XYAP) = ar (ΔCXY) + ar(quad XYQA)

$\Rightarrow$ ar(ΔABP) = ar(ΔACQ).

SE. 15

ABC is a triangle in which D is the mid-point of BC and E is the mid-point of AD. Prove that the area of

ΔBED = $\frac{1}{4}$ area of ΔABC.



Space for Notes :

Ans. Given : A ΔABC in which D is the mid-point of BC and E is the mid-point of AD.

To prove : ar(ΔBED) = $\frac{1}{4}$ ar(ΔABC)

Proof : ∵ AD is median of ΔABC

∴ ar(ΔABD) = ar(ΔADC)

(∵ Median of a Δ divides it into two Δs of equal area)

⇒ ar(ΔABD) = $\frac{1}{2}$ ar(ΔABC)

Again, E is median of ΔABD,

∴ ar(ΔBEA) = ar(ΔBED)

(∵ Median of a Δ divides it into two Δs of equal area)

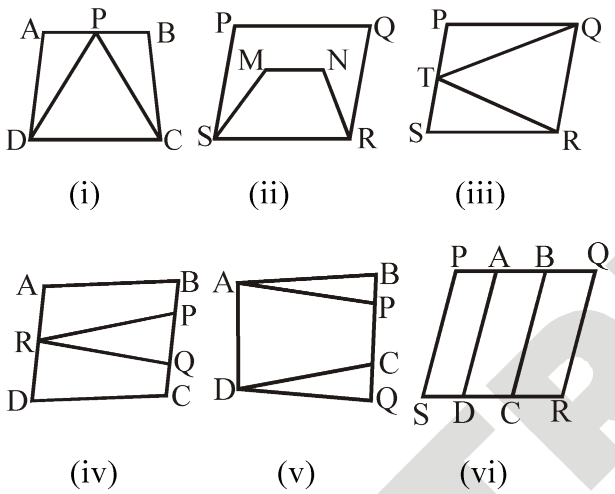
⇒ ar(ΔBEA) = $\frac{1}{2}$ ar(ΔABD)

= $\frac{1}{2} \cdot \frac{1}{2}$ ar(ΔABC) = $\frac{1}{4}$ ar(ΔABC)

EXERCISE 9.1

NS. 1

Which of the following figures lie on the same base and between the same parallels. IN such a case, write the common base and the two parallels.



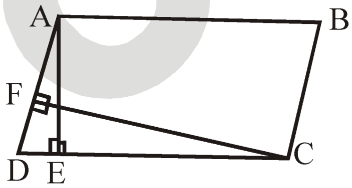
Ans. The figures (i), (iii) and (v) lie on the same base and between the same parallels.

	Base	Two parallel
Fig. (i)	DC	DC and AB
Fig. (iii)	QR	QR and PS
Fig. (v)	AD	AD and BQ

EXERCISE 9.2

NS. 1

In the given figure ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Ans. We have AE ⊥ DC and AB = 16 cm

∵ AB = CD [Opposite sides of parallelogram]

∴ CD = 16 cm

Now, area of parallelogram ABCD = CD × AE

Since, CF ⊥ AD

∴ Area of parallelogram ABCD = AD × CF

⇒ AD × CF = 128

⇒ AD × 10 = 128 [CF = 10 cm]

$\Rightarrow AD = \dfrac{128}{10} = 12.8$

Thus, the required length of AD is 12.8 cm

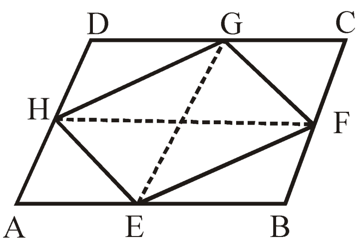
NS. 2

If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that

$ar(EFGH) = \dfrac{1}{2}\ ar(ABCD)$.

Ans. Join GE and HF, where GE || BC || DA and HF || AB || DC

(∵ E, F, G and H are the mid points of a ||^{gm} ABCD).



If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

Now, ΔEFG and parallelogram EBCG are on the same base EG, and between the same parallels EG and BC.

$\therefore ar(\Delta EFG) = \dfrac{1}{2} ar(||^{gm} EBCG)$ ....(i)

Similarly, $ar(\Delta EHG) = \dfrac{1}{2} ar(||^{gm} AEGD)$ ....(ii)

Adding (1) and (2), we get

ar(ΔEFG) + ar(ΔEHG)

$= \frac{1}{2}$ [ar($\|^{gm}$EBCG) + ar($\|^{gm}$ABGD)]

Thus, ar($\|^{gm}$EFGH) = $\frac{1}{2}$ ar($\|^{gm}$ABCD)

NS. 3

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar(APB) = ar(BQC).

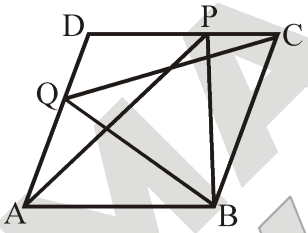
Ans. ∵ ABCD is a parallelogram

∴ AB || CD and BC || AD.

Now, ΔAPB and parallelogram ABCD are on the same base AB and between the same parallels AB and CD.

∴ ar(ΔAPB)= $\frac{1}{2}$ ar($\|^{gm}$ABCD) ...(1)

Also, ΔBQC and parallelogram ABCD are on the same base BC and between the same parallels BC and AD.



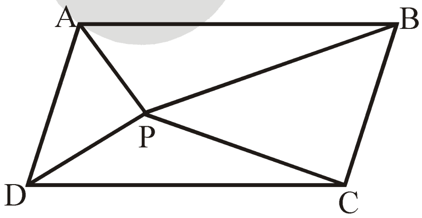
∴ ar(ΔBQC) = $\frac{1}{2}$ ar($\|^{gm}$ ABCD) ...(2)

From (1) and (2),

we have ar(ΔAPB) = ar(ΔBQC).

NS. 4

In the figure, P is a point in the interior of a parallelogram ABCD. Show that



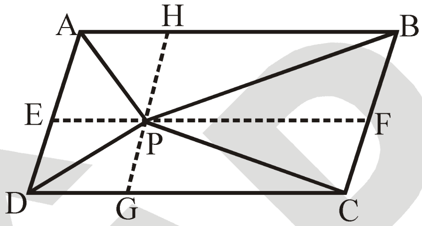
(i) ar(ΔAPB)+ar(ΔPCD) = $\frac{1}{2}$ ar(ABCD)

(ii) ar(ΔAPD) + ar(ΔPBC) = ar(ΔAPB) + ar(ΔPCD)

[Hint: Through P, draw a line parallel to AB]

Ans. We have parallelogram ABCD, i.e., AB || CD and BC || AD.

[Let us draw EF || AB and HG || AD through P.]



(i) ΔAPB and $\|^{gm}$ AEFB are on the same base AB and between the same parallels AB and EF.

∴ ar(ΔAPB= $\frac{1}{2}$ ar($\|^{gm}$ AEFB) ...(1)

Also, ΔPCD and parallelogram CDEF are on the same base CD and between the same parallels CD and EF.

∴ ar(ΔPCD) = $\frac{1}{2}$ ar($\|^{gm}$CDEF) ...(2)

Adding (1) and (2), we have

ar (ΔAPB) + ar(ΔPCD) = $\frac{1}{2}$

ar($\|^{gm}$AEFB) + $\frac{1}{2}$ ar($\|^{gm}$CDEF)

⇒ar(ΔAPB)+ar(ΔPCD) = $\frac{1}{2}$ ar(||gmABCD)...(3)

(ii) ΔAPD and $\|^{gm}$ ADGH are on the same base AD and between the same parallels AD and GH.

∴ ar(ΔAPD) = $\frac{1}{2}$ ar($\|^{gm}$ ADGH) ...(4)

Similarly, ar(ΔPBC) = $\frac{1}{2}$ ar($\|^{gm}$BCGH) ...(5)

Adding (4) and (5), we have

ar(ΔAPD) + ar (ΔPBC)=

$\frac{1}{2}$ ar($\|^{gm}$ ADGH)+ $\frac{1}{2}$ ar($\|^{gm}$BCGH)

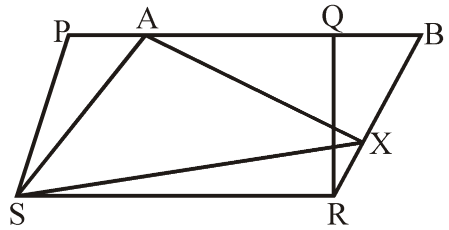
$ar(\Delta APD)+ar(\Delta PBC)=\frac{1}{2}ar(||^{gm}ABCD)$ ...(6)]

From (3) and (6), we have

$ar(\Delta APD) + ar(\Delta PBC) = ar(\Delta APB) + ar(\Delta PCD)$

NS. 5

In the figure, PQRS and ABRS are parallelogram and X is any point on side BR. Show that



(i) ar(PQRS) = ar(ABRS)

(ii) $ar(AXS) = \frac{1}{2}ar(PQRS)$

Ans. (i) Parallelogram PQRS and parallelogram ABRS are on the same base RS and between the same parallels RS and PB.

$\therefore$ ar(PQRS) = ar(ABRS)

(ii) ΔAXS and ||gm ABRS are on the same base AS and between the same parallels AS and BR.

$\therefore ar(AXS) = \frac{1}{2}ar(ABRS)$ ...(1)

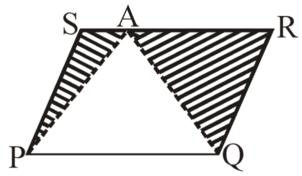
But ar (PQRS) = ar(ABRS) [Proved in (i) part]..(2)

From (1) and (2),

we have $ar(AXS) = \frac{1}{2}ar(PQRS)$

NS. 6

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?



Ans. The farmer is having the field in the form of parallelogram PQRS and a point A is situated on RS. Join AP and AQ.

Clearly, the field is divided into three parts i.e. in ΔAPS, ΔPAQ and ΔQAR.

Since, ΔPAQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.

$\therefore ar(\Delta PAQ) = \frac{1}{2}ar(||^{gm}PQRS)$ ...(1)

$\Rightarrow ar(||^{gm} PQRS) - ar(\Delta PAQ)$

$= ar(||^{gm} PQRS) - \frac{1}{2}ar(||^{gm}PQRS)$

$\Rightarrow [ar(\Delta APS) + ar(\Delta QAR)] = \frac{1}{2}ar(||^{gm} PQRS)$ ...(2)

From (1) and (2), we have

$ar(\Delta PAQ) = ar[(\Delta APS) + (\Delta QAR)]$

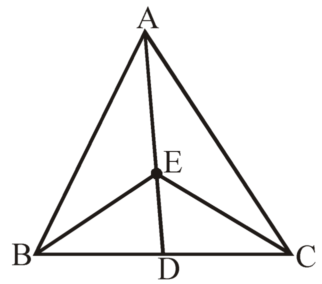
Thus, the farmer can sow wheat in (ΔPAQ) and pulses in [(ΔAPS) + (ΔQAR)]

or wheat in [ΔAPS) + (ΔQAR)] and pulses in (ΔPAQ).

EXERCISE 9.3

NS. 1

In the figure, E is any point on median AD of a ΔABC. Show that ar(ABE) = ar(ACE).



Ans. We have a ΔABC such that AD is a median.

$\therefore$ ar(ΔABD) = ar(ΔADC)

[$\because$ A median divides the trangle into two triangles of equal areas]

Similarly, in ΔBEC,we have

ar(ΔBED) = ar (ΔDEC) ..(2)

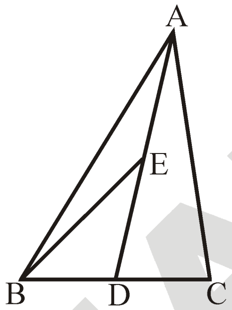
Subtracting (2) from (1), we have

ar(ΔABD) – ar(ΔBED) = ar (ΔADC) – ar(ΔDEC)

$\Rightarrow$ ar (ΔABE) = ar(ΔACE).

NS. 2

In a triangle ABC, E is the mid-point of median AD. Show tha ar(BED) = $\frac{1}{4}$ ar(ABC).



Ans. We have a ΔABC and and its median AD.

Since, a median divides the triangle of equal area.

$\therefore$ ar(ΔABD) = $\frac{1}{2}$ ar(ΔABC) ...(1)

Let us join B and E.

Now, in ΔABD, BE is a median.

[$\because$ E is the mid-point of AD]

$\therefore$ ar(ΔBED) = $\frac{1}{2}$ ar(ΔABD) ...(2)

From (1) and (2), we have

ar(ΔBED) = $\frac{1}{2}\left[\frac{1}{2} ar(\Delta ABC)\right]$

$\Rightarrow$ ar(ΔBED) = $\frac{1}{4}$ ar(ΔABC)

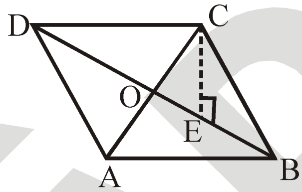
NS. 3

Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Ans. We have a parallelogram ABCD such that its diagonals intersect at O.

$\because$ Diagonals of a parallelogram bisect each other.

$\therefore$ AO = OC and BO = OD



Let us draw CE $\perp$ BD.

Now, ar (ΔBOC) = $\frac{1}{2}$ BO × CE

and ar(ΔDOC) = $\frac{1}{2}$ OD × CE

Since, BO = OD

$\therefore$ ar(ΔBOC) = ar(ΔDOC) ...(1)

Similarly, ar(ΔAOD) = ar (ΔDOC) ...(2)

and ar(ΔAOB) = ar(ΔBOC) ...(3)

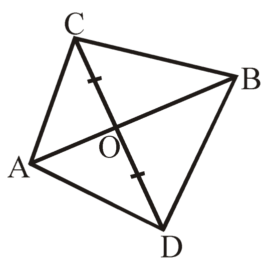
From (1), (2) and (3), we have

ar(ΔAOB) = ar(ΔBOC) = ar(ΔCOD) = ar (ΔAOD)

Thus, the diagonals of a parallelogram divide it into four triangles of equal area.

NS. 4

In figure, ABC and ABD are two triangles on the same base AB. If line segment CD is bisected by AB at O, show that ar(ABC) = ar(ABD).



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Ans. We have ΔABC and ΔABD are on the same base AB.

∵ CD is bisected at O [Given]

∴ CO = DO

Now, in ΔACD, AO is a median

∴ ar(ΔOAC) = ar(ΔOAD) ...(1)

Again, in ΔBCD, BO is a median

∴ ar(ΔOBC) = ar (ΔODB) ...(2)

Adding (1) and (2), we have

ar(ΔOAC) + ar(ΔOBC) = ar(ΔOAD) + ar(ΔODB)

⇒ ar(ΔABC) = ar(ΔABD)

NS. 5

D, E and F are respectively tha mid-points of the sides BC, CA and AB of a ΔABC. Show that

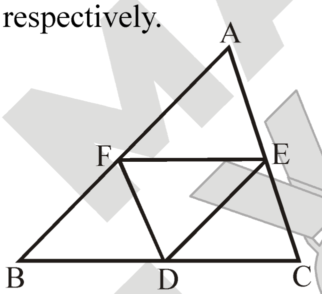
(i) BDEF is a parallelogram

(ii) ar(DEF) = $\frac{1}{4}$ ar(ABC)

(iii) ar(BDEF) = $\frac{1}{2}$ ar(ABC)

Ans. We have ΔABC such that D, E and F are the mid-points of BC, CA and AB respectively.

(i) In ΔABC, E and F are the mid-points of AC and AB respectively.



∴ EF || BC [Mid-point theorem]

⇒ EF || BD Also, EF = $\frac{1}{2}$ BC

⇒ EF = BD [D is the mid-point of BC]

Since BDEF is a quadrilateral whose one pair of opposite sides is parallel and of equal lengths.

∴ BDEF is a parallelogram.

(ii) We have proved that BDEF is a parallelogram. Similarly, DCEF is parallelogram and DEAF is a parallelogram

Now, parallelogram BDEF and parallelogram DCEF are on the same base EF and between the same parallels BC & EF.

∴ ar(||^{gm} BDEF) = ar(||^{gm} DCEF)

⇒ $\frac{1}{2}$ ar(||^{gm}BDEF) = $\frac{1}{2}$ ar(||^{gm} DCEF)

⇒ ar(ΔBDF) = ar(ΔCDE) ...(1)

[Diagonal of a parallelogram divides it into two triangles of equal area]

Similarly, ar(ΔCDE) = ar(ΔDEF) ...(2)

and ar(ΔAEF) = ar(ΔDEF) ...(3)

From (1), (2) and (3), we have

ar(ΔAEF) = ar(ΔFBD) = ar(ΔDEF) = ar(ΔCDE)

Thus, ar(ΔABC) = ar(ΔAEF) + ar(ΔFBD) + ar(ΔDEF) + ar (ΔCDE) = 4 ar(ΔDEF)

⇒ ar(ΔDEF) = $\frac{1}{4}$ ar(ΔABC)

(iii) We have

ar(||^{gm} BDEF) = ar(ΔBDF) + ar (ΔDEF)

= ar(ΔDEF) + ar(ΔDEF) [∵ ar(ΔDEF) = ar(ΔBDF)]

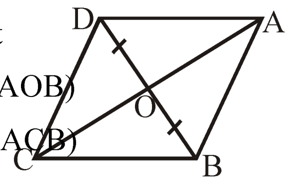
= 2ar(ΔDEF)

$=2\left[\frac{1}{4}ar(\Delta ABC)\right] = \frac{1}{2}$ ar(ΔABC)

Thus, ar(||gm BDEF) = $\frac{1}{2}$ ar(ΔABC).

NS. 6

In the figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that



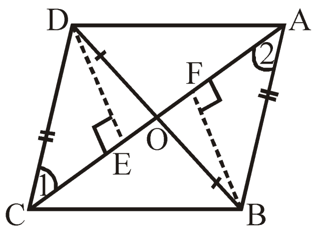
(i) ar(DOC) = ar(AOB)

(ii) ar(DCB) = ar(ACB)

(iii) DA || CB or ABCD is a parallelogram.

[Hint: From D and B, draw perpendiculars to AC]

Ans. We have a quadrilateral ABCD whose diagonals AC and BD intersect at O.



We also have that

OB = OD, AB = CD

Let us draw DE ⊥ AC and BF ⊥ AC

(i) In ΔDEO and ΔBFO, we have

DO = BO [Given]

∠DOE = ∠BOF [Vertically opposite angles]

∠DEO = ∠BFO [Each 90°]

∴ ΔDEO ≅ ΔBFO [By AAS congruency]

⇒ DE = BF [By C.P.C.T]

and ar(ΔDEO) = ar(ΔBFO) ...(1)

Now, in ΔDEC and ΔBFA, we have

∠DEC = ∠BFA [Each 90°]

DE = BF [Proved above]

DC = BA [Given]

∴ ΔDEC ≅ ΔBFA [By RHS congruency]

⇒ ar(ΔDEC) = ar(ΔBFA) ...(2)

and ∠1 = ∠2 ...(3) [By C.P.C.T.]

Adding (1) and (2), we have

ar(ΔDEO) + ar(ΔDEC) = ar(ΔBFO) + ar(ΔBFA)

⇒ ar(ΔDOC) = ar(ΔAOB)

(ii) Since ar(ΔDOC) = ar(ΔAOB) [Proved above]

Adding ar(ΔBOC), on both sides, we have

ar(ΔDOC) + ar(ΔBOC) = ar(ΔAOB) + ar(ΔBOC)

⇒ ar(ΔDCB) = ar(ΔACB)

(iii) Since, ΔDCB and ΔACB are on the same base CB and having equal areas.

∴ They lie between the same parallels CB and DA.

⇒ CB || DA

Also, ∠1 = ∠2 [By (3)]

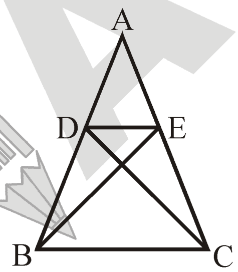
which forms an alternate interior angles.

So, AB || CD

Hence, ABCD is a parallelogram

NS. 7

D and E are points on sides AB and AC respectively of ΔABC such that ar(DBC) = ar(EBC). Prove that DE || BC.



Ans. We have ΔABC and points D and E are such that ar(DBC) = ar(EBC)

Since ΔDBC and ΔEBC are on the same base BC and having same area.

∴ They lie between the same parallels DE and BC.

Hence. DE || BC

NS. 8

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that ar(ABE) = ar(ACF).

Ans. We have a ΔABC such that XY || BC, BE || AC and CF || AB.

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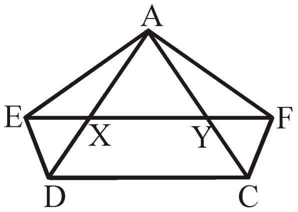


Since, XY || BC and BE || CY

∴ BCYE is a parallelogram. Now, the parallelogram BCYE and ΔABE are on the same base BE and between the same parallels BE and AC.

$\therefore ar(\Delta ABE) = \frac{1}{2} ar(||^{gm} BCYE)$ ...(1)

Again, CF || AB [Given]

Also, XY || BC [Given]

⇒ CF || AB and XF || BC

∴ BCFX is a parallelogram.

Now, ΔACF and parallelogram BCFX are on the same base CF and between the same paralles AB and FC.

$\therefore ar(\Delta ACF) = \frac{1}{2} ar(||^{gm} BCFX)$ ...(2)

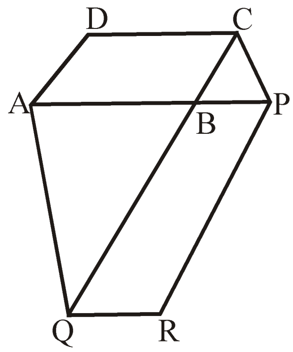
Also parallelogram BCFX and parallelogram BCYE are on the same base BC and between the same parallels BC and EF.

$\therefore ar(||^{gm} BCFX) = ar(||^{gm} BCYE)$ ...(3)

From (1), (2) and (3) we get ar(ΔABE) = ar (ΔACF)

NS. 9

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (seen figrue). Show tha ar(ABCD = ar(PBQR). [Hint : Join AC and PQ. Now compare ar (ACQ) and ar(APQ)]



Ans. Let us join AC and PQ.

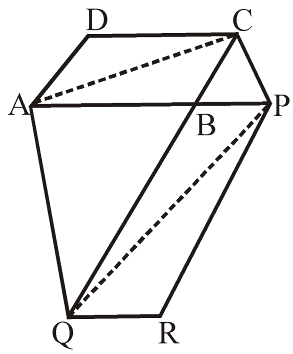
ABCD is a parallelogram [Given]

and AC is its diagonal, we know that diagonal of a parallelogram divides it into to triangles of equal areas.

$\therefore ar(\Delta ABC) = \frac{1}{2} ar(||^{gm} ABCD)$ ...(1)

Also, PBQR is a parallelogram [Given] and QP is its diagonal.

$\therefore ar(\Delta BPQ) = \frac{1}{2} ar(||^{gm} ABCD)$ ...(2)



Since, ΔACQ and ΔAPQ are on the same parallels AQ and CP.

∴ ar(ΔACQ) = ar(ΔAPQ)

⇒ ar(ΔACQ) – ar(ΔABQ) = ar(ΔAPQ) – ar(ΔABQ)

[Subtracting ar(ΔABQ) from both sides]

⇒ ar(ΔABC) = ar(ΔBPQ) ...(3)

From (1) , (2) and (3), we get

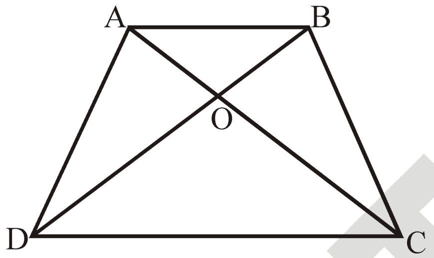
$\frac{1}{2} ar(||^{gm} ABCD) = \frac{1}{2} ar(||^{gm} PBQR)$

$\Rightarrow ar(||^{gm} ABCD) = ar(||^{gm} PBQR)$

NS. 10

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar(AOD) = ar(BOC).

Ans. We have a trapezium ABCD having AB || CD and its diagonals AC and BD are joined. Since, triangles on the same base and between the same parallels have equal areas. ΔABD and ΔABC are on the same base AB and betwen the same parallels AB and DC



∴ ar(ΔABD) = ar(ΔABC)

Subtracting ar(ΔAOB) from both sides, we get

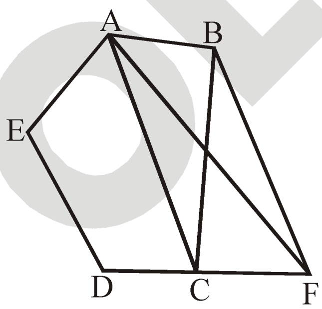
ar(ΔABD) – ar(ΔAOB) = ar(ΔABC) – ar(ΔAOB)

⇒ ar(ΔAOD) = ar(ΔBOC)

NS. 11

In the figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F.

Show that



(i) ar(ACB) = ar(ACF)

(ii) ar(AEDF) = ar(ABCDE)

Ans. We have a pentagon ABCDE in which BF || AC and DC is produced to F.

(i) Since, the triangle between the same paralleles and on the same base are equal in area.

ΔACB and ΔACF are on the same base AC and between the same parallels AC and BF.

∴ ar(ΔACB) = ar(ΔACF)

(ii) Since ar(ΔACB) = ar(ΔACF) [Proved above]

Adding ar(quad. AEDC) to both sides, we get

⇒ ar(ΔACB) + ar(quad. AEDC)

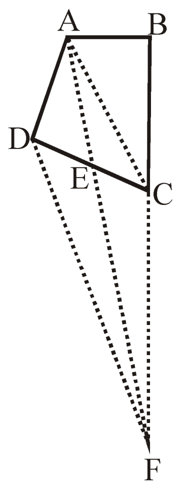
=ar(ΔACF) + ar (quad. AEDC)

∴ ar(ABCDE) = ar(AEDF)

NS. 12

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portin of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Ans. We have a plot in the form of a quadrilateral ABCD.



let us draw DF || AC and join A and F.

Now, ΔDAF and ΔDCF are on the same base DF and between the same parallels AC and DF.

∴ ar(ΔDAF) = ar(ΔDCF)

Subtracting ar(ΔDEF) from both sides, we get

ar(ΔDAF) – ar(ΔDEF)

= ar(ΔDCF) – ar(ΔDEF)

⇒ ar(ΔADE) = ar(ΔCEF)

The portion ΔADE can be taken over by the Gram Panchyat by adding the land (ΔCEF) to his (Itwaari) land so as to form a triangular plot, i.e. ΔABF.

Let us prove that ar(ΔABF) = ar(quad. ABCD), we have

ar(ΔCEF) = ar(ΔADE) [Proved above]

Adding ar(quad. ABCE) to both sides, we get

ar(ΔCEF) + ar(quad. ABCE)

= ar(ΔADE) + ar(quad. ABCE)

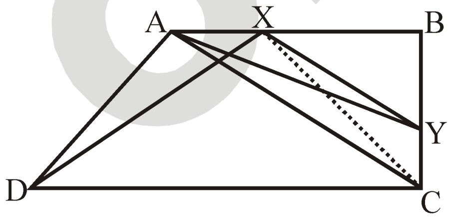
⇒ ar(ΔABF) = ar(quad. ABCD)

NS. 13

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar(ADX) = ar(ACY). [Hint : Join CX]

Ans. We have a trapezium ABCD such that AB || DC. XY || AC meets AB at X and BC at Y.

Let us join CX



∵ ΔADX and ΔACX are on the same base AX and between the same parallels AB and DC.

∴ ar(ΔADX) = ar(ΔACX) ...(1)

∵ ΔACX and ΔACY are on the same base AC and between the same parallels AC and XY.

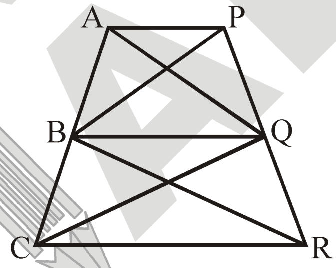
∴ ar(ΔACX) = ar(ΔACY) ...(2)

From (1) and (2), we have ar(ΔADX) = ar(ΔACY)

NS. 14

In the figure, AP || BQ || CR. Prove tha ar(AQC) =ar(PBR).

Ans. We have AP || BQ || CR



∵ ΔBCQ and ΔBQR are on the same base BQ and between the same parallels BQ and CR.

∴ ar(ΔBCQ) = ar(ΔBQR) ...(1)

∵ ΔABQ and ΔPBQ are on the same base BQ and between the same parallels AP and BQ.

∴ ar(ΔABQ) = ar(ΔPBQ) ...(2)

Adding (1) and (2), we have

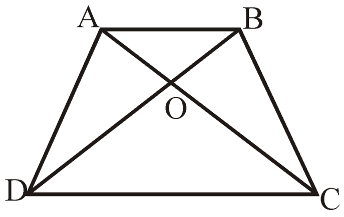
ar(ΔBCQ) + ar(ΔABQ) = ar(ΔBQR) + ar (ΔPBQ)

⇒ ar(ΔAQC) = ar(ΔPBR)

NS. 15

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar(AOD) = ar(BOC). Prove that ABCD is a trapezium.

Ans. We have a quadrilateral ABCD and its diagonals AC and BD intersect at O such that ar(ΔAOD) = ar(ΔBOC) [Given]



Adding ar(ΔAOB) to both sides, we have

ar(ΔAOD) + ar(ΔAOB) = ar(ΔBOC) + ar(ΔAOB)

⇒ ar(ΔABD) = ar(ΔABC)

Also they are on the same base AB.

Since, the triangles on the same base and having equal area.

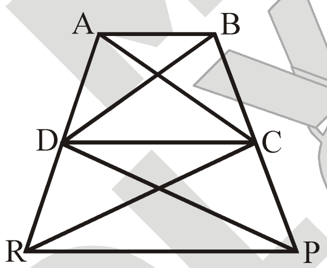
∴ They lie between the same parallels.

∴ AB || DC

Now, ABCD is a quadrilateral having a pair of opposite sides parallel. So, ABCD is a trapezium.

NS. 16

In the figure, ar(DRC) = ar(DPC) and ar(BDP) = ar(ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Ans. We have ar(ΔDRC) = ar(ΔDPC) [Given]

And they are on the same base DC.

∴ ΔDRC and ΔDPC must lie between the same paralles.

So, DC || RP i.e., a pair of opposite sides of quadrilateral DCPR is parallel.

∴ Quadrilateral DCPR is a trapezium.

Again, we have

ar(ΔBDP) = ar(ΔARC) [Given] ...(1)

Also ar(ΔDPC) = ar(ΔDRC) [Given] ...(2)

Subtracting (2) from (1), we get

ar(ΔBDP) – ar (ΔDPC) = ar(ΔARC) –ar(ΔDRC)

⇒ ar(ΔBDC) = ar(ΔADC)

And they are on the same base DC.

∴ ΔBDC and ΔADC must lie between the same parallels.

So, AB || DC i.e.a pair of opposite sides of quadrilateral ABCD is parallel.

∴ Quadrilateral ABCD is a trapezium.

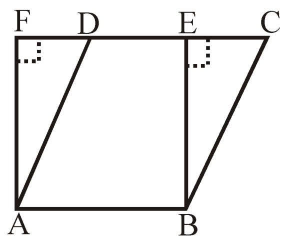
EXERCISE 9.4

NS. 1

Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Ans. We have a parallelogram ABCD and rectangle ABEF such that ar(||^{gm}ABCD) = ar(rect. ABEF)

AB = CD



[Opposite sides of parallelogram]

and AB = EF [Opposite sides of a rectangle]

⇒ CD = EF

⇒ AB + CD = AB + EF ...(1)

∵ BE < BC and AF < AD

[In a right triangle, hypotenuse is the longest side]

⇒ (BC + AD) > (BE + AF) ...(2)

Form (1) and (2), we have

$(AB + CD) + (BC + AD) > (AB + EF) + (BE + AF)$

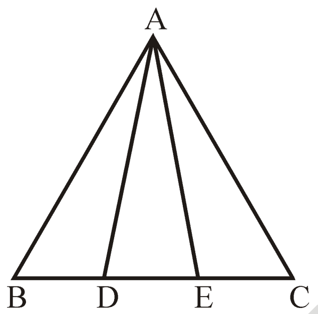
$\Rightarrow (AB + BC + CD + DA) > (AB + BE + EF + FA)$

$\Rightarrow$ Perimeter of parallelogram

ABCD > Perimeter of rectangle ABEF

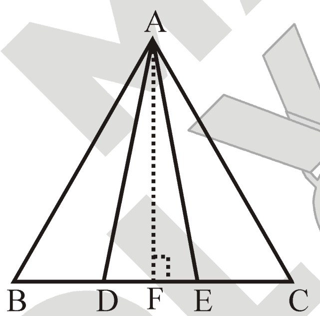
NS. 2

In the figure, D and E are two points on BC such that BD = ED = EC. Show that ar(ABD) = ar(ADE) = ar(AEC).



Ans. let us draw AF, perpendicular to BC such that AF is the height of ΔABD, ΔADE and ΔAEC.

Area of a triangle = $\frac{1}{2}$ × base × height



$\therefore$ ar(ΔABD) = $\frac{1}{2}$ × BD × AF

Similarly, ar(ΔADE) = $\frac{1}{2}$ ×DE ×AF,

ar(ΔAEC) = $\frac{1}{2}$ × EC × AF

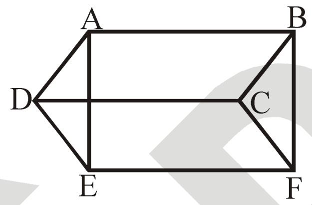
Since, BD = DE = EC

$$\therefore \left(\frac{1}{2}\times BD\times AF\right)=\left(\frac{1}{2}\times DE\times AF\right)=\left(\frac{1}{2}\times EC\times AF\right)$$

$\Rightarrow$ ar(ΔABD) = ar(ΔADE) = ar(ΔAEC).

NS. 3

In the figure, ABCD, DCFE and ABFE are parallelograms. Show that ar(ADE) = ar(BCF).



Ans. We have paralleograms ABCD, DCFE and ABFE.

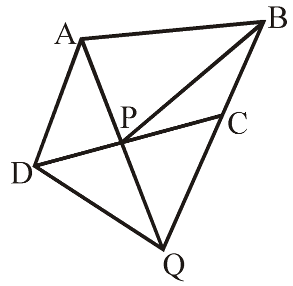
ABCD is a parallelogram [Given]

$\therefore$ Its opposite sides are parallel and equal. Now, ΔADE and ΔBCF are on equal bases AD = BC [from (1)] and between the same parallels AB and EF So, ar(ΔADE) = ar(ΔBCF).

NS. 4

In the figure, ABCD is a parallelogram and BC is produced to a pont Q such that AD = CQ. If AQ intersect DC at P, show that ar(BPC) = ar(DPQ).

[Hint: Join AC]



Ans. We have a parallelogram ABCD and AD = CQ. Let us join AC. We know tha triangles on the same base and between the same parallels are equal in area. Since ΔQAC and ΔQDC are on the same base QC and between the same parallels AD and BQ, ar(ΔQAC) = ar(ΔQDC)

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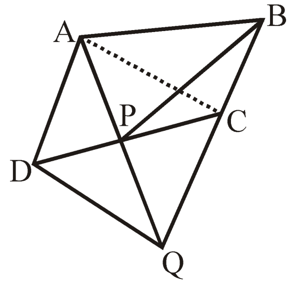
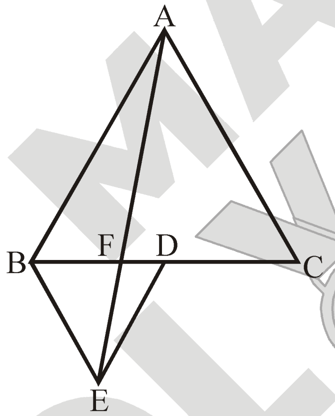


Subtracting ar(ΔQPC) from both sides, we have
ar(ΔQAC) – ar(ΔQPC)
= ar(ΔQDC) – ar(ΔQPC)
⇒ ar(ΔPAC) = ar(ΔQDP) ...(1)
Since, ΔPAC and ΔPBC are on the same base PC and between the same parallels AB and CD
∴ ar(ΔPAC) = ar(ΔPBC) ...(2)
Form (1) and (2), we get ar(ΔPBC) = ar(ΔQDP)

NS. 5

In figure, ABC and BDE are two equilateral triangles such that D is teh mid-point of BC. If AE intersects BC at F, show that



(i) ar(BDE) = $\frac{1}{4}$ ar(ABC)

(ii) ar(BDE) = $\frac{1}{2}$ ar(BAE)

(iii) ar(ABC) = 2 ar(BEC)

(iv) ar(BFE) = ar(AFD)

(v) ar(BFE) = 2 ar(FED)

(vi) ar(FED) = $\frac{1}{8}$ ar(AFC)

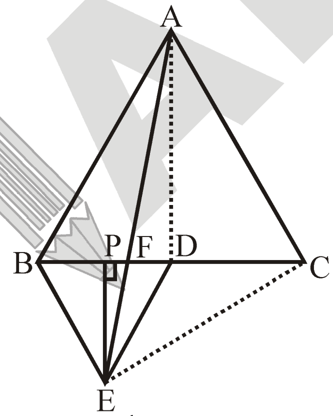
[Hint : Join EC and AD. Show that BE || AC and DE || AB, etc.]

Ans. Let us join EC and AD. Draw EP ⊥ BC.

Let AB = BC = CA = a, then BD = $\frac{a}{2}$ = DE = BE

(i) ar(ΔABC) = $\frac{\sqrt{3}}{4}$ a^2 and

ar(ΔBDE) = $\frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{\sqrt{3}}{16}a^2$



⇒ ar(ΔBDE) = $\frac{1}{4}$ ar(ΔABC)

(ii) Since ΔABC and ΔBED are equilateral triangles ⇒ ∠ACB = ∠DBE = 60° ⇒ BE || AC
ΔBAE and ΔBEC are on the same base BE and between the same paralles BE and AC
∴ ar(ΔBAE) = ar(ΔBEC)
⇒ ar(ΔBAE) = 2ar(ΔBDE)
[DE is median of ΔEBC,
∴ ar(ΔBEC) = 2 ar(ΔBDE)]

⇒ ar(ΔBDE) = $\frac{1}{2}$ ar(ΔBAE)

(iii) ar(ΔABC) = 4 ar(ΔBDE) [Prove in (i) part]
ar(ΔBEC) = 2 ar(ΔBDE) [DE is medina of ΔBEC]
⇒ ar(ΔABC) = 2 ar(ΔBEC)
(iv) Since ΔABC and ΔBDE are equilateral

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triangles

$\Rightarrow \angle ABC = \angle BDE = 60° \Rightarrow AB \parallel DE$

ΔBED and ΔAED are on the same base ED and between same parallels AB and DE]

$\therefore$ ar(ΔBED) = ar(ΔAED)

Subtracting ar(ΔEFD) from both sides, we get

$\Rightarrow$ ar(ΔBED) – ar(ΔEFD) = ar(ΔAED) – ar(ΔEFD)

$\Rightarrow$ ar(ΔBEF) = ar(ΔAFD)

(v) In right angled ΔABD, we get

$AD^2 = AB^2 - BD^2$

$$\Rightarrow AD^2 = a^2 - \frac{a^2}{4} = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4} \Rightarrow AD = \frac{\sqrt{3}a}{2}$$

In right angled ΔPED, $EP^2 = DE^2 - DP^2$

$$\Rightarrow EP^2 = \left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2 = \frac{a^2}{4} - \frac{a^2}{16} = \frac{3a^2}{16} \Rightarrow EP = \frac{\sqrt{3}a}{4}$$

$$\therefore \text{ar}(\Delta AFD) = \frac{1}{2} \times FD \times AD = \frac{1}{2} \times FD \times \frac{\sqrt{3}}{2} a \quad ...(1)$$

$$\text{and ar}(\Delta EFD) = \frac{1}{2} \times FD \times EP = \frac{1}{2} \times FD \times \frac{\sqrt{3}}{4} a \quad ...(2)$$

Form (1) and (2), we get ar(ΔAFD) = 2 ar(ΔEFD)

ar (ΔAFD) = ar(ΔBEF) [from (iv) part]

$\Rightarrow$ ar(ΔBFE) = 2 ar(ΔEFD)

(vi) ar(ΔAFC) = ar(ΔAFD) + ar(ΔADC)

$= \text{ar}(\Delta BFE) + \frac{1}{2}\text{ar}(\Delta ABC)$ [By (iv)]

$= \text{ar}(\Delta BFE) + \frac{1}{2} \times 4 \times \text{ar}(\Delta BDE)$ [By (i) part]

= ar(ΔBFE) + 2ar(ΔBDE) .....(5)

= 2ar(ΔFED) + 2[ar(ΔBFE) + ar(ΔFED)]

= 2ar(ΔFED) + 2[2ar(ΔFED) + ar(ΔFED)] [By (v) part]

= 2ar(ΔFED) + 2[3ar(ΔFED)]

= 2ar(ΔFED) + 6ar(ΔFED) = 8ar(ΔFED)

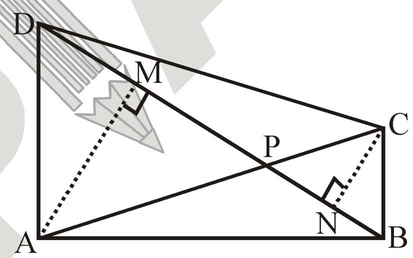
$\therefore \frac{1}{8}\text{ar}(\Delta AFC) = \text{ar}(\Delta FED)$

NS. 6

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that [ar(APB) × ar(CPD) = ar(APD) × ar(BPC).

[Hint : From A and C, draw perpendiculars to BD.]

Ans. We have a quadrilateral ABCD such that its diagonals AC and BD intersect at P. Let us draw AM ⊥ BD and CN ⊥ BD.



Now, $\text{ar}(\Delta APB) = \frac{1}{2} \times BP \times AM$ and

$\text{ar}(\Delta CDP) = \frac{1}{2} \times DP \times CN$

$\therefore$ ar(ΔAPB) × ar (ΔCPD)

$$= \left(\frac{1}{2} \times BP \times AM\right) \times \left(\frac{1}{2} \times DP \times CN\right)$$

$$= \frac{1}{4} \times BP \times DP \times AM \times CN \quad ...(1)$$

Similarly, ar(ΔAPD) × ar(ΔBPC)

$$= \left(\frac{1}{2} \times DP \times AM\right) \times \left(\frac{1}{2} \times BP \times CN\right)$$

$$= \frac{1}{4} \times BP \times DP \times AM \times CN \quad ...(2)$$

From (1) and (2), we get

ar(ΔAPB) × ar(ΔCPD) = ar(ΔAPD) × ar(ΔBPC)

NS. 7

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-Point of AP, show that

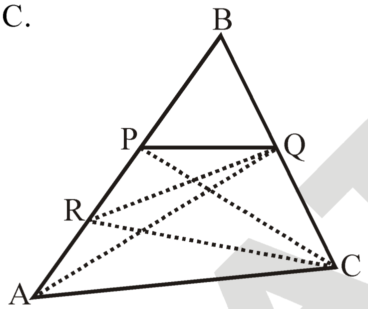
(i) ar (PRQ) = $\frac{1}{2}$ ar(ARC)

(ii) ar(RQC) = $\frac{3}{8}$ ar(ABC)

(iii) ar(PBQ) = ar(ARC)

Ans. We have a ΔABC such that P is the mid-point of AB and Q is the mid-point of BC.

Also, R is the mid-point of AP. Let us join AQ and PC.



(i) In ΔAPQ, R is the mid-point of AP [Given]

∴ RQ is a median of ΔAPQ

$\Rightarrow$ ar(ΔPRQ) = $\frac{1}{2}$ ar(ΔAPQ) ...(1)

In ΔABQ, P is the mid-point of AB

$\Rightarrow$ QP is a median of ABQ

∴ ar(ΔAPQ) = $\frac{1}{2}$ ar(ΔABQ) ...(2)

From (1) and (2), we get

ar(ΔPRQ) = $\frac{1}{2} \times \frac{1}{2}$ ar(ΔABQ) = $\frac{1}{4}$ ar(ΔABQ)

= $\frac{1}{4} \times \frac{1}{2}$ ar(ΔABC)

[∵ AQ is a median of Δ ABC]

= $\frac{1}{8}$ ar(ΔABQ) ...(3)

Now, ar(ΔARC) = $\frac{1}{2}$ ar(ΔAPC)

[∵ CR is a median of Δ APC]

= $\frac{1}{2} \times \frac{1}{2}$ ar(ΔABC) [∵ CP is a median of Δ ABC]

= $\frac{1}{4}$ ar(ΔABC) ...(4)

Now, from (3) and (4), we get,

ar(ΔPRQ) = $\frac{1}{8}$ ar(ΔABC) = $\frac{1}{2} \times \left(\frac{1}{4} ar\Delta ABC\right)$

= $\frac{1}{2}$ ar(ΔARC)

Thus, ar(ΔPRQ) = $\frac{1}{2}$ ar(ΔARC)

(ii) In ΔRBC, RQ is a median.

∴ ar(ΔRQC) = ar(ΔRBQ) = ar(ΔPRQ) + ar(ΔBPQ)

= $\frac{1}{8}$ ar(ΔABC) + ar(ΔBPQ) [From part (3)]

= $\frac{1}{8}$ ar(ΔABC) + $\frac{1}{2}$ ar(ΔPBC)

[∵ PQ is the median of ΔBPC]

= $\frac{1}{8}$ ar(ΔABC)+ $\frac{1}{2} \cdot \frac{1}{2}$ ar(ΔABC)

[∵ CP is the median of ΔABC]

= $\frac{1}{8}$ ar(ΔABC) + $\frac{1}{4}$ ar(ΔABC)

= $\left(\frac{1}{8}+\frac{1}{4}\right)$ ar(ΔABC) = $\frac{3}{8}$ ar(ΔABC)

(iii) QP is a median of ΔABQ.

∴ ar(ΔPBQ) = $\frac{1}{2}$ (ΔABQ) = $\frac{1}{2} \times \frac{1}{2}$ ar(ΔABC)

[∵ AQ is the median of ΔABC]

= $\frac{1}{4}$ ar(ΔABC) = ar (ΔARC) [From (4)]

Thus, ar(ΔPBQ) = ar(ΔARC)

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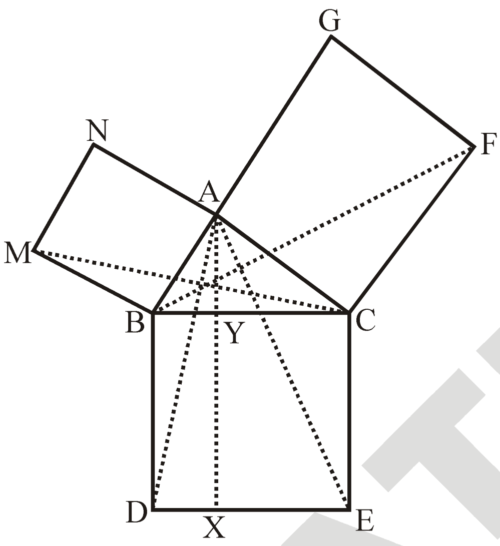
NS. 8

In the figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that:

(i) $\Delta MBC \cong \Delta ABD$



(ii) ar(BYXD) ≅ 2ar(ΔMBC)

(iii) ar(BYXD) = ar(ABMN)

(iv) $\Delta FCB \cong \Delta ACE$

(v) ar(CYXE) = 2ar(FCB)

(vi) ar(CYXE) = ar(ACFG)

(vii) ar(BCED) = ar(ABMN) + ar(ACFG)

Ans. We have a right ΔABC such that BCED, ACFG and ABMN are squares on its sides BC, CA and AB respectively. Line segment AX ⊥ DE is also drawn such that it meets BC at Y.

(i) In ΔABD and ΔMBC, we

$\left.\begin{array}{l} AB = MB \\ BD = BC \end{array}\right\}$ [Sides of a square]

$\angle CBD = \angle MBA$ [Each 90°]

$\therefore \angle CBD + \angle ABC = \angle MBA + \angle ABC$

[By adding ∠ABC on both sides]

or $\angle MBC = \angle ABD$

$\therefore \Delta MBC \cong \Delta ABD$ [By SAS congruency]

(ii) Since parallelogram BYXD and ΔABD are on the same base BD and between the same parallels BD and AX.

$\therefore$ ar(ABD)= $\frac{1}{2}$ ar(BYXD)

But ar(ΔABD) = ar(ΔMBC)

[Congruent triangles have equal areas]

$\Rightarrow$ ar(ΔMBC) = $\frac{1}{2}$ ar($\|^{gm}$BYXD)

$\Rightarrow$ ar($\|^{gm}$BYXD) = 2ar(ΔMBC)

(iii) Since ar(BYXD) = 2ar(ΔMBC) ...(1)

[From (ii) part]

and ar(square ABMN) = 2ar(ΔMBC)..(2)

[ABMN and ΔMBC are on the same base MB and between the same parallels MB and NC]

From (1) and (2), we have

ar(BYXD) = ar(ABMN)

(iv) In ΔFCB and ΔACE,

FC = AC [Sides of a square]

CB = CE [Sides of a square]

$\angle FCA = \angle BCE$ [Each 90°]

or $\angle FCA + \angle ACB = \angle BCE + \angle ACB$

[By adding ∠ACB on both sides.]

$\Rightarrow \angle FCB = \angle ACE$

$\Rightarrow \Delta FCB \cong \Delta ACE$ [By SAS congruency]

(v) Since $\|^{gm}$ CYXE and ΔACE are on the same base CE and between the same parallels CE and AX.

$\therefore$ ar($\|^{gm}$CYXE) = 2ar(ΔACE)

But $\Delta ACE \cong \Delta FCB$

Since, congruent triangles are equal in areas.

$\therefore$ ar($\|^{gm}$CYXE) = 2ar(ΔFCB)

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(vi) Since, ar($||^{gm}$CYXE) = 2ar(ΔFCB) ...(3)

[From (v) part]

Also (quad. ACFG) and ΔFCB are on the same base FC and between the same parallels FC and BG.

⇒ ar(quad. ACFG) = 2 ar(ΔFCB) ...(4)

From (3) and (4), we get

ar(qaud. CYXE) = ar(quad. ACFG) ...(5)

(vii) We have ar(quad. BCED)

= ar(quad. CYXE) + ar(quad.BYXD)

= ar(quad. CYXE) + ar(quad. ABMN)

[From (iii) part]

Now, ar(quad. BCED)

= ar(quad. ABMN) + ar(quad. CYXE)

Thus, ar(quad. BCED)

= ar(quad. ABMN) + ar(quad. ACFG)

[Form (5)]

Space for Notes :

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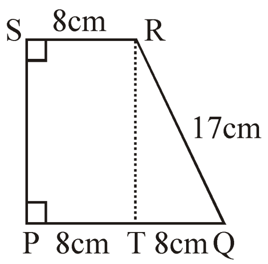
ONLY ONE CORRECT TYPE

1. The area of a rhombus is 20 cm^2. If one of its diagonals is 5 cm, the other diagonal is :

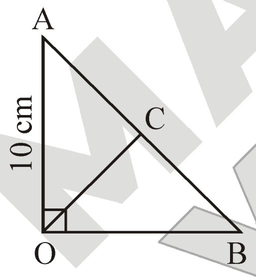
(A) 5 cm (B) 6 cm
(C) 8 cm (D) 10 cm

2. The area of trapezium PQRS in the adjoining figure is



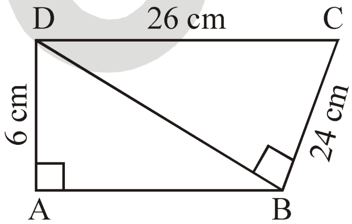
(A) 112 cm^2 (B) 120 cm^2
(C) 160 cm^2 (D) 180 cm^2

3. In the adjoining figure, $\angle AOB = 90°$, AC = BC, OA = 10 cm and OC = 13 cm. The area of ΔAOB is



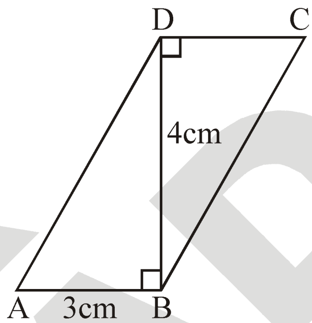
(A) 120 cm^2 (B) 135 cm^2
(C) 140 cm^2 (D) 148 cm^2

4. In the adjoining figure, the area of quadrilateral ABCD is



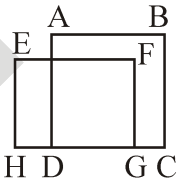
(A) 148 cm^2 (B) 144 cm^2
(C) 120 cm^2 (D) 122 cm^2

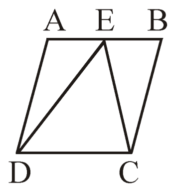
5. In the adjoining figure, ABCD is a parallelogram. Then its area is equal to

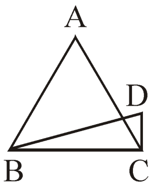


(A) 9 cm^2 (B) 12 cm^2
(C) 15 cm^2 (D) 36 cm^2

6. Which of the following figures lie on the same base and between the same parallels ?

(A) 

(B) 

(C) 

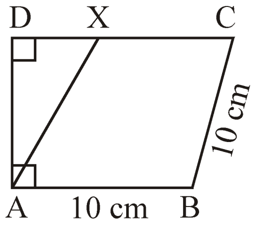
(D) All of these

7. In a parallelogram ABCD, AB = 12 cm and the altitude corresponding to AB is 8 cm. If AD = 10 cm, then the altitude corresponding to AD is equal to

(A) 8.5 cm (B) 9 cm
(C) 9.6 cm (D) 10.8 cm

MATRIX OLYMPIAD

8. In the given figure, ∠BAD = ∠ADC = 90° and AX || BC. If AB = BC = 10 cm and DC = 16 cm, then the area of ABCX is

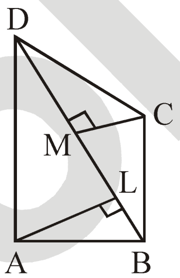


(A) 80 cm^2 (B) 40cm^2
(C) 20 cm^2 (D) 42 cm^2

9. The area of a rhombus whose lengths of diagonals are 16 cm and 24 cm, is
(A) 180 cm^2 (B) 184 cm^2
(C) 198 cm^2 (D) 192 cm^2

10. The area of a trapezium whose parallel sides are 9 cm and 16 cm and the distance between these sides is 8 cm, is
(A) 60 cm^2 (B) 72 cm^2
(C) 56 cm^2 (D) 100 cm^2

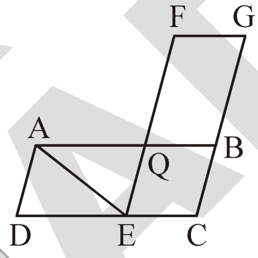
11. In the adjoining figure, ABCD is a quadrilateral in which diagonal BD = 14 cm. If AL ⊥ BD and CM ⊥ BD such that AL = 8 cm and CM = 6 cm, then area of quadrilateral ABCD is



(A) 60 cm^2 (B) 72 cm^2
(C) 84 cm^2 (D) 98 cm^2

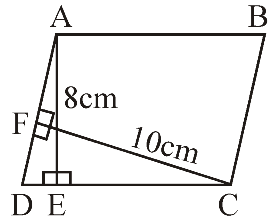
12. If P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD, then ar(ΔBQC) is equal to
(A) ar(ΔAPB) (B) ar(ΔPBC)
(C) ar(ΔAPD) (D) None of these

13. In figure ABCD and FECG are parallelograms equal in area. If ar(ΔAQE) = 12 cm^2, then ar(||^{gm} FGBQ) is equal to



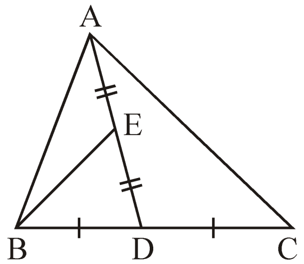
(A) 12 cm^2 (B) 20 cm^2
(C) 24 cm^2 (D) 36 cm^2

14. In figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AD = 12 cm, AE = 8 cm and CF = 10 cm, find CD.

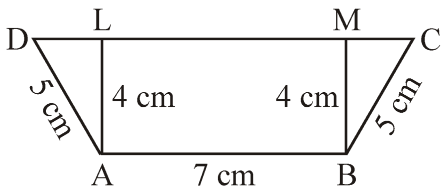


(A) 17 cm (B) 12 cm
(C) 10 cm (D) 15 cm

15. ABC is a triangle in which D is the mid-point of BC and E is the mid-point of AD, such that the area of ΔBED = K area of ΔABC. Find K.
(A) 2 (B) $\frac{1}{4}$
(C) 4 (D) $\frac{1}{2}$

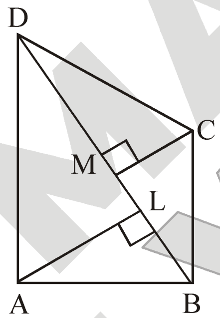


16. In figure, ABCD is a trapezium in which AB || DC. Find the length of DC.



(A) 17 cm (B) 11 cm
(C) 13 cm (D) 15 cm

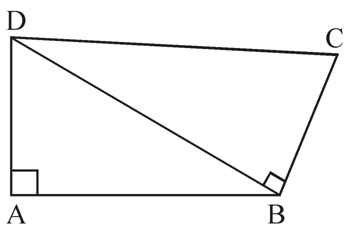
17. In the figure, ABCD is a quadrilateral BD = 20 cm. If AL ⊥ BD and CM ⊥ BD such that AL = 10 cm and CM = 5 cm, find the area of quad. ABCD.



(A) 150 cm^2 (B) 180 cm^2
(C) 100 cm^2 (D) 140 cm^2

18. In the given figure, AB ⊥ AD, BC ⊥ BD and AD = 9 cm, BC = 8 cm and CD = 17 cm. Find AB.

(A) 14 cm (B) 12 cm
(C) 9 cm (D) 17 cm



19. ABCD is a parallelogram. E is a point on BA such that BE = 2EA and F is a point on DC such that DF = 2FC. If ar(AECF) = k[ar(ABCD)], then k equals

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{4}{3}$ (D) $\frac{3}{4}$

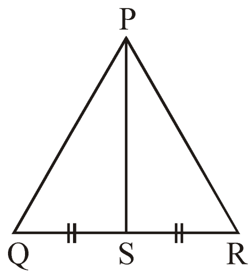
20. PQRS is an isosceles trapezium in which PS = 10 cm, PQ = SR = 13 cm and the distance between PS and QR is 12 cm. Find the area of the trapezium.

(A) 180 cm^2 (B) 160 cm^2
(C) 176 cm^2 (D) 194 cm^2

21. Parallelograms on the same base and between the same parallels are equal in

(A) Perimeter (B) Volume
(C) Area (D) Weight

22. If PS is median of the triangle PQR, then ar(ΔPQS) : ar(ΔQRP) is



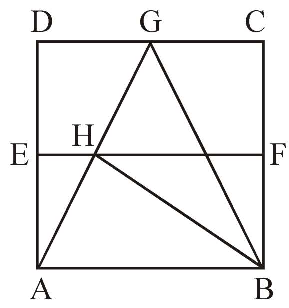
(A) 1 : 1 (B) 2 : 1
(C) 1 : 2 (D) Can't be determined

23. ABCD is a trapezium with parallel sides AB = a cm and DC = b cm, E and F are the midpoints of the non-parallel sides, find the ratio of ar(ABFE) and ar(EFCD).

(A) (3b + a) : (3a + b)
(B) (3a + b) : (3b + a)
(C) (2a + 3b) : (3a + b)
(D) (3a + 2b) : (2a + 3b)

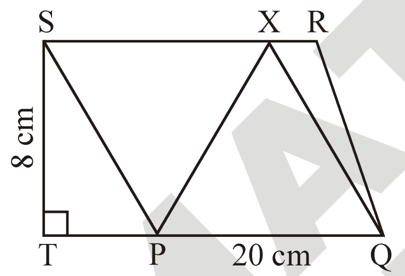
MATRIX OLYMPIAD

24. In the figure, ABCD is a square. E and F are midpoints of AD and BC respectively. The ratio of areas of ΔGAB and ΔHAB is :



(A) 4 : 1 (B) 1 : 4
(C) 1 : 2 (D) 2 : 1

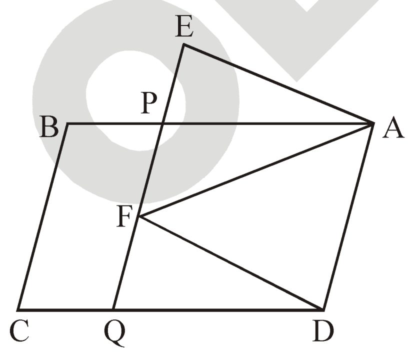
25. In the given figure, PQRS is parallelogram, then find the area of ΔPQX.



(A) 80 cm^2 (B) 40 cm^2
(C) 120 cm^2 (D) 60 cm^2

PARAGRAPH TYPE

PASSAGE – I : In the given figure, ABCD and AEFD are two parallelograms.



26. PE =
(A) BP (B) FQ
(C) AP (D) CQ

27. $\dfrac{ar(\Delta APE)}{ar(\Delta PFA)} =$

(A) $\dfrac{ar(\Delta QFD)}{ar(\Delta PFD)}$ (B) $\dfrac{ar(\Delta AEF)}{ar(\Delta PFD)}$

(C) $\dfrac{ar(\Delta QFD)}{ar(\Delta AEF)}$ (D) None of these

28. $ar(\Delta PEA) =$
(A) ar ($||^{gm}$ ABCD) (B) $ar(\Delta PFD)$
(C) $ar(\Delta QFD)$ (D) ar($||^{gm}$ CQPB)

PASSAGE–II : In a ΔABC, P and Q are respectively the midpoints of AB and BC and R is the midpoint of AP. Then

29. $ar(\Delta PBQ) =$
(A) $ar(\Delta PRQ)$ (B) $ar(\Delta RQC)$
(C) $ar(\Delta ABP)$ (D) $ar(\Delta ARC)$

30. $ar(\Delta PRQ) =$

(A) $ar(\Delta PBQ)$ (B) $\left(\dfrac{1}{2}\right) ar(\Delta ARC)$

(C) $\left(\dfrac{1}{2}\right) ar(\Delta RQC)$ (D) $\left(\dfrac{1}{4}\right) ar(\Delta PBQ)$

31. If $ar(\Delta RQC) = k\, ar(\Delta ABC)$. Then k equals

(A) $\dfrac{3}{8}$ (B) $\dfrac{3}{2}$

(C) $\dfrac{1}{2}$ (D) $\dfrac{3}{4}$

MATCH THE COLUMN TYPE

In this section each question has two matching lists. Choices for the correct combination of elements from Column – I and Column – II are given as options (A), (B), (C) and (D) out of which one is correct.

MATRIX OLYMPIAD

32. In ΔABC, if L and M are points on AB and AC respectively such that LM || BC. Then match equal areas in both lists.

Column – I	**Column – II**
(P) ar(ΔLCM)	(i) ar(ΔACL)
(Q) ar(ΔLBC)	(ii) ar(ΔLBM)
(R) ar(ΔABM)	(iii) ar(ΔMOC)
(S) ar(ΔLOB)	(iv) ar(ΔMBC)

(A) (P) → (iv), (Q) → (ii), (R) → (iii), (S) → (i)
(B) (P) → (ii), (Q) → (iv), (R) → (i), (S) → (iii)
(C) (P) → (iii), (Q) → (ii), (R) → (iv), (S) → (i)
(D) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)

33. ABCD is a parallelogram. G is the point on AB such that AG = 2GB, E is a point on DC such that CE = 2DE and F is the point on BC such that BF = 2FC. Then match equal areas in both lists.

Column – I	**Column – II**
(P) ar(ADEG)	(i) $\frac{1}{6}$ ar(ABCD)
(Q) ar(ΔEGB)	(ii) ar(ΔCBG)
(R) ar(ΔEFC)	(iii) ar(GBCE)
(S) ar(ΔADE)	(iv) $\frac{1}{2}$ ar(ΔEBF)

(A) (P) → (iv), (Q) → (ii), (R) → (i), (S) → (iii)
(B) (P) → (i), (Q) → (ii), (R) → (iii), (S) → (iv)
(C) (P) → (ii), (Q) → (iii), (R) → (i), (S) → (iv)
(D) (P) → (iii), (Q) → (i), (R) → (iv), (S) → (ii)

Space for Notes :

MATRIX OLYMPIAD

EXERCISE – II

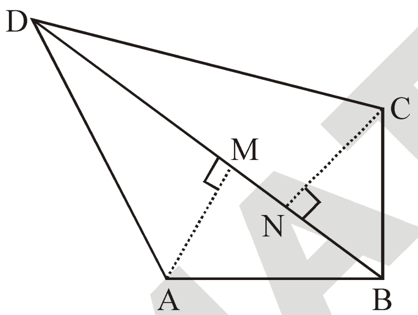
VERY SHORT ANSWER TYPE

1. If P is any point in the interior of a parallelogram ABCD, then prove that area of the triangle APB is less than half the area of parallelogram.

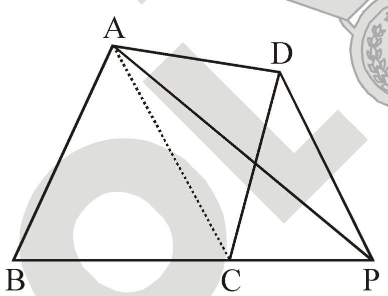
2. ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that
(i) ar(ΔADO) = ar(ΔCDO)
(ii) ar(ΔABP) = ar(ΔCBP)

3. BD is one of the diagonals of a quadrilateral ABCD. AM and CN are the perpendiculars from A and C, respectively on BD. Show that

$$ar(quad.ABCD) = \frac{1}{2} BD \cdot (AM + CN)$$



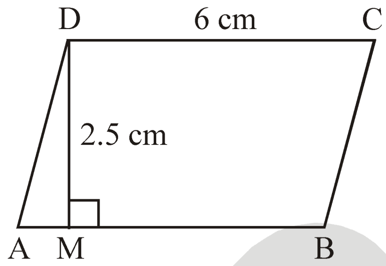
4. ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced at P as shown in figure. Prove that ar(ΔABP) = ar(quad. ABCD).



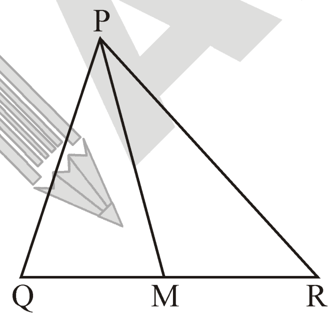
5. If ar(ΔABC) = 16 cm^2, then find the area of the triangle formed by joining the mid points of the sides of ΔABC.

6. Find the area of a rhombus with length of diagonals as 8 cm and 14 cm.

7. In the adjoining figure, find the area of the parallelogram ABCD.



8. In the given figure PM is the median of ΔPQR. The area of ΔPMQ is k times the area of ΔPQR, find k.

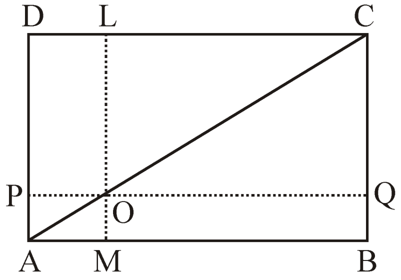


9. What is the ratio of areas of two parallelograms on equal bases and between the same parallels ?

10. What can you say about the area of two congruent figures ?

SHORT ANSWER TYPE

1. In the given figure ABCD is a $||^{gm}$. O is any point on AC. PQ || AB and LM || AD. Prove that ar($||^{gm}$ DLOP) = ar($||^{gm}$ BMOQ).



MATRIX OLYMPIAD

2. In ΔABC, D is the midpoint of AB. P is any point of BC. CQ || PD meets AB at Q. Show that $ar(\Delta BPQ) = \frac{1}{2} ar(\Delta ABC)$

3. ABCD is a trapezium in which AB || DC. DC is produced to E such that CE = AB, prove that $ar(\Delta ABD) = ar(\Delta BCE)$.

4. Prove that the area of an equilateral triangle is equal to $\frac{\sqrt{3}}{4}a^2$, where a is the side of the triangle.

5. In a parallelogram ABCD, E, F are any two points on the sides AB and BC respectively. Show that $ar(\Delta ADF) = ar(\Delta DCE)$.

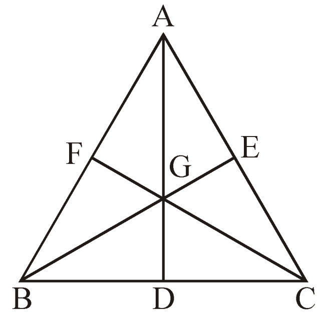
LONG ANSWER TYPE

1. ABCD is a parallelogram. X and Y are the midpoints of BC and CD respectively. Prove that $ar(\Delta AXY) = \frac{3}{8} ar(\|^{gm} ABCD)$

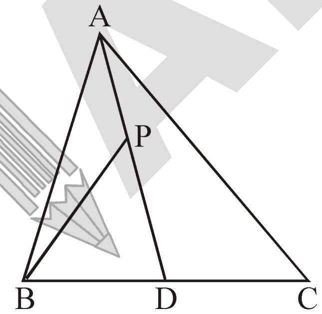
2. The medians BE and CF of a triangle ABC intersect at G. Prove that area of ΔGBC = area of quadrilateral AFGE.

3. A point O inside a rectangle ABCD is joined to the vertices. Prove that the sum of the areas of a pair of opposite triangles so formed is equal to the sum of the other pair of triangles.

4. For 'Sarva Shiksha Abhiyan' a rally was organised by a school. Students were given triangular cardboard pieces to write slogans. They divided the triangular shape into three equal parts by drawing medians as shown. Prove that $ar(\Delta AGC) = ar(\Delta AGB) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$



5. In ΔABC, AD is the median and P is a point on AD such that AP : PD = 1 : 2, then find the area of ΔABP if area of ΔABC is 12 square units.



TRUE / FALSE TYPE

1. If P is any point on the median AD of a ΔABC, then $ar(ABP) \neq ar(ACP)$.

2. ABCD is a parallelogram and X is the mid-point of AB. If ar (AXCD) = 24 cm^2, then ar (ABC) = 24 cm^2.

3. PQRS is a parallelogram whose area is 180 cm^2 and A is any point on the diagonal QS. The area of ΔASR = 90 cm^2.

4. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then ar (BDE) = $\frac{1}{4}$ ar (ABC).

5. A quadrilateral formed by joining the mid-point of the sides of a quadrilateral in order is a parallelogram

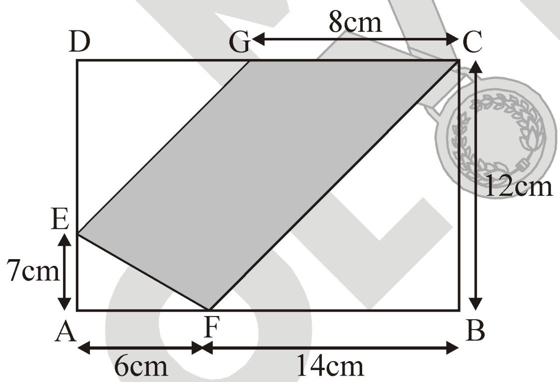
FILL IN THE BLANKS

1. ΔABC, E is the mid-point of median AD then ar (ΔBED) = _______ar (ΔABC)
2. The median of a tringle divides in into _______triangles of equal area.
3. The perimeter of an isosceles right triangle is 2x , then its area is _______.
4. If the points D divides the side BC of ΔABC in the ratio p:q then ar (ΔABD) : ar (ΔADC) = _______.
5. If the area , base & corresponding altitude of a parallelogram are $2t^2+2t$, 2t-3 & t+4 respectively, then the value of t is _ _ _ _

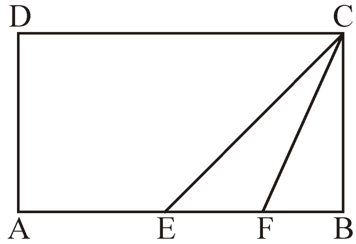
ANALYTICAL PROBLEMS & BRAIN TEASER

1. In the given figure ABCD is a rectangle and all measurements are in centimeters. Find the area of the shaded region.

   (A) 240 cm^2 (B) 205 cm^2
   (C) 105 cm^2 (D) 95 cm^2

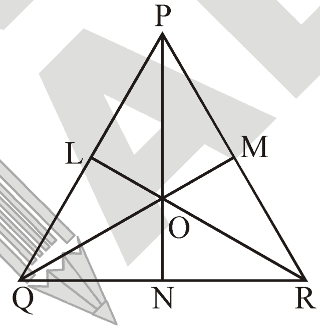


2. In the figure ABCD is a rectangle with AE = EF = FB, the ratio of the areas of triangle CEF and that of rectangle ABCD is



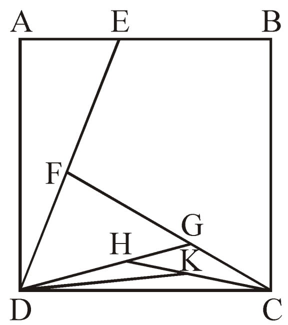
   (A) 1 : 6 (B) 1 : 8
   (C) 1 : 9 (D) 1 : 10

3. If the medians of ΔPQR intersect at O, then ar(ΔPOQ) =



   (A) ar(ΔQOR) (B) $\frac{1}{3}$ ar(ΔPQR)
   (C) Both (A) and (B) (D) Neither (A) nor (B)

4. In the figure, the area of square ABCD is 4 cm^2 and E is mid point of AB. F, G, H and K are the mid points of DE, CF, DG and CH respectively. The area of ΔKDC is



   (A) $\frac{1}{4}$ cm^2 (B) $\frac{1}{8}$ cm^2
   (C) $\frac{1}{16}$ cm^2 (D) $\frac{1}{32}$ cm^2

MATRIX OLYMPIAD

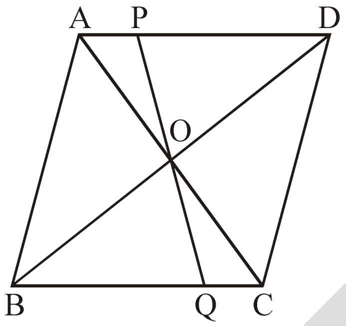
5. The diagonals of a parallelogram ABCD intersect at a point O. Through O, if a line is drawn to intersect AD at P and BC at Q, then PQ divides the parallelogram into ________.

(A) Two parts of equal area

(B) Two parts of area in 2 : 1

(C) Two parts of area in 1 : 3

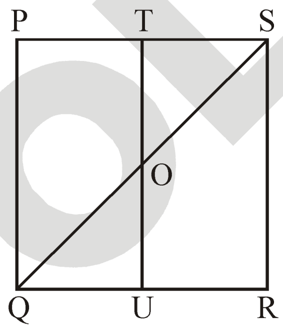
(D) Two parts of area in 4 : 3



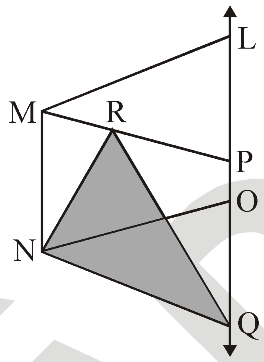
NUMERICAL PROBLEMS

1. In parallelogram ABCD, AB = 10 cm. The altitudes corresponding to the sides AB and AD are respectively 7 cm and 8 cm. If AD is k cm. Then value of 4 k is

2. In the given figure, PQRS is a square and T and U are respectively, the mid-points of PS and QR. Then what is the area of ΔOTS if PQ = 8 cm ?

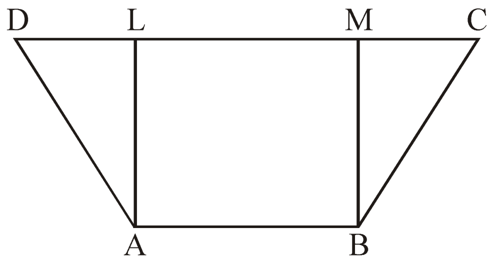


3. In the given figure LMNO and PMNQ are two parallelograms. R is any point on side MP. If ar(ΔNRQ) = k[ar($||^{gm}$ LMNO)], then 2k equals



4. D is the mid-point of side BC of ΔABC and E is the mid-point of BD. If O is the mid-point of AE, then ar(ΔBOE) = $\frac{1}{k}$ ar(ΔABC). Then k equals

5. In the given figure, ABCD is a trapezium in which AB = 7 cm, AD = BC = 5 cm, DC = x cm, and distance between AB and DC is 4 cm. The value of area of trapezium ABCD is



Answer Key

EXERCISE–I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
C	D	A	B	B	B	C	A	D	D	D	A	C	D	B
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
C	A	B	A	A	C	C	B	D	A	B	A	C	D	B
31	32	33												
A	B	D												

EXERCISE II

VERY SHORT ANSWER TYPE

5. 4 cm^2 **6.** 56 cm^2 **7.** 15 cm^2 **8.** $\frac{1}{2}$ **9.** 1 : 1

10. Two congruent figures are equal in area.

LONG ANSWER TYPE

5. 2 sq. units

TRUE / FALSE

1. F **2.** F **3.** F **4.** T 5. T

FILL IN THE BLANKS

1. $\frac{1}{4}$ **2.** 2 **3.** $\left(3-2\sqrt{2}\right)x^2$ **4.** p:q **5.** 4

ANALYTICAL PROBLEMS & BRAIN TEASER

1. C **2.** A **3.** C **4.** B **5.** A

NUMERICAL PROBLEMS

1. 35 **2.** 8 **3.** 1 **4.** 8 **5.** 40

MATRIX OLYMPIAD

SELF PROGRESS ASSESSMENT FRAMEWORK

(CHAPTER : AREAS OF PARALLELOGRAM AND TRIANGLES)

CONTENT	STATUS	DATE OF COMPLETION	SELF SIGNATURE
Theory			
In-Text Examples			
Solved Examples			
NCERT Exercises			
Exercise I			
Exercise II			
Short Note-1			
Revision - 1			
Revision - 2			
Revision - 3			
Remark			

MATRIX OLYMPIAD

NOTES :

1. In the status, put "completed" only when you have thoroughly worked through this particular section.

2. Always remember to put down the date of completion correctly. It will help you in future at the time of revision.

Space for Notes :